Example 1

An 18-gauge copper wire has a nominal diameter of 1.02 mm. This wire carries a constant current of 1.67 A to a 200 W lamp. The density of free electrons is 8.5 x 10^{28} electrons per cubic meter. Find the magnitude of

a.) the current density

b.) the drift velocity

Example 1: $D = 1.2$ mm, $I = 1.67$ A, and $N = 8.5 \times 10^{28}\ \frac{\text{electrons}}{\text{m}^3}$

a.) $J = ?$

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{1.67\ \text{A}}{\pi\left(0.51 \times 10^{-3}\ \text{m}\right)^2}$$

$$\boxed{J = 2.04 \times 10^6\ \frac{\text{A}}{\text{m}^2}}$$

b.) $v_d = ?$

$$I = Nev_d A$$

$$\frac{I}{A} = Nev_d = J \quad \text{so} \quad v_d = \frac{J}{Ne}$$

$$v_d = \frac{2.04 \times 10^6\ \frac{\text{A}}{\text{m}^2}}{\left(8.5 \times 10^{28}\ \frac{\text{electrons}}{\text{m}^3}\right)\left(1.6 \times 10^{-19}\ \frac{\text{C}}{\text{electron}}\right)}$$

$$\boxed{v_d = 1.5 \times 10^{-4}\ \frac{\text{m}}{\text{s}}}$$

Example 2

A 50 m long copper wire ($\rho = 1.72 \times 10^{-8}\ \Omega\cdot$m) has a diameter of 1.0 mm and is connected to a 6 V battery with an internal resistance of 0.5 Ω. Find:

a.) the resistance of the wire.

b.) the current through the wire.

c.) the terminal voltage of the battery.

Example 2:

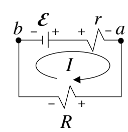
$\ell = 50$ m, $D = 1.0$ mm, $\mathcal{E} = 6$ V, $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$, and $r = 0.5\ \Omega$

a.) $R = ?$

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2} = \frac{\left(1.72 \times 10^{-8}\ \Omega \cdot \text{m}\right)(50\ \text{m})}{\pi\left(0.5 \times 10^{-3}\ \text{m}\right)^2}$$

$$\boxed{R = 1.1\ \Omega}$$

b.) $I = ?$



$$\mathcal{E} = Ir + IR = I(r + R)$$

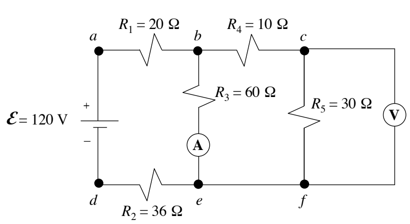
$$I = \frac{\mathcal{E}}{r + R} = \frac{6\ \text{V}}{0.5\ \Omega + 1.1\ \Omega}$$

$$\boxed{I = 3.75\ \text{A}}$$

c.) $V_{ab} = ?$

$$V_{ab} = \mathcal{E} - Ir = 6\ \text{V} - (3.75\ \text{A})(0.5\ \Omega)$$

$$\boxed{V_{ab} = 4.125\ \text{V}}$$



Example 3

For the circuit shown above:

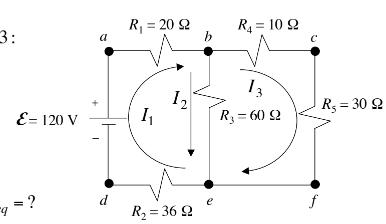
a.) Find the equivalent resistance.

b.) The current and voltage for each resistor.

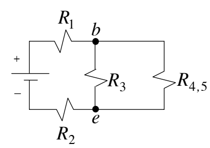
c.) The readings on the ammeter and on the voltmeter.

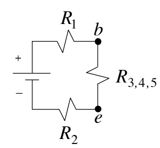
d.) The voltages V_{ae} and V_{db}.

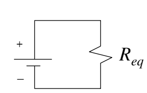
Example 3:



a.) $R_{eq} = ?$







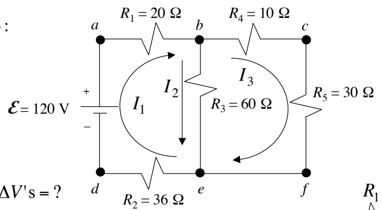
$$R_{4,5} = R_4 + R_5$$
$$R_{4,5} = 10\ \Omega + 30\ \Omega$$
$$R_{4,5} = 40\ \Omega$$

$$R_{3,4,5} = \left(\frac{1}{R_3} + \frac{1}{R_{4,5}}\right)^{-1}$$

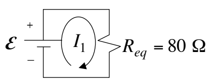
$$R_{3,4,5} = \left(\frac{1}{60\ \Omega} + \frac{1}{40\ \Omega}\right)^{-1} = 24\ \Omega$$

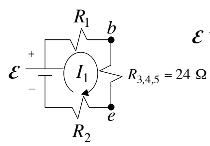
$$R_{eq} = R_1 + R_2 + R_{3,4,5}$$
$$R_{eq} = 20\ \Omega + 36\ \Omega + 24\ \Omega$$
$$\boxed{R_{eq} = 80\ \Omega}$$

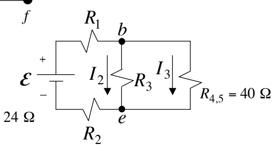
Example 3:



b.) I's and ΔV's = ?





