Example 1

A 50 m long copper wire (ρ = 1.72 x 10^{-8} Ω·m) has a diameter of 1.0 mm and is connected to a 6 V. Find:

a.) the resistance of the wire.

b.) the current through the wire.

Example 1:

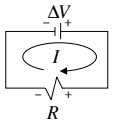
$\ell = 50$ m, $D = 1.0$ mm, $\Delta V = 6$ V, and $\rho = 1.72 \times 10^{-8}\ \Omega \cdot$ m

a.) $R = ?$

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2} = \frac{\left(1.72 \times 10^{-8}\ \Omega \cdot \text{m}\right)(50\ \text{m})}{\pi\left(0.5 \times 10^{-3}\ \text{m}\right)^2}$$

$$\boxed{R = 1.1\ \Omega}$$

b.) $I = ?$



$$\Delta V = IR$$

$$I = \frac{\Delta V}{R} = \frac{6\ \text{V}}{1.1\ \Omega}$$

$$\boxed{I = 5.45\ \text{A}}$$

Example 2:

A 24 Ω resistor is connected to a 6.0 V battery.

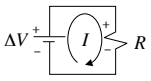
a.) Find the current through the resistor.

b.) Find the power delivered to the resistor.

c.) How much energy is dissipated in the resistor in 1.0 hour?

Example 2: $R = 24\ \Omega$ and $\Delta V = 6.0$ V

a.) $I = ?$



$$\Delta V = IR$$

$$I = \frac{\Delta V}{R} = \frac{6.0\ \text{V}}{24\ \Omega}$$

$$\boxed{I = 0.25\ \text{A}}$$

b.) $P = ?$

$$P = I\Delta V = (0.25\ \text{A})(6.0\ \text{V}) \qquad P = I^2 R = (0.25\ \text{A})^2(24\ \Omega) = 1.5\ \text{W}$$

$$\boxed{P = 1.5\ \text{W}}$$

$$P = \frac{\Delta V^2}{R} = \frac{(6.0\ \text{V})^2}{24\ \Omega} = 1.5\ \text{W}$$

c.) $t = 1$ h, $E = ?$

$$E = Pt = (1.5\ \text{W})(3600\ \text{s})$$

$$\boxed{E = 5400\ \text{J}}$$

Example 3:

A 120 Ω, a 60 Ω, and a 40 Ω resistor are connected in series with a 110 V power source.

a.) Draw a schematic diagram.

b.) What is the equivalent resistance of the circuit?

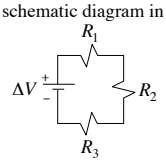
c.) What is the current from the power source?

d.) What is the current through each resistor?

e.) What is the voltage drop across each resistor?

Example 3: $R_1 = 120\ \Omega$, $R_2 = 60\ \Omega$, $R_3 = 40\ \Omega$, and $\Delta V = 110$ V

a.) schematic diagram in series



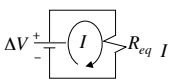
b.) $R_{eq} = ?$

$$R_{eq} = R_1 + R_2 + R_3$$

$$R_{eq} = 120\ \Omega + 60\ \Omega + 40\ \Omega$$

$$\boxed{R_{eq} = 220\ \Omega}$$

c.) $I = ?$



$$\Delta V = IR_{eq} \qquad I = \frac{\Delta V}{R_{eq}} = \frac{110\ \text{V}}{220\ \Omega}$$

$$\boxed{I = 0.50\ \text{A}}$$

d.) I's = ? for each resistor

Resistors in series have the same current and the same current as their equivalent so:

$$\boxed{I_1 = I_2 = I_3 = I = 0.50\ \text{A}}$$

e.) ΔV's = ? for each resistor

$$\Delta V_1 = I_1 R_1 \qquad \Delta V_2 = I_2 R_2 \qquad \Delta V_3 = I_3 R_3$$

$$\Delta V_1 = (0.5\ \text{A})(120\ \Omega) \qquad \Delta V_2 = (0.5\ \text{A})(60\ \Omega) \qquad \Delta V_3 = (0.5\ \text{A})(40\ \Omega)$$

$$\boxed{\Delta V_1 = 60\ \text{V}} \qquad \boxed{\Delta V_2 = 30\ \text{V}} \qquad \boxed{\Delta V_3 = 20\ \text{V}}$$

Example 4:

A 120 Ω, a 60 Ω, and a 40 Ω resistor are connected parallel to a 110 V power source.

a.) Draw a schematic diagram.

b.) What is the equivalent resistance of the circuit?

