Vibrations and Waves

Waves

- **A *wave* is a disturbance that propagates through a medium or space.**
- **Waves *transfer energy* in the direction of propagation *without the bulk transfer of matter*.**

Examples:
Sound waves, light waves, water waves, earthquakes, string vibrations....

Mechanical Waves

***Mechanical waves* require a medium such as water, air or rock. The motion of these waves are governed by Newton's laws.**

1.) *Transverse waves* cause the particles of the medium to vibrate *perpendicularly* to the direction of wave motion.

2.) *Longitudinal waves* cause the particles of the medium to vibrate *parallel* to the direction of wave motion.

Electromagnetic Waves

- ***Electromagnetic waves* do not require a medium to exist.**
- **They all travel at the speed of light (3.00×10^8 m/s).**
- **They exist only as *transverse waves*.**

Describing Waves

1.) *period* (T) - the shortest time interval during which the wave motion repeats itself. (measured in seconds)

2.) *frequency* (f) - the number of complete vibrations per second measured at a fixed location. (measured in Hertz = s^{-1})

3.) *angular frequency* (ω) - is 2π times the frequency and represents the rate of change of an angular quantity. (measured in radians/s)

$$\boxed{T = \frac{1}{f} = \frac{2\pi}{\omega}}$$

Still Describing Waves

4.) *wavelength* (λ) - the shortest distance between points where the wave pattern repeats. (measured in meters)

5.) *wave number* (k) $\quad k = \frac{2\pi}{\lambda} \left(\frac{\text{rad}}{\text{m}}\right)$

6.) *amplitude* (A) - is the maximum displacement from the rest or equilibrium position. It is an indication of the intensity of the wave.

7.) *velocity* (v) - the speed of the traveling wave (m/s).

$$v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k} \qquad \boxed{\lambda = \frac{v}{f}}$$

Wave Equation

For sinusoidal waves a traveling wave can be described by the following equations:

1.) For waves traveling in the positive *x*-direction:

$$y(x,t) = A\sin(kx - \omega t)$$

2.) For waves traveling in the negative *x*-direction:

$$y(x,t) = A\sin(kx + \omega t)$$

Wave Interference

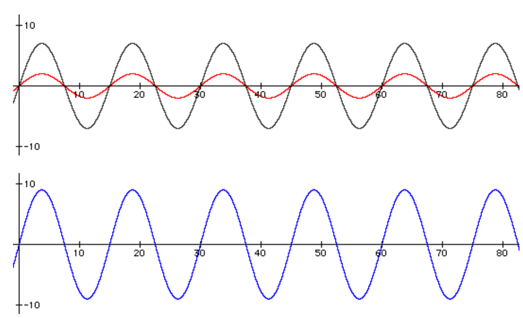
Principle of Superposition **- the displacement of the medium by two or more waves is the *sum* of the displacements caused by the individual waves.**

The resultant of the superposition of two or more waves is called *interference*.

Constructive Wave Interference

Constructive interference **occurs when the wave displacements are in the *same direction*, resulting in a wave with a larger amplitude than the individual components.**

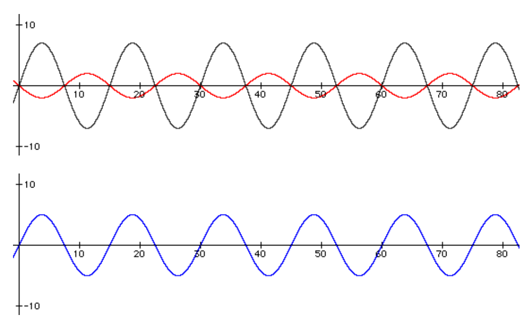
Constructive Wave Interference



Destructive Wave Interference

Destructive interference **occurs when the wave displacements are in the *opposite directions*, resulting in a wave with a smaller amplitude than the individual components.**

