Example 1:

The minute hand of a clock has a length of 0.15 m. What distance does the tip of the minute hand sweep through in 15 minutes?

$$r = 0.15 \text{ m},\ \Delta t = 15.0 \text{ min},\ \Delta s = ?$$

$$\Delta t = 15.0 \text{ min so } \Delta\theta = \frac{\pi}{2} \text{ rad}$$

$$\Delta s = r\Delta\theta$$

$$\Delta s = (0.15 \text{ m})\left(\frac{\pi}{2} \text{ rad}\right)$$

$$\boxed{\Delta s = 0.236 \text{ m}}$$

Example 2:

What is the angular speed of the tip of the minute hand on a clock, in rad/s?

$$\Delta\theta = 2\pi \text{ rad},\ \Delta t = 60 \text{ min } = 3600 \text{ s},\ \alpha = 0,\ \omega = ?$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{3600 \text{ s}}$$

$$\boxed{\omega = 1.7\times 10^{-3} \frac{\text{rad}}{\text{s}}}$$

Example 3:

A motor starts from rest and accelerates at 20 rev/s^2. After 30 seconds,

a.) What is the angular displacement in radians?

b.) What is the angular velocity in rad/s?

Example 3: $\alpha = 20 \frac{\text{rev}}{\text{s}^2},\ t = 30 \text{ s},\ \omega_0 = 0$

a.) $\Delta\theta = ?$ (in rad)

$$\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t$$

$$\Delta\theta = \frac{1}{2}\left(20 \frac{\text{rev}}{\text{s}^2}\right)(30 \text{ s})^2 + 0$$

$$\Delta\theta = 9000 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$

$$\boxed{\Delta\theta = 56{,}549 \text{ rad}}$$

b.) $\omega = ?$ (in rad/s)

$$\omega = \alpha t + \omega_0$$

$$\omega = \left(20 \frac{\text{rev}}{\text{s}^2}\right)(30 \text{ s}) + 0$$

$$\omega = 600 \frac{\text{rev}}{\text{s}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$

$$\boxed{\omega = 3770 \frac{\text{rad}}{\text{s}}}$$

Example 4:

A wheel turns 40 revolutions while accelerating from an angular velocity of 15 rad/s to an angular velocity of 35 rad/s.

a.) What is the angular acceleration of the wheel?

b.) How many seconds did the increase in angular velocity take?

Example 4: $\omega_0 = 15 \frac{\text{rad}}{\text{s}},\ \omega = 35 \frac{\text{rad}}{\text{s}},\ \Delta\theta = 40 \text{ rev}$

$$\Delta\theta = 40 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 251 \text{ rad}$$

a.) $\alpha = ?$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta}$$

$$\alpha = \frac{\left(35 \frac{\text{rad}}{\text{s}}\right)^2 - \left(15 \frac{\text{rad}}{\text{s}}\right)^2}{2(251 \text{ rad})}$$

$$\boxed{\alpha = 2.0 \frac{\text{rad}}{\text{s}^2}}$$

b.) $\Delta\theta = ?$ (in revolutions)

$$\Delta\theta = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$t = \frac{2\Delta\theta}{\omega + \omega_0}$$

$$t = \frac{2(251 \text{ rad})}{\left(35 \frac{\text{rad}}{\text{s}} + 15 \frac{\text{rad}}{\text{s}}\right)}$$

$$\boxed{t = 10.0 \text{ s}}$$

Example 5:

A turntable rotates initially at 33 rpm (rev/min) is switched off and takes 20 s to come to a complete stop.

a.) What is the angular acceleration in rad/s^2?

b.) How many revolutions does the turntable make before coming to rest?

c.) If the radius is 14 cm, what is the initial linear speed of a bug riding on the rim?

d.) What is the tangential acceleration at a distance of 7 cm from the center while the turntable is slowing down?