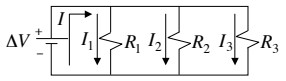
c.) What is the current from the power source?

d.) What is the voltage drop across each resistor?

e.) What is the current through each resistor?

Example 4 : $R_1 = 120\ \Omega$, $R_2 = 60\ \Omega$, $R_3 = 40\ \Omega$, and $\Delta V = 110$ V

a.) schematic diagram in parallel



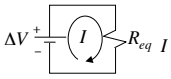
b.) $R_{eq} = ?$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$R_{eq} = \left(\frac{1}{120\ \Omega} + \frac{1}{60\ \Omega} + \frac{1}{40\ \Omega}\right)^{-1}$$

$$\boxed{R_{eq} = 20\ \Omega}$$

c.) $I = ?$



$$\Delta V = IR_{eq}$$

$$I = \frac{\Delta V}{R_{eq}} = \frac{110\ \text{V}}{20\ \Omega}$$

$$\boxed{I = 5.5\ \text{A}}$$

d.) V's = ? for each resistor

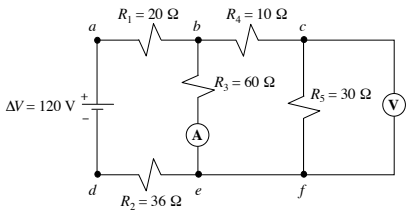
Resistors in parallel have the same voltage and the same voltage as their equivalent so :

$$\boxed{\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V = 110\ \text{V}}$$

e.) I's = ? for each resistor

$$I_1 = \frac{\Delta V_1}{R_1} = \frac{110\ \text{V}}{120\ \Omega} \qquad I_2 = \frac{\Delta V_2}{R_2} = \frac{110\ \text{V}}{60\ \Omega} \qquad I_3 = \frac{\Delta V_3}{R_3} = \frac{110\ \text{V}}{40\ \Omega}$$

$$\boxed{I_1 = 0.9167\ \text{A}} \qquad \boxed{I_2 = 1.833\ \text{A}} \qquad \boxed{I_3 = 2.75\ \text{A}}$$



Example 5:

For the circuit shown above:

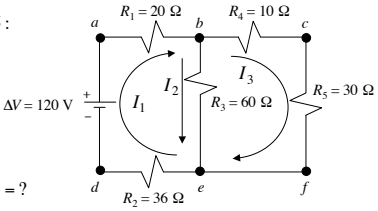
a.) Find the equivalent resistance.

b.) The current and voltage for each resistor.

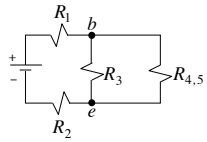
c.) The readings on the ammeter and on the voltmeter.

d.) The voltages V_{ae} and V_{fb}.

Example 5 :



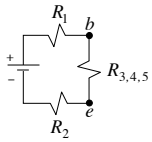
a.) $R_{eq} = ?$



$$R_{4,5} = R_4 + R_5$$

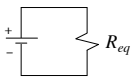
$$R_{4,5} = 10\ \Omega + 30\ \Omega$$

$$R_{4,5} = 40\ \Omega$$



$$R_{3,4,5} = \left(\frac{1}{R_3} + \frac{1}{R_{4,5}}\right)^{-1}$$

$$R_{3,4,5} = \left(\frac{1}{60\ \Omega} + \frac{1}{40\ \Omega}\right)^{-1} = 24\ \Omega$$

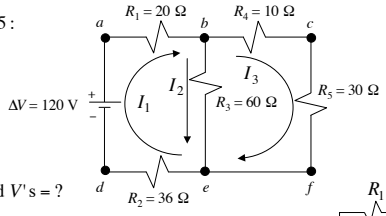


$$R_{eq} = R_1 + R_2 + R_{3,4,5}$$

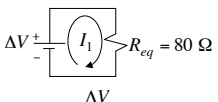
$$R_{eq} = 20\ \Omega + 36\ \Omega + 24\ \Omega$$

$$\boxed{R_{eq} = 80\ \Omega}$$

Example 5 :



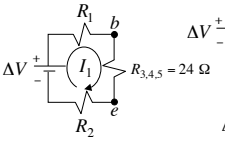
b.) I's and V's = ?