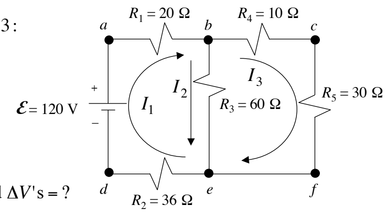


$$I_1 = \frac{\mathcal{E}}{R_{eq}}$$
$$I_1 = \frac{120\text{ V}}{80\ \Omega}$$
$$I_1 = 1.5\text{ A}$$

$$V_{be} = I_1 R_{3,4,5}$$
$$V_{be} = (1.5\text{ A})(24\ \Omega)$$
$$V_{be} = 36\text{ V}$$

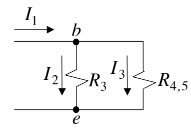
$$\Delta V_3 = \Delta V_{4,5} = V_{be} = 36\text{ V}$$
$$I_2 = \frac{\Delta V_3}{R_3} = \frac{36\text{ V}}{60\ \Omega} = 0.6\text{ A}$$
$$I_3 = \frac{\Delta V_{4,5}}{R_{4,5}} = \frac{36\text{ V}}{40\ \Omega} = 0.9\text{ A}$$

Example 3:



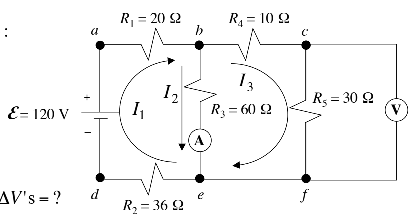
b.) I's and ΔV's = ?

alternatively (Current Divider):



$$I_2 = I_1 \frac{R_{4,5}}{R_3 + R_{4,5}} = 1.5\text{ A}\frac{40\ \Omega}{60\ \Omega + 40\ \Omega} = 0.6\text{ A}$$
$$I_3 = I_1 \frac{R_3}{R_3 + R_{4,5}} = 1.5\text{ A}\frac{60\ \Omega}{60\ \Omega + 40\ \Omega} = 0.9\text{ A}$$

Example 3:



$I_1 = 1.5$ A
$I_2 = 0.6$ A
$I_3 = 0.9$ A

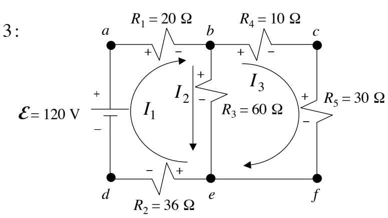
b.) I's and ΔV's = ?

Resistor	Current	Voltage ($\Delta V = IR$)
$R_1 = 20\ \Omega$	$I_1 = 1.5$ A	$\Delta V_1 = 30$ V
$R_2 = 36\ \Omega$	$I_1 = 1.5$ A	$\Delta V_2 = 54$ V
$R_3 = 60\ \Omega$	$I_2 = 0.6$ A	$\Delta V_3 = 36$ V
$R_4 = 10\ \Omega$	$I_3 = 0.9$ A	$\Delta V_4 = 9$ V
$R_5 = 30\ \Omega$	$I_3 = 0.9$ A	$\Delta V_5 = 27$ V

c.) Meter readings?

Ammeter reads current $I_2 = 0.6$ A
Voltmeter reads voltage $V_5 = 27$ V

Example 3:



$\Delta V_1 = 30$ V
$\Delta V_2 = 54$ V
$\Delta V_3 = 36$ V
$\Delta V_4 = 9$ V
$\Delta V_5 = 27$ V

d.) V_{ae} = ? and V_{db} = ?

$$V_{ae} = V_a - V_e$$

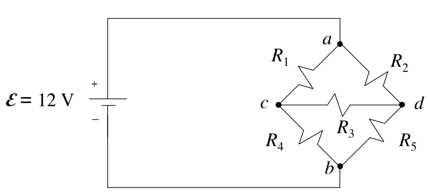
Using point e as a reference $(V_e = 0)$

$$V_d = V_e - \Delta V_2 = 0 - 54\text{ V} = -54\text{ V}$$
$$V_a = V_d + \mathcal{E} = -54\text{ V} + 120\text{ V} = 66\text{ V}$$
$$V_{ae} = V_a - V_e = 66\text{ V} - 0$$
$$\boxed{V_{ae} = 66\text{ V}}$$

$$V_{db} = V_d - V_b$$

Using point b as a reference $(V_b = 0)$

$$V_e = V_b - \Delta V_3 = 0 - 36\text{ V} = -36\text{ V}$$
$$V_d = V_e - \Delta V_2 = -36\text{ V} - 54\text{ V} = -90\text{ V}$$
$$V_{db} = V_d - V_b = -90\text{ V} - 0$$
$$\boxed{V_{db} = -90\text{ V}}$$