Destructive Wave Interference



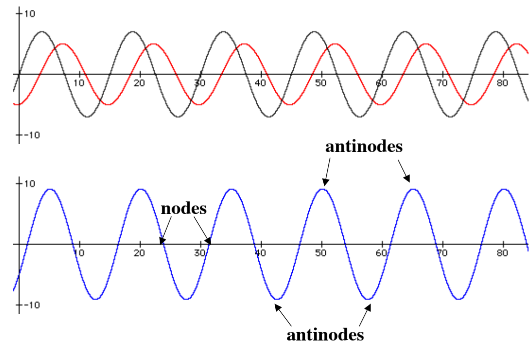
Wave Interference

In general, the interference of two or more waves is a combination of constructive and destructive interference.

A *node* occurs when there is complete destructive interference.

An *antinode* occurs at the point of maximum constructive interference.

Wave Interference



Wave Interactions with Medium Changes

When waves encounter medium changes, the *frequency does not change*.

The *velocity changes* and as a consequence the *wavelength changes*.

Reflection of Waves

***Reflection* occurs when a wave is incident upon a medium change.**

When a pulse encounters a *more dense medium*, the reflected pulse is *inverted*.

When a pulse encounters a *less dense medium*, the reflected pulse is *not inverted*.

Standing Waves

Standing waves are the result of the interference of 2 waves traveling in opposite directions with the same amplitude, wavelength, and frequency.

***Standing waves* have stationary nodes and antinodes.**

Standing Waves

Consider the interference of 2 waves moving in opposite directions. The equations are:

$$y_1 = A\sin(kx - \omega t) \quad \text{and} \quad y_2 = A\sin(kx + \omega t)$$

The interference is the sum of these two waves:

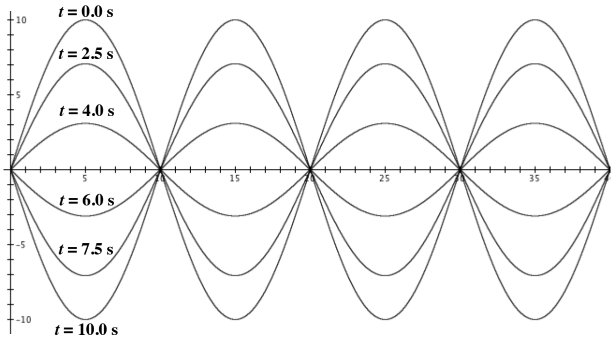
$$y_{sw} = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$y_{sw} = A\sin(kx)\cos(\omega t) - A\cos(kx)\sin(\omega t) + A\sin(kx)\cos(\omega t) + A\cos(kx)\sin(\omega t)$$

$$y_{sw} = 2A\sin(kx)\cos(\omega t)$$

Standing Waves

$$y_{sw} = 10\sin(0.3142x)\cos(0.3142t)$$



Node Locations on Standing Waves

$$y_{sw} = 2A\sin(kx)\cos(\omega t)$$

***Nodes* occur when $\sin(kx) = 0$.**

$$kx = n\pi \;\; ; \;\; n = 0, 1, 2, 3, ...$$

Spacial locations are: $x = \dfrac{n\pi}{k}$

and since $k = \dfrac{2\pi}{\lambda}$ **it follows that** $x = n\dfrac{\lambda}{2}$

NODE locations are: $x = 0,\ \dfrac{\lambda}{2},\ \lambda,\ \dfrac{3\lambda}{2},\ 2\lambda,\ ...$

Antinode Locations on Standing Waves

$$y_{sw} = 2A\sin(kx)\cos(\omega t)$$

***Antinodes* occur when** $\sin(kx) = \pm 1$

$$kx = \left(n + \tfrac{1}{2}\right)\pi \;\; : \;\; n = 0,\ 1,\ 2,\ 3,\ ...$$

Spacial locations are: $x = \left(n + \tfrac{1}{2}\right)\dfrac{\pi}{k}$

and since $k = \dfrac{2\pi}{\lambda}$ **it follows that** $x = \left(n + \tfrac{1}{2}\right)\dfrac{\lambda}{2}$