$$I_1 = \frac{\Delta V}{R_{eq}}$$

$$I_1 = \frac{120\ \text{V}}{80\ \Omega}$$

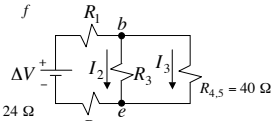
$$I_1 = 1.5\ \text{A}$$



$$V_{be} = I_1 R_{3,4,5}$$

$$V_{be} = (1.5\ \text{A})(24\ \Omega)$$

$$V_{be} = 36\ \text{V}$$

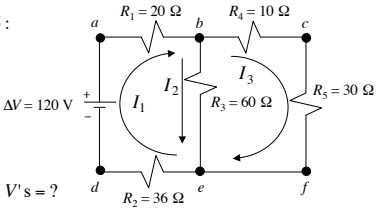


$$\Delta V_3 = \Delta V_{4,5} = V_{be} = 36\ \text{V}$$

$$I_2 = \frac{\Delta V_3}{R_3} = \frac{36\ \text{V}}{60\ \Omega} = 0.6\ \text{A}$$

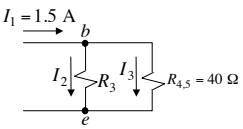
$$I_3 = \frac{\Delta V_{4,5}}{R_{4,5}} = \frac{36\ \text{V}}{40\ \Omega} = 0.9\ \text{A}$$

Example 5 :



b.) I's and V's = ?

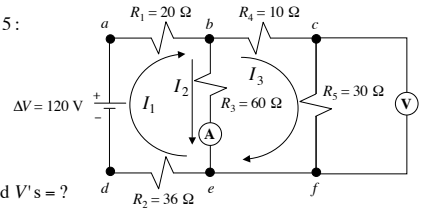
alternatively (Current Divider) :



$$I_2 = I_1 \frac{R_{4,5}}{R_3 + R_{4,5}} = 1.5\ \text{A}\,\frac{40\ \Omega}{60\ \Omega + 40\ \Omega} = 0.6\ \text{A}$$

$$I_3 = I_1 \frac{R_3}{R_3 + R_{4,5}} = 1.5\ \text{A}\,\frac{60\ \Omega}{60\ \Omega + 40\ \Omega} = 0.9\ \text{A}$$

Example 5 :



$I_1 = 1.5$ A
$I_2 = 0.6$ A
$I_3 = 0.9$ A

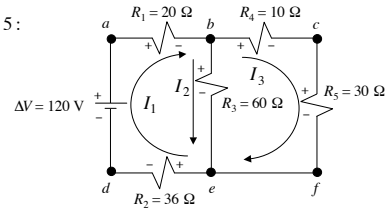
b.) I's and V's = ?

Resistor	Current	Voltage ($\Delta V = IR$)
$R_1 = 20\ \Omega$	$I_1 = 1.5$ A	$\Delta V_1 = 30$ V
$R_2 = 36\ \Omega$	$I_1 = 1.5$ A	$\Delta V_2 = 54$ V
$R_3 = 60\ \Omega$	$I_2 = 0.6$ A	$\Delta V_3 = 36$ V
$R_4 = 10\ \Omega$	$I_3 = 0.9$ A	$\Delta V_4 = 9$ V
$R_5 = 30\ \Omega$	$I_3 = 0.9$ A	$\Delta V_5 = 27$ V

c.) Meter readings?

Ammeter reads current $I_2 = 0.6$ A
Voltmeter reads voltage $\Delta V_5 = 27$ V

Example 5 :



$\Delta V_1 = 30$ V
$\Delta V_2 = 54$ V
$\Delta V_3 = 36$ V
$\Delta V_4 = 9$ V
$\Delta V_5 = 27$ V

d.) V_{ae} = ? and V_{fb} = ?

