Example 1:

A car traveling at 44 m/s is uniformly accelerated to a velocity of 22 m/s over an 11 second interval.

a.) What is the car's acceleration?

b.) What is its displacement during this time?

Example 1:

$$v_i = 44\ \frac{\text{m}}{\text{s}},\ v = 22\ \frac{\text{m}}{\text{s}},\ \text{and } t = 11.0\ \text{s}$$

a.) $a = ?$

$$v = at + v_i$$

$$a = \frac{v - v_i}{t}$$

$$a = \frac{22\ \frac{\text{m}}{\text{s}} - 44\ \frac{\text{m}}{\text{s}}}{11\ \text{s}}$$

$$\boxed{a = -2\ \frac{\text{m}}{\text{s}^2}}$$

b.) $\Delta x = ?$

$$\Delta x = \left(\frac{v_i + v}{2}\right)t$$

$$\Delta x = \left(\frac{44\ \frac{\text{m}}{\text{s}} + 22\ \frac{\text{m}}{\text{s}}}{2}\right)(11\ \text{s})$$

$$\boxed{\Delta x = 363\ \text{m}}$$

Example 1:

$$v_i = 44\ \frac{\text{m}}{\text{s}},\ v = 22\ \frac{\text{m}}{\text{s}},\ \text{and } t = 11.0\ \text{s}$$

b.) $\Delta x = ?$

$$\Delta x = \frac{1}{2}at^2 + v_i t$$

$$\Delta x = \frac{1}{2}\left(-2\ \frac{\text{m}}{\text{s}^2}\right)(11\ \text{s})^2 + \left(44\ \frac{\text{m}}{\text{s}}\right)(11\ \text{s})$$

$$\boxed{\Delta x = 363\ \text{m}}$$

b.) $\Delta x = ?$

$$v^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{v^2 - v_i^2}{2a}$$

$$\Delta x = \frac{\left(22\ \frac{\text{m}}{\text{s}}\right)^2 - \left(44\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(-2\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$\boxed{\Delta x = 363\ \text{m}}$$

Example 2:

An airplane must reach a velocity of 70 m/s for takeoff. If the runway is 1.0 km long, what must the constant acceleration be?

Example 2:

$$v_i = 0,\ v = 70\ \frac{\text{m}}{\text{s}},\ \text{and } \Delta x = 1000\ \text{m}$$

$$a = ?$$

$$v^2 = v_i^2 + 2a\Delta x$$

$$a = \frac{v^2 - v_i^2}{2\Delta x}$$

$$a = \frac{\left(70\ \frac{\text{m}}{\text{s}}\right)^2 - (0)^2}{2(1000\ \text{m})}$$

$$\boxed{a = 2.45\ \frac{\text{m}}{\text{s}^2}}$$

Example 3:

A bike rider accelerates constantly from a velocity of 2.5 m/s for 4.0 s. The displacement is 20 m.

a.) What is the final velocity of the bike?

b.) What is the acceleration of the bike?

Example 3 :

$$v_i = 2.5\ \frac{\text{m}}{\text{s}},\ t = 4\ \text{s, and}\ \Delta x = 20\ \text{m}$$

a.) $v = ?$

$$\Delta x = \left(\frac{v_i + v}{2}\right)t$$

$$v = \frac{2\Delta x}{t} - v_i$$

$$v = \frac{2(20\ \text{m})}{4\ \text{s}} - 2.5\ \frac{\text{m}}{\text{s}}$$

$$\boxed{v = 7.5\ \frac{\text{m}}{\text{s}}}$$

b.) $a = ?$

$$v = at + v_i$$

$$a = \frac{v - v_i}{t}$$

$$a = \frac{7.5\ \frac{\text{m}}{\text{s}} - 2.5\ \frac{\text{m}}{\text{s}}}{4\ \text{s}}$$

$$\boxed{a = 1.25\ \frac{\text{m}}{\text{s}^2}}$$

Example 4:

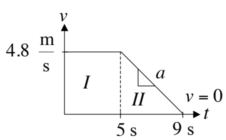
Rat is riding her bike at a constant velocity of 4.8 m/s. After riding for 5.0 seconds she begins to slow down at a rate of -1.2 m/s^2 until she comes to a complete stop.

a.) How many seconds does it take Rat to stop (from the time she starts braking)?

b.) How many meters did Rat travel?