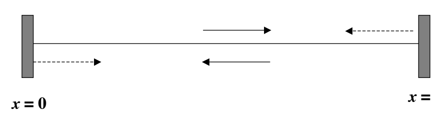
ANTINODE locations are: $x = \dfrac{\lambda}{4},\ \dfrac{3\lambda}{4},\ \dfrac{5\lambda}{4},\ \dfrac{7\lambda}{4},\ ...$

Waves on Strings

Consider a string of length L fixed at both ends.

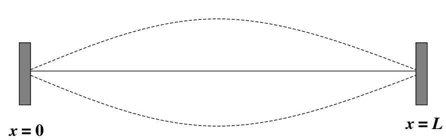
A *standing wave pattern forms* due to interference of incident and reflected waves from the fixed ends of the strings.

Because the ends are fixed, *nodes* must be present at these locations.



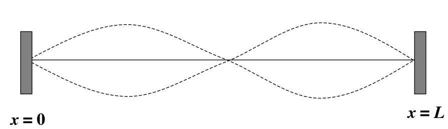
Normal Modes of a String

$\lambda_1 = 2L$



fundamental, first harmonic

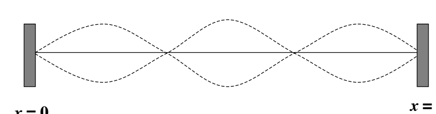
$\lambda_2 = L$



second harmonic

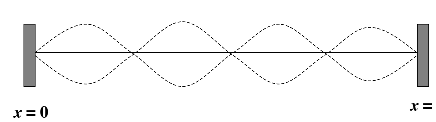
Normal Modes of a String

$\lambda_3 = \dfrac{2L}{3}$



third harmonic

$\lambda_4 = \dfrac{L}{2}$



fourth harmonic

Waves on Strings

The standing wave patterns generated occur for waves of certain wavelengths that result in nodes at $x = 0$ and $x = L$.

Therefore, the allowed wavelengths are restricted to:

$$\lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots)$$

$$\text{or} \quad \lambda_n = 2L, L, \frac{2L}{3}, \frac{L}{2}, \frac{2L}{5}, \ldots$$

Frequencies of String Modes

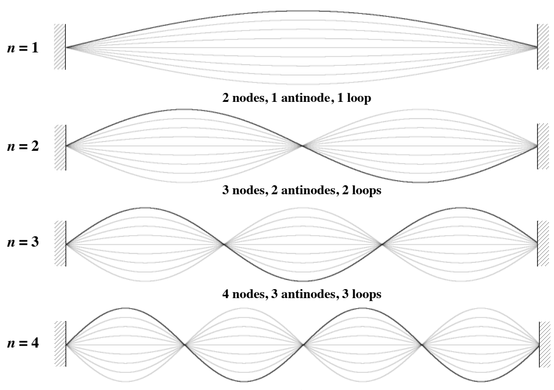
The frequency f of these modes depends upon the wave velocity.

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1$$

The lowest mode is when $n = 1$ and is called the *fundamental frequency*. All other modes are integral multiples of the fundamental frequency.

$$f_n = f_1,\ 2f_1,\ 3f_1,\ 4f_1, \ldots \quad \text{(harmonic overtones)}$$

Normal Modes of a String



Nodes, Anitnodes, and Loops

The n^{th} mode has $n + 1$ nodes.

The n^{th} mode has n antinodes.

Antinodes are also referred to as *loops*.

Velocity of Waves on Strings

The *velocity* of a wave on a string depends upon the tension T and the linear density μ of the string.

$$v = \sqrt{\frac{T}{\mu}} \qquad \left(\mu = \frac{m}{L}\right)$$

where:

T = Tension of string (N)

μ = Linear density of string (kg/m)