$V_{ae} = V_a - V_e$

Using point e as a reference $(V_e = 0)$

$V_d = V_e - \Delta V_2 = 0 - 54\ \text{V} = -54\ \text{V}$

$V_a = V_d + \Delta V = -54\ \text{V} + 120\ \text{V} = 66\ \text{V}$

$V_{ae} = V_a - V_e = 66\ \text{V} - 0$

$\boxed{V_{ae} = 66\ \text{V}}$

$V_{fb} = V_f - V_b$

Using point b as a reference $(V_b = 0)$

$V_e = V_b - \Delta V_3 = 0 - 36\ \text{V} = -36\ \text{V}$

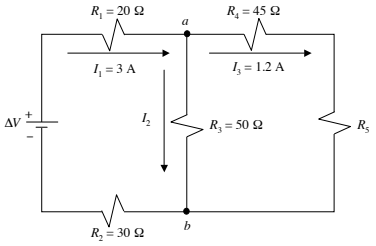
$V_f = V_e = -36\ \text{V}$

$V_{fb} = V_f - V_b = -36\ \text{V} - 0$

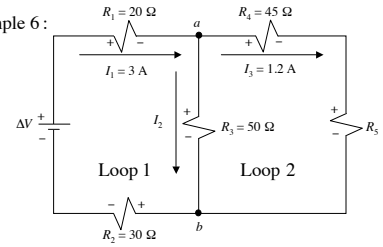
$\boxed{V_{fb} = -36\ \text{V}}$

Example 6:

Find the current I_2, the resistance R_5, and the voltage ΔV.



Example 6 :



I_2 = ?

@ junction a :

$I_1 = I_2 + I_3$

$I_2 = I_1 - I_3$

$I_2 = 3\ \text{A} - 1.2\ \text{A}$

$\boxed{I_2 = 1.8\ \text{A}}$

R_5 = ?

around Loop 2

$-\Delta V_4 - \Delta V_5 + \Delta V_3 = 0$

$-I_3 R_4 - I_3 R_5 + I_2 R_3 = 0$

$R_5 = \dfrac{-I_3 R_4 + I_2 R_3}{I_3}$

$R_5 = \dfrac{-(1.2\ \text{A})(45\ \Omega) + (1.8\ \text{A})(50\ \Omega)}{1.2\ \text{A}}$

$\boxed{R_5 = 30\ \Omega}$

ΔV = ?

around Loop 1

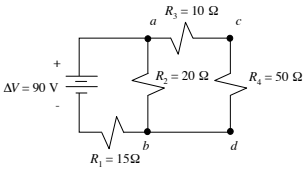
$\Delta V - \Delta V_1 - \Delta V_3 - \Delta V_2 = 0$

$\Delta V = \Delta V_1 + \Delta V_3 + \Delta V_2$

$\Delta V = I_1 R_1 + I_2 R_3 + I_1 R_2$

$\Delta V = (3\ \text{A})(20\ \Omega) + (1.8\ \text{A})(50\ \Omega) + (3\ \text{A})(30\ \Omega)$

$\boxed{\Delta V = 240\ \text{V}}$



Example 7:

For the circuit shown above:

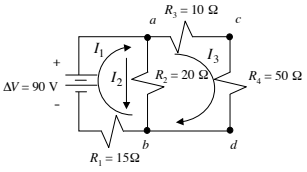
a.) Find the equivalent resistance.

b.) The current and voltage for each resistor.

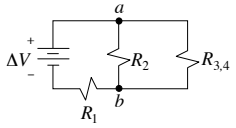
c.) The power dissipated in the circuit.

d.) The voltages V_{ad} and V_{bc}.

Example 7 :



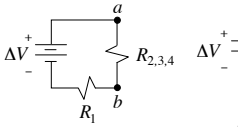
a.) R_{eq} = ?



$R_{3,4} = R_3 + R_4$

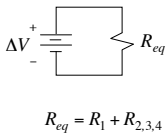
$R_{3,4} = 10\ \Omega + 50\ \Omega$

$R_{3,4} = 60\ \Omega$



$R_{2,3,4} = \left(\dfrac{1}{R_2} + \dfrac{1}{R_{3,4}}\right)^{-1}$

$R_{2,3,4} = \left(\dfrac{1}{20\ \Omega} + \dfrac{1}{60\ \Omega}\right)^{-1} = 15\ \Omega$

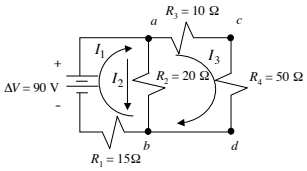


$R_{eq} = R_1 + R_{2,3,4}$

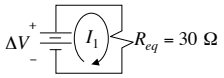
$R_{eq} = 15\ \Omega + 15\ \Omega$

$\boxed{R_{eq} = 30\ \Omega}$

Example 7 :