Example 4 :

$$v_i = 4.8\ \frac{\text{m}}{\text{s}},\ t_1 = 5\ \text{s},\ a = -1.2\ \frac{\text{m}}{\text{s}^2},\ \text{and}\ v = 0$$



a.) $\Delta t = ?$

$$v = a\Delta t + v_i$$

$$\Delta t = \frac{v - v_i}{a}$$

$$\Delta t = \frac{0 - 4.8\ \frac{\text{m}}{\text{s}}}{-1.2\ \frac{\text{m}}{\text{s}^2}}$$

$$\boxed{\Delta t = 4\ \text{s}}$$

b.) $\Delta x = ?$

$$\Delta x = \text{Area}$$

$$\Delta x = A_I + A_{II}$$

$$\Delta x = (5\ \text{s})\left(4.8\ \frac{\text{m}}{\text{s}}\right) + \frac{1}{2}(4\ \text{s})\left(4.8\ \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{\Delta x = 33.6\ \text{m}}$$

Example 5:

A car travels at 20 m/s for 10 s. The car then uniformly accelerates to 30 m/s in a 5.0 s time interval. After reaching 30 m/s it continues at that speed for 20 s and then slows down at a constant rate of -5.0 m/s^2 until it stops.

a.) How far does the car move during this motion?

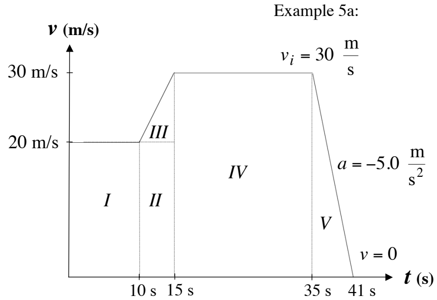
b.) What is the car's average acceleration

i.) for the first 15 s?

ii.) for the first 25 s?

iii.) the entire trip?

Example 5a:



$$v_i = 30\ \frac{\text{m}}{\text{s}}$$

$$a = -5.0\ \frac{\text{m}}{\text{s}^2}$$

$$v = a\Delta t + v_i$$

$$\Delta t = \frac{v - v_i}{a}$$

$$\Delta t = \frac{0 - 30\ \frac{\text{m}}{\text{s}}}{-5.0\ \frac{\text{m}}{\text{s}^2}} = 6\ \text{s}$$

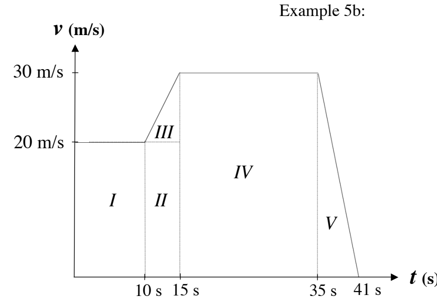
$$\Delta x = \text{Area}(v - t) = A_I + A_{II} + A_{III} + A_{IV} + A_V$$

$$\Delta x = (10\ \text{s})\left(20\ \frac{\text{m}}{\text{s}}\right) + (5\ \text{s})\left(20\ \frac{\text{m}}{\text{s}}\right) + \frac{1}{2}(5\ \text{s})\left(10\ \frac{\text{m}}{\text{s}}\right) + (20\ \text{s})\left(30\ \frac{\text{m}}{\text{s}}\right) + \frac{1}{2}(6\ \text{s})\left(30\ \frac{\text{m}}{\text{s}}\right)$$

$$\Delta x = 200\ \text{m} + 100\ \text{m} + 25\ \text{m} + 600\ \text{m} + 90\ \text{m}$$

$$\boxed{\Delta x = 1015\ \text{m}}$$

Example 5b:



$$a_{av} = ?$$

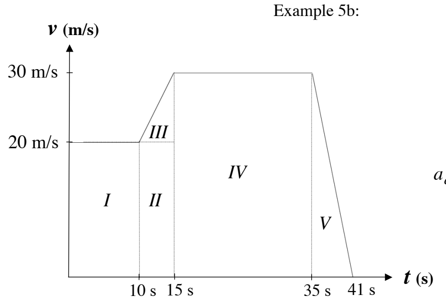
$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v - v_i}{t - t_i}$$

i.) for the first 15 s

$$a_{av} = \frac{30\ \frac{\text{m}}{\text{s}} - 20\ \frac{\text{m}}{\text{s}}}{15\ \text{s} - 0}$$

$$\boxed{a_{av} = 0.67\ \frac{\text{m}}{\text{s}^2}}$$

Example 5b:



$a_{av} = ?$

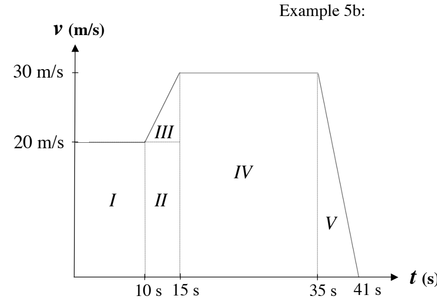
$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v - v_i}{t - t_i}$$

ii.) for the first 25 s

$$a_{av} = \frac{30\ \frac{m}{s} - 20\ \frac{m}{s}}{25\ s - 0}$$

$$\boxed{a_{av} = 0.40\ \frac{m}{s^2}}$$

Example 5b:



$a_{av} = ?$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v - v_i}{t - t_i}$$

iii.) the entire trip

$$a_{av} = \frac{0 - 20\ \frac{m}{s}}{41\ s - 0}$$

$$\boxed{a_{av} = -0.49\ \frac{m}{s^2}}$$

Example 6:

Two cars are 150 m apart and traveling towards each other. Car 1 is traveling east with an acceleration of 2.0 m/s^2 and an initial speed of 5.0 m/s. Car 2 is traveling west with an acceleration of -4.0 m/s^2 and an initial speed of 10.0 m/s. Find the time and position when the two cars pass one another.

Example 6

W ← $t = 0$ → … ← $t = 0$ → E

$x_{1_i} = 0$ | $x_1 = x_2 = ?$ | $x_{2_i} = 150$ m

$t = ?$

$v_{1i} = 5.0\ \frac{m}{s}$ | $v_{2_i} = -10.0\ \frac{m}{s}$

$a_1 = 2.0\ \frac{m}{s^2}$ | $a_2 = -4.0\ \frac{m}{s^2}$

$x_1 = \frac{1}{2}a_1t^2 + v_{1_i}t + x_{1i}$ | $x_2 = \frac{1}{2}a_2t^2 + v_{2_i}t + x_{2i}$

$$x_1 = x_2$$

$$\frac{1}{2}a_1t^2 + v_{1_i}t + x_{1i} = \frac{1}{2}a_2t^2 + v_{2_i}t + x_{2i}$$

$$\frac{1}{2}a_1t^2 + v_{1_i}t + x_{1i} - \frac{1}{2}a_2t^2 - v_{2_i}t - x_{2i} = 0$$

$$\frac{1}{2}(a_1 - a_2)t^2 + (v_{1_i} - v_{2_i})t + (x_{1i} - x_{2i}) = 0$$

Example 6

$t = 0$ | $t = 0$

$x_{1_i} = 0$ | $x_1 = x_2 = ?$ | $x_{2_i} = 150$ m

$t = ?$

$v_{1i} = 5.0\ \frac{m}{s}$ | $v_{2_i} = -10.0\ \frac{m}{s}$

$a_1 = 2.0\ \frac{m}{s^2}$ | $a_2 = -4.0\ \frac{m}{s^2}$

$$\frac{1}{2}(a_1 - a_2)t^2 + (v_{1_i} - v_{2_i})t + (x_{1i} - x_{2i}) = 0$$

$$\frac{1}{2}\left(2.0\ \frac{m}{s^2} - \left(-4.0\ \frac{m}{s^2}\right)\right)t^2 + \left(5.0\ \frac{m}{s} - \left(-10.0\ \frac{m}{s}\right)\right)t + (0 - 150\ m) = 0$$

