

Example 1:

A block attached to a spring with unknown spring constant oscillates with a period of 2.00 s. What is the period if

- a.) The mass is doubled?
- b.) The mass is halved?
- c.) The amplitude is doubled?
- d.) The spring constant is doubled?

Example 1:

$$T_s = 2\pi\sqrt{\frac{m}{k}} = 2.00 \text{ s}$$

a.) $m = 2m, T = ?$ $T = 2\pi\sqrt{\frac{2m}{k}} = \sqrt{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \sqrt{2}(2.00 \text{ s})$
 $T = 2.83 \text{ s}$

b.) $m = \frac{m}{2}, T = ?$ $T = 2\pi\sqrt{\frac{\frac{m}{2}}{k}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{\sqrt{2}}(2.00 \text{ s})$
 $T = 1.41 \text{ s}$

c.) $A = 2A, T = ?$ $T = 2\pi\sqrt{\frac{m}{k}}$ $T = 2.00 \text{ s}$

d.) $k = 2k, T = ?$ $T = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{\sqrt{2}}(2.00 \text{ s})$
 $T = 1.41 \text{ s}$

Example 2:

A 2.00 kg object is attached to a horizontal spring of force constant 5000 N/m. The spring is stretched 10.0 cm from equilibrium and released. Find

- a.) the period
- b.) the frequency
- c.) the angular frequency
- d.) the amplitude of motion
- e.) the maximum speed
- f.) the maximum acceleration

Example 2: $m = 2.00 \text{ kg}, k = 5000 \frac{\text{N}}{\text{m}}$, and $A = 10.0 \text{ cm}$

a.) $T = ?$
 $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(2.00 \text{ kg})}{\left(5000 \frac{\text{N}}{\text{m}}\right)}}$
 $T = 0.1257 \text{ s}$

b.) $f = ?$
 $T = \frac{1}{f}$ so $f = \frac{1}{T} = \frac{1}{(0.1257 \text{ s})}$
 $f = 7.96 \text{ Hz}$

c.) $\omega = ?$
 $\frac{2\pi}{\omega} = \frac{1}{f}$ so $\omega = 2\pi f$
 $\omega = 2\pi(7.96 \text{ Hz})$
 $\omega = 50.0 \frac{\text{rad}}{\text{s}}$

d.) $A = ?$ $A = 10.0 \text{ cm}$

e.) $v_{\text{max}} = ?$
 $v = -\omega A \sin(\omega t) = -v_{\text{max}} \sin(\omega t)$
 $v_{\text{max}} = \omega A = \left(50 \frac{\text{rad}}{\text{s}}\right)(10.0 \text{ cm})$
 $v_{\text{max}} = 500 \frac{\text{cm}}{\text{s}}$

f.) $a_{\text{max}} = ?$
 $a = -\omega^2 A \cos(\omega t) = -a_{\text{max}} \sin(\omega t)$
 $a_{\text{max}} = \omega^2 A = \left(50 \frac{\text{rad}}{\text{s}}\right)^2 (10.0 \text{ cm})$
 $a_{\text{max}} = 2500 \frac{\text{cm}}{\text{s}^2}$

Example 3:

The position of a 50.0 g oscillating mass is given by the following equation

$$x = (2.0 \text{ cm})\cos\left(10 \frac{\text{rad}}{\text{s}} \cdot t\right)$$

Find:

- a.) the amplitude
- b.) the period
- c.) the spring constant
- d.) the maximum speed
- e.) the total energy

Example 3:

$$x = (2.0 \text{ cm})\cos\left(10 \frac{\text{rad}}{\text{s}} \cdot t\right) \text{ and } m = 50 \text{ g}$$

a.) $A = ?$ $x = A \cos(\omega t)$
 $A = 2.0 \text{ cm}$

b.) $T = ?$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(10 \frac{\text{rad}}{\text{s}}\right)}$
 $T = 0.63 \text{ s}$

c.) $k = ?$ $T = 2\pi\sqrt{\frac{m}{k}}$
 $T^2 = 4\pi^2 \frac{m}{k}$
 $k = 4\pi^2 \frac{m}{T^2} = 4\pi^2 \frac{(0.050 \text{ kg})}{(0.63 \text{ s})^2}$
 $k = 5.0 \frac{\text{N}}{\text{m}}$

d.) $v_{\text{max}} = ?$
 $v = -\omega A \sin(\omega t) = -v_{\text{max}} \sin(\omega t)$

$$v_{\text{max}} = \omega A = \left(10 \frac{\text{rad}}{\text{s}}\right)(2.0 \text{ cm})$$

 $v_{\text{max}} = 20 \frac{\text{cm}}{\text{s}}$

e.) $E = ?$ $E = U_s(A) = \frac{1}{2}kA^2$
 $E = \frac{1}{2}kA^2$

$$E = \frac{1}{2}\left(5.0 \frac{\text{N}}{\text{m}}\right)(0.020 \text{ m})^2$$

$$E = 0.0010 \text{ J}$$

Example 4:

A 0.40 kg ball is suspended from a spring with spring constant 12 N/m. If the ball is pulled down 0.20 m from the equilibrium position and released, what is the maximum speed of the ball?