Example 5 : $\omega_0 = 33\ \frac{\text{rev}}{\text{min}},\ \omega = 0,\ t = 20\ \text{s}$

$$\omega_0 = 33\ \frac{\text{rev}}{\text{min}}\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 3.46\ \frac{\text{rad}}{\text{s}}$$

a.) $\alpha = ?\ \left(\text{in rad/s}^2\right)$

$$\omega = \alpha t + \omega_0$$

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\alpha = \frac{0 - 3.46\ \frac{\text{rad}}{\text{s}}}{20\ \text{s}}$$

$$\boxed{\alpha = -0.173\ \frac{\text{rad}}{\text{s}^2}}$$

b.) $\Delta\theta = ?$ (in revolutions)

$$\Delta\theta = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$\Delta\theta = \frac{\left(0 + 3.46\ \frac{\text{rad}}{\text{s}}\right)}{2}(20\ \text{s})$$

$$\Delta\theta = 34.6\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)$$

$$\boxed{\Delta\theta = 5.5\ \text{rev}}$$

Example 5 : $\omega_0 = 33\ \frac{\text{rev}}{\text{min}} = 3.46\ \frac{\text{rad}}{\text{s}},\ \omega = 0,\ t = 20\ \text{s}$

c.) $r = 14\ \text{cm},\ v_0 = ?$

$$v_0 = r\omega_0$$

$$v_0 = (0.14\ \text{m})\left(3.46\ \frac{\text{rad}}{\text{s}}\right)$$

$$\boxed{v_0 = 0.484\ \frac{\text{m}}{\text{s}}}$$

d.) $r = 7\ \text{cm},\ a = ?$

$$a = r\alpha$$

$$a = (0.07\ \text{m})\left(-0.173\ \frac{\text{rad}}{\text{s}^2}\right)$$

$$\boxed{a = -0.0121\ \frac{\text{m}}{\text{s}^2}}$$

Example 6:

A solid sphere with radius R = 0.15 m and mass M = 7.5 kg rolls without slipping on a flat surface with a constant rotational speed of 40 rad/s. The moment of inertia for a solid sphere about its center-of-mass is $I = 2/5MR^2$.

a.) Find the linear speed for the center-of-mass of the sphere.

b.) Find the distance the sphere travels in 15 seconds.

c.) Find the total kinetic energy of the sphere.

Example 6: $R = 0.15\ \text{m},\ M = 7.25\ \text{kg},\ \omega = 40\ \frac{\text{rad}}{\text{s}}$

a.) $v = ?$

$$v = r\omega$$

$$v = (0.15\ \text{m})\left(40\ \frac{\text{rad}}{\text{s}}\right)$$

$$\boxed{v = 6.0\ \frac{\text{m}}{\text{s}}}$$

b.) $t = 15\ \text{s},\ d = ?$

$$d = \Delta s = r\Delta\theta$$

$$\Delta s = r\omega t$$

$$\Delta s = (0.15\ \text{m})\left(40\ \frac{\text{rad}}{\text{s}}\right)(15\ \text{s})$$

$$\boxed{\Delta s = 90\ \text{m}}$$

$$d = \Delta x = vt$$

$$\Delta x = \left(6.0\ \frac{\text{m}}{\text{s}}\right)(15\ \text{s})$$

$$\boxed{\Delta x = 90\ \text{m}}$$

Example 6: $R = 0.15\ \text{m},\ M = 7.25\ \text{kg},\ \omega = 40\ \frac{\text{rad}}{\text{s}}$

c.) $K_{total} = ?$

$$K_{total} = K_{trans} + K_{rot}$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{5}MR^2\omega^2$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{5}M(R\omega)^2$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$$

$$K_{total} = \frac{5}{10}Mv^2 + \frac{2}{10}Mv^2$$

$$K_{total} = \frac{7}{10}Mv^2$$

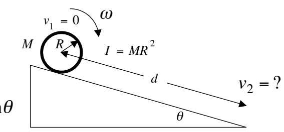
$$K_{total} = \frac{7}{10}(7.25\ \text{kg})\left(6.0\ \frac{\text{m}}{\text{s}}\right)^2$$

$$\boxed{K_{total} = 182.7\ \text{J}}$$

Example 7:

A thin hoop with a radius $R = 0.5$ m and mass $M = 0.25$ kg rolls down an incline plane that makes an angle $\theta = 30^\circ$ with respect to the horizontal. The hoop is released from rest and rolls without slipping a distance $d = 6.0$ m down the incline. Use energy conservation to find the speed of the hoop. The moment of inertia for a thin hoop about its center-of-mass is $I = MR^2$.

