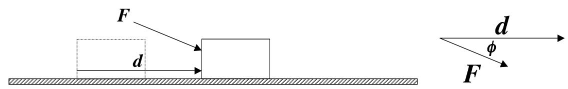
Work and Energy

Work

Work (*W*) the product of the force exerted on an object and the distance the object moves in the direction of the force (constant force only).

$$W = \vec{F} \cdot \vec{d} = Fd\cos\phi \qquad (\mathrm{N \cdot m = J})$$

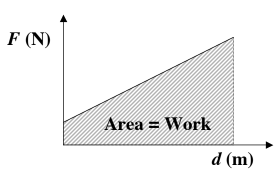
ϕ is the angle between the force and the direction of motion.



Work

Work is done on an object only if the object moves and is accelerating or changing height.

If the force is not constant then a force-displacement graph can be used to determine the work done. The *area under a force-displacement graph is the work done*.



Energy

Energy (*E*) the ability to do work.

Kinetic Energy (K) - the ability of an object to do work because of its motion.

$$W_{net} = F\Delta x = ma\Delta x$$

$$v_f^2 = v_i^2 + 2a\Delta x \quad \text{or} \quad a\Delta x = \frac{v_f^2 - v_i^2}{2}$$

$$W_{net} = m\left(\frac{v_f^2 - v_i^2}{2}\right)$$

$$W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

Kinetic Energy

The last equation gives us the definition of Kinetic Energy (*K*)

$$W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\boxed{K = \tfrac{1}{2}mv^2}$$

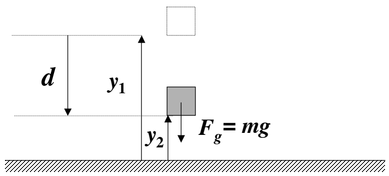
Work-Energy Theorem

The *net* work done by a *net* force acting on an object is equal to the change in the kinetic energy of the object.

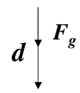
$$W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = K_f - K_i$$

$$\boxed{W_{net} = \Delta K}$$

Gravity and Work



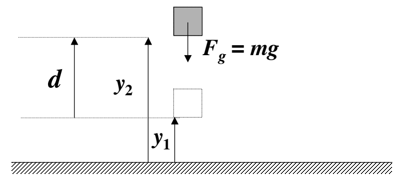
$$W_g = F_g d\cos\phi_g = mgd\cos 0$$
$$= mgd$$



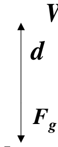
$\phi_g = 0$

Gravity does positive work when things fall.

Gravity and Work



$$W_g = F_g d\cos\phi_g = mgd\cos 180°$$
$$= -mgd$$

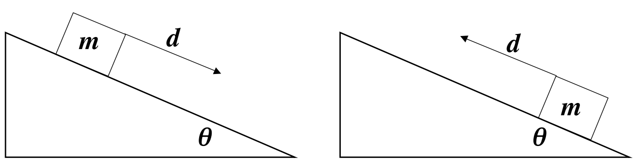


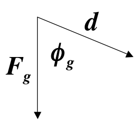
$\phi_g = 180°$

Gravity does negative work when things rise.

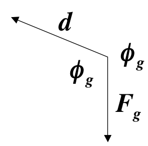
Inclined Planes

On inclined planes, the work done by gravity depends upon the angle of the incline θ and the distance the object moves along the incline d.





$\phi_g = 90° - \theta$



$\phi_g = 90° + \theta$

Gravitational Potential Energy

Potential Energy (U) **- the ability of an object to do work because of its position.**

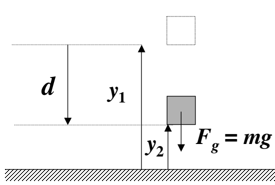
Gravitational Potential Energy (U_g) **- the ability of an object to do work because of its position in a gravitational field.**