Example 4:

$$m = 0.40 \text{ kg}, k = 12 \frac{\text{N}}{\text{m}}, A = 0.20 \text{ m}, v_{\text{max}} = ?$$

$$v = -\omega A \sin(\omega t) = -v_{\text{max}} \sin(\omega t)$$

$$v_{\text{max}} = \omega A$$

$$T = \frac{2\pi}{\omega} \text{ so } \omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{2\pi \sqrt{\frac{m}{k}}} = \sqrt{\frac{k}{m}}$$

$$v_{\text{max}} = \sqrt{\frac{k}{m}} A$$

$$v_{\text{max}} = \sqrt{\frac{12 \frac{\text{N}}{\text{m}}}{0.40 \text{ kg}}} (0.20 \text{ m})$$

$$v_{\text{max}} = 1.1 \frac{\text{m}}{\text{s}}$$

Example 5:

A 3.00 kg object oscillating on a spring of force constant 2000 N/m has a total energy of 0.900 J. (a) What is the amplitude of the motion? (b) What is the maximum speed?

Example 5:

$$m = 3 \text{ kg}, k = 2000 \frac{\text{N}}{\text{m}}, \text{ and } E = 0.900 \text{ J}$$

a.) $A = ?$

$$E = K + U$$

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

when $x = A, v = 0$ and $E = 0 + \frac{1}{2} k A^2 = \frac{1}{2} k A^2$

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.900 \text{ J})}{(2000 \frac{\text{N}}{\text{m}})}}$$

$$A = 0.0300 \text{ m}$$

b.) $v_{\text{max}} = ?$

$$v = -\omega A \sin(\omega t)$$

$$v_{\text{max}} = \omega A = \sqrt{\frac{k}{m}} A = \sqrt{\frac{(2000 \frac{\text{N}}{\text{m}})}{3 \text{ kg}}} (0.03 \text{ m})$$

$$v = -v_{\text{max}} \sin(\omega t)$$

$$v_{\text{max}} = 0.775 \frac{\text{m}}{\text{s}}$$

Example 6:

A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At one instant, the mass is at $x = 5.0 \text{ cm}$ and has $v_x = -30 \text{ cm/s}$. Determine:

- a.) the period
- b.) the amplitude

Example 6:

$$m = 200 \text{ g}, f = 2.0 \text{ Hz}, \text{ at } x = 5.0 \text{ cm}, v_x = -30 \frac{\text{cm}}{\text{s}}$$

a.) $T = ?$

$$T = \frac{1}{f} = \frac{1}{(2.0 \text{ Hz})}$$

$$T = 0.50 \text{ s}$$

b.) $A = ?$

$$x = A \cos(\omega t) \text{ and } v_x = -\omega A \sin(\omega t)$$

$$\frac{v_x}{x} = \frac{-\omega A \sin(\omega t)}{A \cos(\omega t)} = -\omega \tan(\omega t)$$

$$\tan(\omega t) = \frac{-v_x}{x \omega} \text{ so } \omega t = \tan^{-1}\left(\frac{-v_x}{x \omega}\right) = \tan^{-1}\left(\frac{-v_x}{x 2\pi f}\right)$$

$$\omega t = \tan^{-1}\left(\frac{30 \frac{\text{cm}}{\text{s}}}{(5.0 \text{ cm}) 2\pi (2.0 \text{ Hz})}\right) = 0.4455 \text{ rad}$$

$$A = \frac{x}{\cos(\omega t)} = \frac{5.0 \text{ cm}}{\cos(0.4455 \text{ rad})} = 5.5 \text{ cm}$$

Example 7:

A mass on a string of unknown length oscillates as a pendulum with a period of 4.00 s. What is the period if

- The mass is doubled?
- The string length is doubled?
- The string length is halved?
- The amplitude is halved?

Example 7:

$$T_p = 2\pi\sqrt{\frac{\ell}{g}} = 4.00 \text{ s}$$

a.) $m = 2m, T = ?$ $T = 4.00 \text{ s}$

b.) $\ell = 2\ell, T = ?$ $T = 2\pi\sqrt{\frac{2\ell}{g}} = \sqrt{2}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \sqrt{2}(4.00 \text{ s})$

$T = 5.66 \text{ s}$

c.) $\ell = \frac{\ell}{2}, T = ?$ $T = 2\pi\sqrt{\frac{\frac{\ell}{2}}{g}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \frac{1}{\sqrt{2}}(4.00 \text{ s})$

$T = 2.83 \text{ s}$

d.) $A = \frac{A}{2}, T = ?$ $T = 4.00 \text{ s}$

Example 8:

- Find the length of a simple pendulum if the period is 5.0 s at a point where $g = 9.8 \text{ m/s}^2$.
- What would be the period of the pendulum in (a) on the moon, where the acceleration due to gravity is one-sixth that on earth.

Example 8:

a.) $T = 5.0 \text{ s}, g = 9.8 \frac{\text{m}}{\text{s}^2}, \ell = ?$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = 4\pi^2 \frac{\ell}{g}$$

$$\ell = \frac{gT^2}{4\pi^2} = \frac{\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(5.0 \text{ s})^2}{4\pi^2}$$

$\ell = 6.2 \text{ m}$

b.) $g = \frac{g}{6}, T = ?$ $T = 2\pi\sqrt{\frac{\ell}{\frac{g}{6}}} = \sqrt{6}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \sqrt{6}(5.0 \text{ s})$

$T = 12.2 \text{ s}$