Example 7: $R = 0.5 \text{ m},\ M = 0.25 \text{ kg},\ d = 6.0 \text{ m},\ \theta = 30^\circ,\ v_1 = 0,\ v_2 = ?$



$v = R\omega$

$y_1 = d\sin\theta$

$$\cancel{K_1} + U_{g_1} = K_2 + \cancel{U_{g_2}}$$

$v_1 = 0 \qquad y_2 = 0$

$$Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{1}{2}I\omega_2^2$$

$$Mgd\sin\theta = \frac{1}{2}Mv_2^2 + \frac{1}{2}MR^2\left(\frac{v_2}{R}\right)^2$$

$$gd\sin\theta = \frac{1}{2}v_2^2 + \frac{1}{2}R^2\left(\frac{v_2^2}{R^2}\right)$$

$$gd\sin\theta = \frac{1}{2}v_2^2 + \frac{1}{2}v_2^2$$

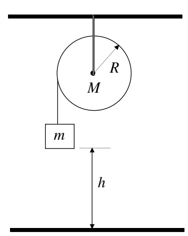
$$gd\sin\theta = v_2^2$$

$$v_2 = \sqrt{gd\sin\theta}$$

$$v_2 = \sqrt{\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(6.0 \text{ m})\sin(30^\circ)}$$

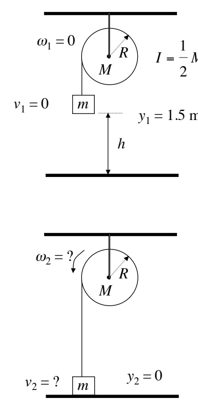
$$\boxed{v_2 = 5.42 \ \frac{\text{m}}{\text{s}}}$$

Example 8



In the figure above, the pulley is a solid disk of mass $M = 50$ kg and radius $R = 0.50$ m. A blocks of mass $m = 25$ kg hangs from one side of the pulley by a light cord. Initially the system is at rest, with $h = 1.5$ m above the floor. Then m is released and allowed to fall. Use energy conservation to find the speed of the block just before it strikes the ground and the angular velocity of the pulley. The moment of inertia of a solid disk is $1/2MR^2$.

Example 8: $R = 0.5 \text{ m},\ M = 50 \text{ kg},\ m = 25 \text{ kg},\ h = 1.5 \text{ m},\ v_1 = 0,\ v_2 = ?,\ \omega_2 = ?$



$$\cancel{K_1} + U_{g_1} = K_2 + \cancel{U_{g_2}}$$

$v_1 = 0 \qquad y_2 = 0$

$\omega_1 = 0$

$$mgy_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$$

$$mgy_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_2}{R}\right)^2$$

$$mgy_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_2^2}{R^2}\right)$$

$$mgy_1 = \frac{1}{4}(2m + M)v_2^2$$

$$v_2 = \sqrt{\frac{4mgy_1}{2m + M}}$$

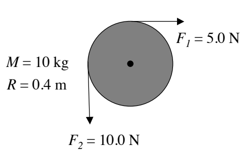
$$v_2 = \sqrt{\frac{4(25 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(1.5 \text{ m})}{2(25 \text{ kg}) + 50 \text{ kg}}} = \boxed{3.83 \ \frac{\text{m}}{\text{s}}}$$

$v_2 = R\omega_2$

$$\omega_2 = \frac{v_2}{R}$$

$$\omega_2 = \frac{3.83 \ \frac{\text{m}}{\text{s}}}{0.5 \text{ m}}$$

$$\boxed{\omega_2 = 7.66 \ \frac{\text{rad}}{\text{s}}}$$



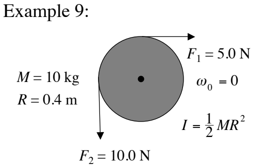
Example 9:

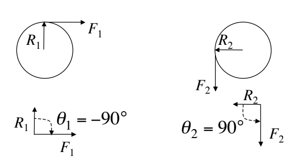
A solid cylinder of mass 10 kg and radius 0.4 m is pivoted about a frictionless axis through its center. Two forces are applied to the cylinder as shown in the figure above.

a.) Find the angular acceleration of the cylinder. (The moment of inertia for a solid cylinder is $\frac{1}{2}MR^2$.)

b.) If the cylinder starts from rest find the angular velocity 2.0 s after the forces are applied.