$$\left(3.0\ \frac{m}{s^2}\right)t^2 + \left(15.0\ \frac{m}{s}\right)t - 150\ m = 0$$

$$t^2 + (5.0\ s)t - 50.0\ s^2 = 0$$

$$(t - 5.0\ s)(t + 10.0\ s) = 0$$

so $\boxed{t = 5\ s}$ or ~~$t = -10$ s~~

Example 6

$t = 0$ | $t = 0$

$x_{1_i} = 0$ | $x_1 = x_2 = ?$ | $x_{2_i} = 150$ m

$t = 5.0$ s

$v_{1i} = 5.0\ \frac{m}{s}$ | $v_{2_i} = -10.0\ \frac{m}{s}$

$a_1 = 2.0\ \frac{m}{s^2}$ | $a_2 = -4.0\ \frac{m}{s^2}$

$$x_1 = \frac{1}{2}a_1t^2 + v_{1_i}t + x_{1i}$$

$$x_1 = \frac{1}{2}\left(2.0\ \frac{m}{s^2}\right)(5.0\ s)^2 + \left(5.0\ \frac{m}{s}\right)(5.0\ s) + 0$$

$$x_1 = 25.0\ m + 25.0\ m$$

$$\boxed{x_1 = 50.0\ m}$$

or

$$x_2 = \frac{1}{2}a_2t^2 + v_{2_i}t + x_{2i}$$

$$x_2 = \frac{1}{2}\left(-4.0\ \frac{m}{s^2}\right)(5.0\ s)^2 + \left(-10.0\ \frac{m}{s}\right)(5.0\ s) + 150\ m$$

$$\boxed{x_2 = 50.0\ m}$$

Example 7: (Free fall)

A ball is dropped off a 250 m building. The magnitude of the acceleration due to gravity is 9.80 m/s^2. Assuming there is no air resistance:

a.) How far does the ball fall after 2.0 s?

b.) How long will the ball take to reach the ground?

c.) What is its velocity just before the ball hits the ground?

Example 7 :

$$v_i = 0,\ y_i = 250\ \text{m, and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$$

a.) $t = 2.0$ s, $\Delta y = ?$

$$\Delta y = \frac{1}{2}at^2 + v_i t$$

$$\Delta y = \frac{1}{2}\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.0\ \text{s})^2 + 0$$

$$\boxed{\Delta y = -19.6\ \text{m}}$$

The ball falls 19.6 m.

b.) $y = 0$, $t = ?$

$$\Delta y = \frac{1}{2}at^2 + \cancel{v_i t} \qquad v_i = 0$$

$$t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(y - y_i)}{a}}$$

$$t = \sqrt{\frac{2(0 - 250\ \text{m})}{-9.8\ \frac{\text{m}}{\text{s}^2}}}$$

$$\boxed{t = 7.14\ \text{s}}$$

Example 7 :

$$v_i = 0,\ y_i = 250\ \text{m, and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$$

c.) $y = 0$ and $t = 7.14$ s , $v = ?$

$$v = at + v_i$$

$$v = \left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(7.14\ \text{s}) + 0$$

$$\boxed{v = -70\ \frac{\text{m}}{\text{s}}}$$

alternatively, since $v^2 = v_i^2 + 2a\Delta y$

$$v = \pm\sqrt{v_i^2 + 2a\Delta y} = -\sqrt{v_i^2 + 2a\Delta y} \quad \text{(since it is going downward)}$$

$$v = -\sqrt{0 + 2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(0 - 250\ \text{m})} = \boxed{-70\ \frac{\text{m}}{\text{s}}}$$

Example 8: (Free fall)

A ball is tossed upward from an initial height (with respect to the ground) of 45 m. The ball reaches its maximum height 2.5 s after it is thrown.

a.) What is the initial velocity of the ball?

b.) What is the maximum height of the ball?

c.) What is the position of the ball after 5.5 s?

d.) What is the velocity of the ball just before it hits the ground?