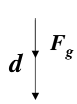
$$U_g = F_g y = mgy$$

Moving through a vertical distance Δy, the change in the gravitational potential energy for a mass (m) is:

$$\Delta U_g = mg\Delta y$$

Gravitational Potential Energy and Work





$\phi_g = 0$

$$W_g = F_g d\cos\phi_g = mgd\cos 0$$
$$= mgd = mg(y_1 - y_2)$$
$$= -mg(y_2 - y_1)$$
$$= -(mgy_2 - mgy_1)$$
$$= -(U_{g_2} - U_{g_1}) = -\Delta U_g$$

Work-Energy Theorem

$$W_{net} = \Delta K$$
$$W_g = \Delta K$$
$$-\Delta U_g = \Delta K$$
$$-(U_{g_f} - U_{g_i}) = K_f - K_i$$
$$-U_{g_f} + U_{g_i} = K_f - K_i$$
$$K_i + U_{g_i} = K_f + U_{g_f}$$

Conservative versus Nonconservative Forces

A *conservative force* is one in which the work done on an object from one point to another does not depend upon the path taken by the object. For conservative forces:

$$\boxed{W_F = -\Delta U_F} \text{ and } \boxed{\Delta K = -\Delta U_F}$$

A *nonconservative force* is one in which the work done on an object from one point to another does depend upon the path taken by the object. Friction and applied forces are examples of a nonconservative forces.

Conservation of Mechanical Energy

The sum of an object's kinetic and potential energies is called the *total mechanical energy E*. Assuming only conservative forces are involved, the total mechanical energy E is:

$$E = K + U$$

If there are no nonconservative forces acting on the system, the initial mechanical energy E_i is equal to the final mechanical energy E_f and:

$$K_i + U_i = K_f + U_f$$

This is a statement of *Conservation of Mechanical Energy*.

$$\text{Note: } \Delta E = 0 = \Delta K + \Delta U \text{ and } \boxed{\Delta K = -\Delta U}$$

Conservation of Energy

If there is also work done by nonconservative forces such as friction and applied forces:

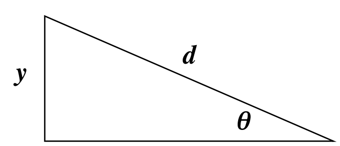
$$\boxed{K_i + U_i + W_{other} = K_f + U_f}$$

W_{other} accounts for the work done by each nonconservative force.

This is a statement of *Conservation of Energy*.

Inclined Planes

On inclined planes, the change in gravitational potential energy depends up the vertical displacement y while moving a distance d along the incline.



$$y = d\sin\theta$$

Power

Power (*P*) - the rate of at which energy is transformed.

$$\boxed{P = \frac{\Delta E}{\Delta t}}$$

$$\frac{\text{J}}{\text{s}} = \text{Watt (W) and 1 hp} = 746 \text{ W}$$

Power also measures the rate of at which energy is transformed as work.

$$\boxed{P = \frac{W}{\Delta t}}$$

Power

$$P = \frac{W}{\Delta t} \quad \text{and} \quad W = Fd = F\Delta x$$

$$P = \frac{F\Delta x}{\Delta t} = F\frac{\Delta x}{\Delta t}$$

$$P = Fv$$

Power

When a force F acts on an object moving with velocity v, the instantaneous power or rate at which the force does work is:

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi$$

Force and Springs

The force required to stretch a spring beyond its unstretched length by an amount x is

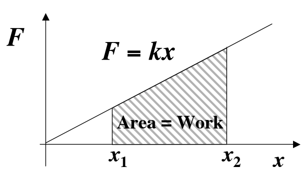
$$F = kx \quad \text{(Hooke's Law)}$$

where k is a constant called the *force constant* (or *spring constant*) of the spring.

Hooke's Law holds for compression as well as stretching, but the force and displacement are in opposite directions.

Work and Springs

To stretch a spring requires work.



$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$

Elastic Potential Energy

Elastic Energy ($U_{elastic}$) **- the potential energy stored in any compressed or stretched object such as a spring.**

$$U_{elastic} = \tfrac{1}{2}kx^2$$