Example 9:





a.) $\alpha = ? \qquad \sum\tau = I\alpha$

$$\tau_1 + \tau_2 = I\alpha$$

$$R_1F_1\sin\theta_1 + R_2F_2\sin\theta_2 = I\alpha$$

$$\alpha = \frac{R_1F_1\sin\theta_1 + R_2F_2\sin\theta_2}{I}$$

$$\alpha = \frac{R_1F_1\sin\theta_1 + R_2F_2\sin\theta_2}{\frac{1}{2}MR^2}$$

$$\alpha = \frac{2(F_1\sin\theta_1 + F_2\sin\theta_2)}{MR}$$

$$\alpha = \frac{2((5.0 \text{ N})\sin(-90^\circ) + (10.0 \text{ N})\sin(90^\circ))}{(10.0 \text{ kg})(0.4 \text{ m})}$$

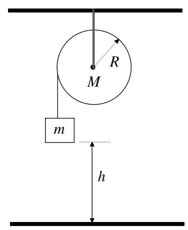
$$\boxed{\alpha = 2.5 \ \frac{\text{rad}}{\text{s}^2}}$$

b.) $t = 2.0 \text{ s},\ \omega = ?$

$$\omega = \alpha t + \omega_0$$

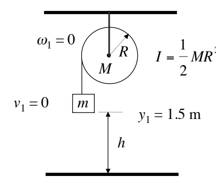
$$\omega = \left(2.5 \ \frac{\text{rad}}{\text{s}^2}\right)(2.0 \text{ s}) + 0 = \boxed{5.0 \ \frac{\text{rad}}{\text{s}}}$$

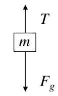
Example 10:

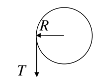


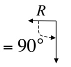
In the figure above, the pulley is a solid disk of mass M = 50 kg and radius R = 0.50 m. A blocks of mass m = 25 kg hangs from one side of the pulley by a light cord. Initially the system is at rest, with h = 1.5 m above the floor. Then m is released and allowed to fall. Use Newton's 2nd law to find the speed of the block just before it strikes the ground. The moment of inertia of a solid disk is 1/2 MR^2.

Example 10: $R = 0.5$ m, $M = 50$ kg, $m = 25$ kg, $h = 1.5$ m, $v_1 = 0$, $v_2 = ?$



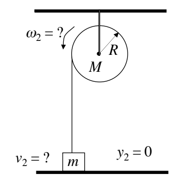






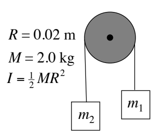
$$\sum F_y = ma$$
$$F_g - T = ma$$
$$(1) \quad mg - T = ma$$

$$\sum \tau = I\alpha$$
$$RT\sin\theta = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$
$$T\sin(90°) = \frac{1}{2}Ma$$
$$(2) \quad T = \frac{1}{2}Ma$$



$$(1+2) \quad mg = ma + \frac{1}{2}Ma$$
$$2mg = (2m + M)a$$
$$a = \frac{2mg}{2m + M}$$

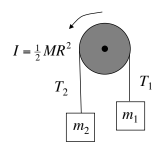
$$a = \frac{2(25\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}{2(25\text{ kg}) + 50\text{ kg}} = 4.9\ \frac{\text{m}}{\text{s}^2}$$
$$v_2^2 = v_1^2 + 2a\Delta y$$
$$v_2 = \sqrt{v_1^2 + 2a\Delta y}$$
$$v_2 = \sqrt{0 + 2\left(4.9\ \frac{\text{m}}{\text{s}^2}\right)(1.5\text{ m})} = \boxed{3.83\ \frac{\text{m}}{\text{s}}}$$

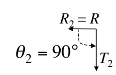


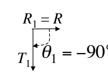
Example 11:

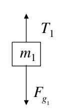
A pulley has a mass of 0.5 kg and a radius of 0.02 m. A cord is wrapped over the pulley and attached to a hanging object on either end. Assume the cord does not slip, the axle is frictionless, and the two hanging objects have masses m_1 = 1.5 kg and m_2 = 2.5 kg. Find the tension in the cord supporting each mass.

