Circular Motion

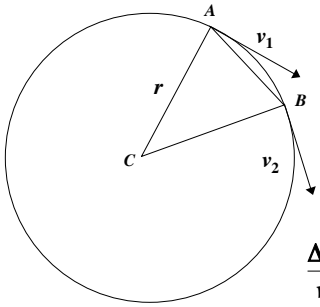
Uniform Circular Motion

An object that moves in a circle at constant speed is said to experience *uniform circular motion*.

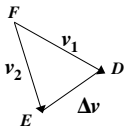
- **The magnitude of the velocity remains constant.**
- **The direction of the velocity is continuously changing as the object moves around the circle.**
- **The object is accelerating because there is a change in velocity.**

This acceleration is called *centripetal acceleration* and it points towards the center of the circle.

Centripetal Acceleration



$\Delta DEF \approx \Delta ABC$



$\Delta v = v_2 - v_1$ or $v_2 = \Delta v + v_1$

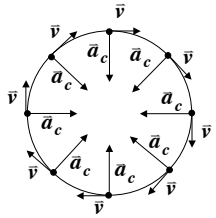
$$a_{rad} = \frac{\Delta v}{\Delta t}$$

$$\frac{\Delta v}{v} = \frac{AB}{r} \text{ and } AB = d = v \cdot \Delta t$$

so $\frac{\Delta v}{v} = \frac{v \cdot \Delta t}{r}$ and $\frac{\Delta v}{\Delta t} = \frac{v^2 \cdot \Delta t}{r \cdot \Delta t} = \boxed{\frac{v^2}{r} = a_c}$

Centripetal Acceleration

$$a_c = \frac{v^2}{r}$$



- **This component always points towards the axis of rotation.**
- **The centripetal acceleration is always perpendicular to tangential motion.**

Forces in Circular Motion

Because an object in uniform circular motion is accelerating, there must be a net force creating this acceleration. Therefore, Newton's second-law can be applied to problems involving circular motion. This net force is called a centripetal force which causes the centripetal acceleration.

$$\sum F_r = ma_c = m\frac{v^2}{r}$$

where $\sum F_r$ is the sum of all forces in the radial direction (towards or away from the center of the circle).

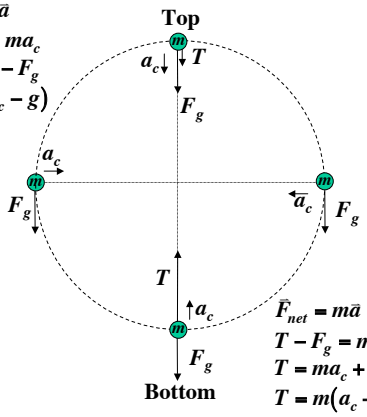
Forces in Circular Motion

Examples of forces that result in circular motion include:

- **Tensions in cords swinging objects in circular paths.**
- **Normal forces on objects in motion on roller coaster loops and Ferris wheels.**
- **Frictional forces on objects moving on curved roads.**
- **Gravitational forces between objects orbiting other objects.**

Vertical Motion of a Mass on a Cord

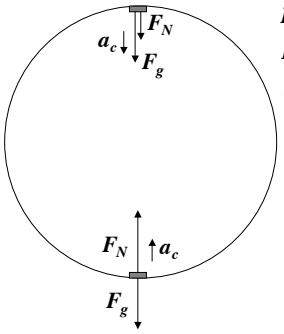
$\vec{F}_{net} = m\vec{a}$
$T + F_g = ma_c$
$T = ma_c - F_g$
$T = m(a_c - g)$



$\vec{F}_{net} = m\vec{a}$
$T - F_g = ma_c$
$T = ma_c + F_g$
$T = m(a_c + g)$

Roller Coaster Loops

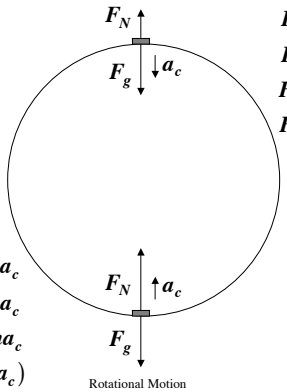
$\vec{F}_{net} = m\vec{a}$
$F_g + F_N = ma_c$
$F_N = ma_c - F_g$
$F_N = ma_c - mg$
$F_N = m(a_c - g)$



$\vec{F}_{net} = m\vec{a}$
$F_N - F_g = ma_c$
$F_N = F_g + ma_c$
$F_N = mg + ma_c$
$F_N = m(g + a_c)$

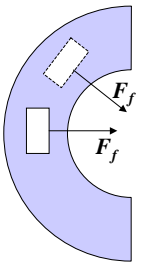
Ferris Wheel

$\vec{F}_{net} = m\vec{a}$
$F_g - F_N = ma_c$
$F_N = F_g - ma_c$
$F_N = mg - ma_c$
$F_N = m(g - a_c)$

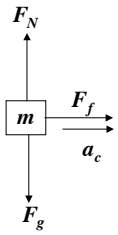


$\vec{F}_{net} = m\vec{a}$
$F_N - F_g = ma_c$
$F_N = F_g + ma_c$
$F_N = mg + ma_c$
$F_N = m(g + a_c)$

Car on a Curved Road



(View from above)



(Side view)

$\vec{F}_{net} = m\vec{a}$
$\sum F_y = ma$
$F_N - F_g = 0$
$F_N = F_g = mg$

$\sum F_x = ma$
$F_f = ma_c$
$\mu F_N = ma_c$
$\mu mg = ma_c$
$\mu g = a_c$