Example 8 :

$$y_i = 45\ \text{m, and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$$

a.) $v = 0$ at $t = 2.5$ s (max height), $v_i = ?$

$$v = at + v_i$$

$$v_i = v - at$$

$$v_i = 0 - \left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.5\ \text{s})$$

$$\boxed{v_i = 24.5\ \frac{\text{m}}{\text{s}}}$$

b.) $v_i = 24.5\ \frac{\text{m}}{\text{s}}$, $v = 0$ at $t = 2.5$ s (max height), $y = ?$

$$\Delta y = \left(\frac{v_i + v}{2}\right)t \qquad \Delta y = \left(\frac{24.5\ \frac{\text{m}}{\text{s}} + 0}{2}\right)(2.5\ \text{s}) = 30.6\ \text{m}$$

$$\Delta y = y - y_i \text{ so } y = y_i + \Delta y = 45\ \text{m} + 30.6\ \text{m} \qquad \boxed{y = 75.6\ \text{m}}$$

Example 8 :

$$y_i = 45\ \text{m, and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$$

b.) $v_i = 24.5\ \frac{\text{m}}{\text{s}}$, $v = 0$ at $t = 2.5$ s (max height), $y = ?$

alternatively, since $v^2 = v_i^2 + 2a\Delta y$

$$\Delta y = \frac{v^2 - v_i^2}{2a} = \frac{0 - \left(24.5\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)} = 30.6\ \text{m}$$

$$\Delta y = y - y_i \text{ so } y = y_i + \Delta y = 45\ \text{m} + 30.6\ \text{m}$$

$$\boxed{y = 75.6\ \text{m}}$$

Example 8 : $y_i = 45\text{ m, and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$

b.) $v_i = 24.5\ \frac{\text{m}}{\text{s}}, v = 0$ at $t = 2.5$ s (max height), $y = ?$

alternatively, since $\Delta y = \frac{1}{2}at^2 + v_i t$

$$\Delta y = \frac{1}{2}\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.5\text{ s})^2 + \left(24.5\ \frac{\text{m}}{\text{s}}\right)(2.5\text{ s}) = 30.6\text{ m}$$

$$y = y_i + \Delta y = 45\text{ m} + 30.6\text{ m}$$

$$\boxed{y = 75.6\text{ m}}$$

Example 8 : $y_i = 45\text{ m, and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$

c.) $v_i = 24.5\ \frac{\text{m}}{\text{s}}$, $t = 5.5$ s, $y = ?$

$$\Delta y = \frac{1}{2}at^2 + v_i t$$

$$y = \frac{1}{2}at^2 + v_i t + y_i$$

$$y = \frac{1}{2}\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(5.5\text{ s})^2 + \left(24.5\ \frac{\text{m}}{\text{s}}\right)(5.5\text{ s}) + 45\text{ m}$$

$$\boxed{y = 31.5\text{ m}}$$

Example 8 : $y_i = 45\text{ m, and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$

d.) $v_i = 24.5\ \frac{\text{m}}{\text{s}}$, $y = 0$, $v = ?$

$$v^2 = v_i^2 + 2a\Delta y$$

$$v = \pm\sqrt{v_i^2 + 2a\Delta y} = -\sqrt{v_i^2 + 2a\Delta y} \quad \text{(since it is going downward)}$$

$$v = -\sqrt{\left(24.5\ \frac{\text{m}}{\text{s}^2}\right)^2 + 2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(0 - 45\text{ m})}$$

$$\boxed{v = -38.5\ \frac{\text{m}}{\text{s}}}$$

Example 9: (Free fall)

A ball is tossed upward with a speed of 15 m/s from a building that is 100 m tall.

a.) How many seconds does it take for the ball to reach its maximum height?

b.) At what two times does the ball have a speed of 10 m/s?

c.) How much time does it take the ball to reach the ground?

d.) What is the velocity of the ball just before it hits the ground?