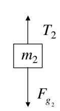
Example 11:
$R = 0.5$ m, $M = 2.0$ kg, $m_1 = 1.5$ kg, $m_2 = 2.5$ kg, $v_1 = 0$, $T_1 = ?$, $T_2 = ?$

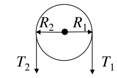












$$\sum F_y = ma$$
$$T_1 - F_{g1} = m_1 a$$
$$(1) \quad T_1 - m_1 g = m_1 a$$

$$\sum F_y = ma$$
$$F_{g2} - T_2 = m_2 a$$
$$(2) \quad m_2 g - T_2 = m_2 a$$

$$\sum \tau = I\alpha$$
$$R_2 T_2 \sin\theta_2 + R_1 T_1 \sin\theta_1 = I\alpha$$
$$RT_2\sin(90°) + RT_1\sin(-90°) = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$
$$(3) \quad T_2 - T_1 = \frac{1}{2}Ma$$

Example 11:
$R = 0.5$ m, $M = 2.0$ kg, $m_1 = 1.5$ kg, $m_2 = 2.5$ kg, $v_1 = 0$, $T_1 = ?$, $T_2 = ?$

$$(1) \quad T_1 - m_1 g = m_1 a \qquad (2) \quad m_2 g - T_2 = m_2 a \qquad (3) \quad T_2 - T_1 = \frac{1}{2}Ma$$

$$(1+2+3) \quad m_2 g - m_1 g = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$
$$2g(m_2 - m_1) = (2m_1 + 2m_2 + M)a$$
$$a = \frac{2g(m_2 - m_1)}{2m_1 + 2m_2 + M}$$
$$a = \frac{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.5\text{ kg} - 1.5\text{ kg})}{2(1.5\text{ kg}) + 2(2.5\text{ kg}) + 2.0\text{ kg}}$$
$$a = 1.96\ \frac{\text{m}}{\text{s}^2}$$

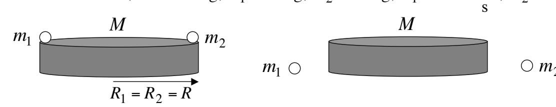
$$(1) \quad T_1 = m_1 a + m_1 g$$
$$T_1 = m_1(a + g)$$
$$T_1 = (1.5\text{ kg})\left(1.96\ \frac{\text{m}}{\text{s}^2} + 9.8\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\boxed{T_1 = 17.64\text{ N}}$$

$$(2) \quad T_2 = m_2 g - m_2 a$$
$$T_2 = m_2(g - a)$$
$$T_2 = (2.5\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2} - 1.96\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\boxed{T_2 = 19.6\text{ N}}$$

Example 12:

A merry-go-round ($I = \frac{1}{2}MR^2$) has a radius of 14 m and a mass of 150 kg. Two children, each with a mass of 25 kg, are riding on the outer edge while it is rotating at a constant angular speed of 0.25 rad/s. Find the angular speed of the merry-go-round after both children jump off.

Example 12: $R = 14\text{ m},\ M = 125\text{ kg},\ m_1 = 25\text{ kg},\ m_2 = 25\text{ kg},\ \omega_1 = 0.25\ \frac{\text{rad}}{\text{s}},\ \omega_2 = ?$



$$I_1 = I_M + I_1 + I_2$$
$$I_1 = \frac{1}{2}MR^2 + m_1R_1^2 + m_2R_2^2$$

$$I_2 = I_M$$
$$I_2 = \frac{1}{2}MR^2$$

$$\sum L_i = \sum L_f$$
$$I_1\omega_1 = I_2\omega_2$$
$$\omega_2 = \frac{I_1}{I_2}\omega_1$$
$$\omega_2 = \frac{\frac{1}{2}MR^2 + m_1R_1^2 + m_2R_2^2}{\frac{1}{2}MR^2}\omega_1$$
$$\omega_2 = \frac{\frac{1}{2}(150\text{ kg})(14\text{ m})^2 + (25\text{ kg})(14\text{ m})^2 + (25\text{ kg})(14\text{ m})^2}{\frac{1}{2}(150\text{ kg})(14\text{ m})^2}\left(0.25\ \frac{\text{rad}}{\text{s}}\right) = \boxed{0.45\ \frac{\text{rad}}{\text{s}}}$$