Projectile Motion

Equations of Motion for Projectiles

For *projectiles* (cannon balls, bullets, footballs) the motion of the object in the horizontal and vertical direction are independent of one another.

Therefore, the motion of projectile can be described by separate equations of motion for the *x* and *y* directions.

Projectile Motion in Two Dimensions

Horizontal Motion (*x*-direction)

No acceleration in the *x*-direction so there is constant velocity and the equations of motion become

$$x = v_{x_0} t + x_0 \quad (x\text{ - position})$$

$$v_x = v_{x_0} \quad (x\text{ - velocity})$$

Projectile Motion in Two Dimensions

Vertical Motion (*y*-direction)

There is gravitational acceleration in the *y*-direction so the equations of motion are those for uniform acceleration

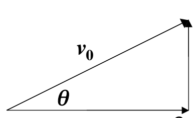
$$y = -\tfrac{1}{2} gt^2 + v_{y_0} t + y_0 \quad (y\text{ - position})$$

$$v_y = -gt + v_{y_0} \quad (y\text{ - velocity})$$

$$\textit{On Earth:} \quad g = 9.80 \ \tfrac{\text{m}}{\text{s}^2}$$

Determining Initial Velocity Components

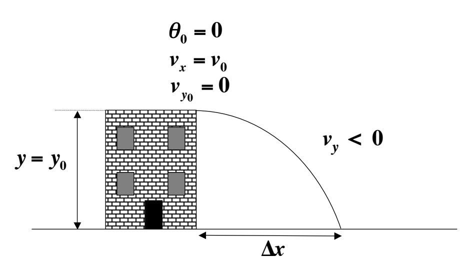
The initial velocities in the *x* and *y* directions are found from the initial velocity of the object and the angle at which the object is launched.



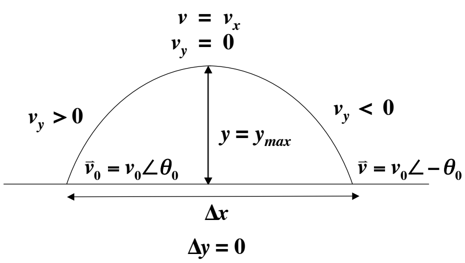
$$v_{y_0} = v_0 \sin\theta$$

$$v_{x_0} = v_0 \cos\theta$$

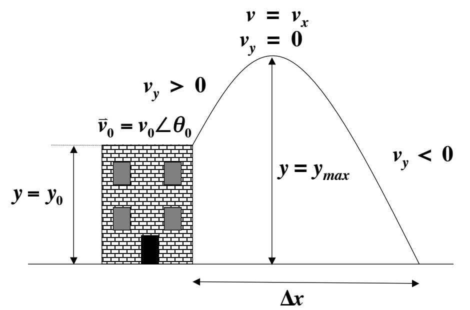
Horizontally Launched Projectile



Ground-to-Ground Projectile

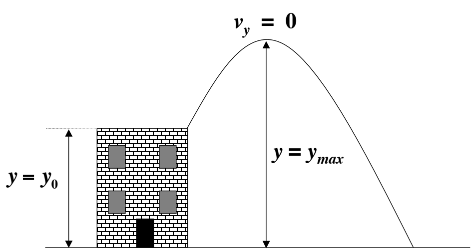


Projectile Launched From a Height at an Angle



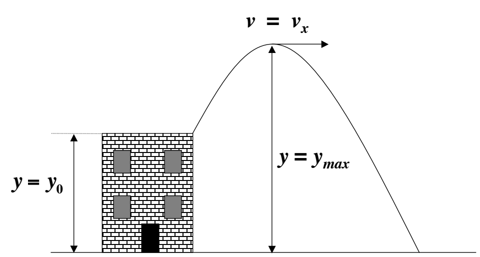
Maximum Height of a Projectile

When a projectile reaches its maximum height, the vertical (*y*) velocity is zero.



Maximum Height of a Projectile

At the maximum height the speed is equal to the *x*-component of the initial velocity.



More Equations for Projectiles

Because projectiles are uniformly accelerating in the *y*-direction:

$$y = \left(\frac{v_{y_0} + v_y}{2}\right)t + y_0$$

$$v_y^2 = v_{y_0}^2 - 2g\left(y - y_0\right)$$

Circular Motion

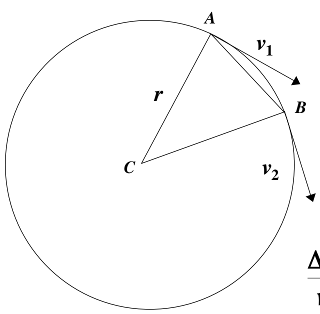
Uniform Circular Motion

An object that moves in a circle at constant speed is said to experience *uniform circular motion*.

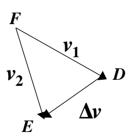
- **The magnitude of the velocity remains constant.**
- **The direction of the velocity is continuously changing as the object moves around the circle.**
- **The object is accelerating because there is a change in velocity.**

This acceleration is called *centripetal acceleration* and it points towards the center of the circle.

Centripetal Acceleration



$$\Delta DEF \approx \Delta ABC$$



$$\Delta v = v_2 - v_1 \text{ or } v_2 = \Delta v + v_1$$

$$a_{rad} = \frac{\Delta v}{\Delta t}$$

$$\frac{\Delta v}{v} = \frac{AB}{r} \text{ and } AB = d = v \cdot \Delta t$$

$$\text{so } \frac{\Delta v}{v} = \frac{v \cdot \Delta t}{r} \text{ and } \frac{\Delta v}{\Delta t} = \frac{v^2 \cdot \Delta t}{r \cdot \Delta t} = \boxed{\frac{v^2}{r} = a_c = a_{rad}}$$

Centripetal Acceleration

$$a_{rad} = \frac{v^2}{r}$$

In terms of the period of revolution (*T*)

$$v = \frac{2\pi \cdot r}{T} \text{ and } a_{rad} = \frac{\left(\frac{2\pi \cdot r}{T}\right)^2}{r} = \frac{4\pi^2 \cdot r}{T^2}$$

$$a_{rad} = \frac{4\pi^2 \cdot r}{T^2}$$

Non-Uniform Circular Motion

If the *speed also varies*, there is a *tangential component* to the acceleration in addition to the radial component.

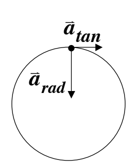
The tangential component is parallel to the path of motion.

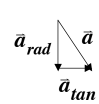
$$a_{tan} = \frac{d|\vec{v}|}{dt}$$

Non-Uniform Circular Motion

The *vector acceleration* of a particle moving in a circle with varying speed is the *vector sum of the radial and tangential components* of acceleration.

$$\vec{a} = \vec{a}_{rad} + \vec{a}_{tan}$$





$$a = \sqrt{a_{rad}^2 + a_{tan}^2}$$