Example 9 : $v_i = 15\ \frac{\text{m}}{\text{s}}$, $y_i = 100$ m, and $a = -9.8\ \frac{\text{m}}{\text{s}^2}$

a.) $v = 0$ (max height), $t = ?$

$$v = at + v_i$$

$$t = \frac{v - v_i}{a}$$

$$t = \frac{0 - 15\ \frac{\text{m}}{\text{s}}}{-9.8\ \frac{\text{m}}{\text{s}^2}}$$

$$\boxed{t = 1.53\text{ s}}$$

Example 9 : $v_i = 15\ \frac{\text{m}}{\text{s}}$, $y_i = 100$ m, and $a = -9.8\ \frac{\text{m}}{\text{s}^2}$

b.) $v = \pm 10\ \frac{\text{m}}{\text{s}}$ (going up and coming down), $t = ?$

$$v = at + v_i$$

$$t = \frac{v - v_i}{a}$$

$$t_1 = \frac{10\ \frac{\text{m}}{\text{s}} - 15\ \frac{\text{m}}{\text{s}}}{-9.8\ \frac{\text{m}}{\text{s}^2}} \qquad t_2 = \frac{-10\ \frac{\text{m}}{\text{s}} - 15\ \frac{\text{m}}{\text{s}}}{-9.8\ \frac{\text{m}}{\text{s}^2}}$$

$\boxed{t_1 = 0.51\text{ s}}$ (going up) $\boxed{t_2 = 2.55\text{ s}}$ (coming down)

Example 9: $v_i = 15\ \frac{\text{m}}{\text{s}},\ y_i = 100\ \text{m},\ \text{and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$

c.) $y = 0,\ t = ?$

$$\Delta y = \frac{1}{2}at^2 + v_i t$$

$$0 = \frac{1}{2}at^2 + v_i t - \Delta y \quad \text{(Solution requires Quadratic Formula)}$$

$$t = \frac{-v_i \pm \sqrt{v_i^2 - 4\left(\frac{1}{2}a(-\Delta y)\right)}}{2\left(\frac{1}{2}a\right)} = \frac{-v_i \pm \sqrt{v_i^2 + 2a\Delta y}}{a} = \frac{-v_i \pm \sqrt{v_i^2 + 2a(y - y_i)}}{a}$$

$$t = \frac{-15\ \frac{\text{m}}{\text{s}} \pm \sqrt{\left(15\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(0 - 100\ \text{m})}}{-9.8\ \frac{\text{m}}{\text{s}^2}}$$

~~$t = -3.24$ s~~ or $\boxed{t = 6.30\ \text{s}}$

Example 9: $v_i = 15\ \frac{\text{m}}{\text{s}},\ y_i = 100\ \text{m},\ \text{and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$

c.) $y = 0,\ t = ?$

Another approach is to find v first and use it to find t.

since $v^2 = v_i^2 + 2a\Delta y$

$$v = \pm\sqrt{v_i^2 + 2a\Delta y} = -\sqrt{v_i^2 + 2a\Delta y} \quad \text{(since it is going downward)}$$

$$v = -\sqrt{\left(15\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(0 - 100\ \text{m})} = -46.7\ \frac{\text{m}}{\text{s}}$$

$$v = at + v_i \ \text{ so } t = \frac{v - v_i}{a} \quad \text{and } t = \frac{-46.7\ \frac{\text{m}}{\text{s}} - 15\ \frac{\text{m}}{\text{s}}}{-9.8\ \frac{\text{m}}{\text{s}^2}}$$

$$\boxed{t = 6.30\ \text{s}}$$

Example 9: $v_i = 15\ \frac{\text{m}}{\text{s}},\ y_i = 100\ \text{m},\ \text{and } a = -9.8\ \frac{\text{m}}{\text{s}^2}$

d.) $y = 0,\ v = ?$

If v is found before t, $\boxed{v = -46.7\ \frac{\text{m}}{\text{s}}}$

If t is found before v, $v = at + v_i$

$$v = \left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)(6.30\ \text{s}) + 15\ \frac{\text{m}}{\text{s}}$$

$$\boxed{v = -46.7\ \frac{\text{m}}{\text{s}}}$$