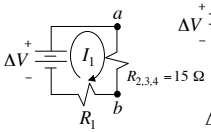
b.) I's and V's = ?



$$I_1 = \frac{\Delta V}{R_{eq}}$$

$$I_1 = \frac{90\text{ V}}{30\ \Omega}$$

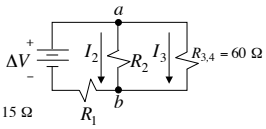
$$I_1 = 3.0\text{ A}$$



$$V_{ab} = I_1 R_{2,3,4}$$

$$V_{ab} = (3.0\text{ A})(15\ \Omega)$$

$$V_{ab} = 45\text{ V}$$

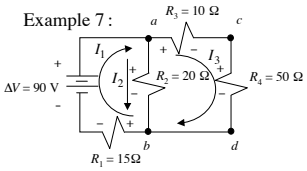


$$\Delta V_2 = \Delta V_{3,4} = V_{ab} = 45\text{ V}$$

$$I_2 = \frac{\Delta V_2}{R_2} = \frac{45\text{ V}}{20\ \Omega} = 2.25\text{ A}$$

$$I_3 = \frac{\Delta V_{3,4}}{R_{3,4}} = \frac{45\text{ V}}{60\ \Omega} = 0.75\text{ A}$$

Example 7 :



Resistor	Current	Voltage
$R_1 = 15\ \Omega$	$I_1 = 3.0$ A	$\Delta V_1 = 45$ V
$R_2 = 20\ \Omega$	$I_2 = 2.25$ A	$\Delta V_2 = 45$ V
$R_3 = 10\ \Omega$	$I_3 = 0.75$ A	$\Delta V_3 = 7.5$ V
$R_4 = 60\ \Omega$	$I_3 = 0.75$ A	$\Delta V_4 = 37.5$ V

c.) P = ?

$$P = \Delta V I_1 = (90\text{ V})(3.0\text{ A})$$

$$\boxed{P = 270\text{ W}}$$

d.) V_{ad} = ? and V_{bc} = ?

$$V_{ad} = V_a - V_d$$

Using point d as a reference $(V_d = 0)$

$$V_b = V_d = 0$$

$$V_a = V_b + \Delta V_2 = 0 + 45\text{ V} = 45\text{ V}$$

$$V_{ad} = V_a - V_d = 45\text{ V} - 0$$

$$\boxed{V_{ad} = 45\text{ V}}$$

$$V_{bc} = V_b - V_c$$

Using point c as a reference $(V_c = 0)$

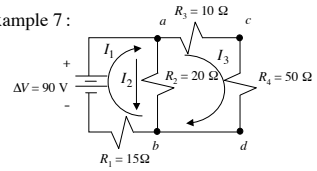
$$V_d = V_c - \Delta V_4 = 0 - 37.5\text{ V} = -37.5\text{ V}$$

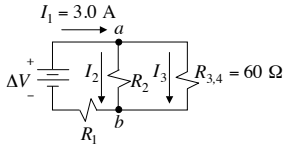
$$V_b = V_d = -37.5\text{ V}$$

$$V_{bc} = V_b - V_c = -37.5\text{ V}$$

$$\boxed{V_{bc} = -37.5\text{ V}}$$

Example 7 :





b.) I's and V's = ? alternatively (Current Divider) :

$$I_2 = I_1 \frac{R_{3,4}}{R_2 + R_{3,4}} = 3.0\text{ A} \frac{60\ \Omega}{20\ \Omega + 60\ \Omega} = 2.25\text{ A}$$

$$I_3 = I_1 \frac{R_2}{R_2 + R_{3,4}} = 3.0\text{ A} \frac{20\ \Omega}{20\ \Omega + 60\ \Omega} = 0.75\text{ A}$$

Resistor	Current	Voltage $(\Delta V = IR)$
$R_1 = 15\ \Omega$	$I_1 = 3.0$ A	$\Delta V_1 = 45$ V
$R_2 = 20\ \Omega$	$I_2 = 2.25$ A	$\Delta V_2 = 45$ V
$R_3 = 10\ \Omega$	$I_3 = 0.75$ A	$\Delta V_3 = 7.5$ V
$R_4 = 60\ \Omega$	$I_3 = 0.75$ A	$\Delta V_4 = 37.5$ V