$\mathcal{E} = 12$ V

Example 4

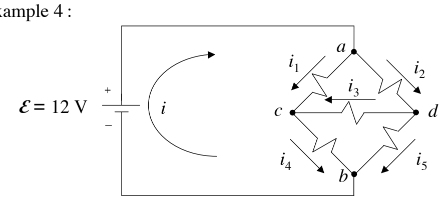
In the circuit shown above: $R_1 = 6\ \Omega$, $R_2 = 3\ \Omega$, $R_4 = R_5 = 4\ \Omega$, and $R_3 = 2\ \Omega$.

a.) Find the equivalent resistance.

b.) The current and voltage for each resistor.

c.) Suppose R_5 is variable and can have any resistance. What value will result in no current passing through R_3?

Example 4:



Loop Rule Equations	Junction Rule Equations
$\mathcal{E} = i_1R_1 + i_4R_4$	$i = i_1 + i_2$ at a
$0 = -i_1R_1 + i_2R_2 + i_3R_3$	$i_2 = i_3 + i_5$ at d
$0 = -i_3R_3 - i_4R_4 + i_5R_5$	$i_1 + i_3 = i_4$ at c

Example 4 :

$$\mathcal{E} = i_1 R_1 + i_4 R_4 \qquad i = i_1 + i_2 \text{ at } a$$

$$0 = -i_1 R_1 + i_2 R_2 + i_3 R_3 \qquad i_2 = i_3 + i_5 \text{ at } d$$

$$0 = -i_3 R_3 - i_4 R_4 + i_5 R_5 \qquad i_1 + i_3 = i_4 \text{ at } c$$

$$\begin{pmatrix} 0 & R_1 & 0 & 0 & R_4 & 0 \\ 0 & -R_1 & R_2 & R_3 & 0 & 0 \\ 0 & 0 & 0 & -R_3 & -R_4 & R_5 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} i \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{pmatrix} = \begin{pmatrix} \mathcal{E} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

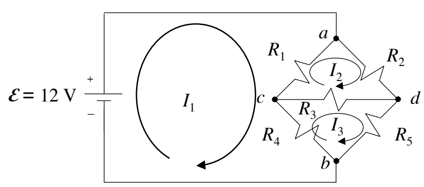
Example 4 :

$$\begin{pmatrix} 0 & 6 & 0 & 0 & 4 & 0 \\ 0 & -6 & 3 & 2 & 0 & 0 \\ 0 & 0 & 0 & -2 & -4 & 4 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} i \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} i \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{pmatrix} = \begin{pmatrix} 2.97\text{ A} \\ 1.06\text{ A} \\ 1.91\text{ A} \\ 0.34\text{ A} \\ 1.40\text{ A} \\ 1.57\text{ A} \end{pmatrix}$$

Example 4 :

Loop Method



Loop Equations

$$\mathcal{E} = (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3$$

$$0 = -R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3$$

$$0 = -R_4 I_1 - R_3 I_2 + (R_3 + R_4 + R_5) I_3$$

Example 4 :

Loop Method

$$\mathcal{E} = (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3$$

$$0 = -R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3$$

$$0 = -R_4 I_1 - R_3 I_2 + (R_3 + R_4 + R_5) I_3$$

$$\begin{pmatrix} R_1 + R_4 & -R_1 & -R_4 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \\ -R_4 & -R & R_3 + R_4 + R_5 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} \mathcal{E} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 10\ \Omega & -6\ \Omega & -4\ \Omega \\ -6\ \Omega & 11\ \Omega & -2\ \Omega \\ -4\ \Omega & -2\ \Omega & 10\ \Omega \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12\text{ V} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 2.97\text{ A} \\ 1.91\text{ A} \\ 1.57\text{ A} \end{pmatrix}$$

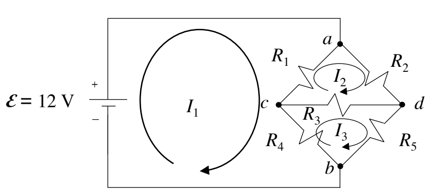
Example 4 :

a.) $R_{eq} = ?$

$$R_{eq} = \frac{\mathcal{E}}{I_1} = \frac{12\text{ V}}{2.97\text{ A}} = \boxed{4.04\ \Omega}$$

b.) I's and ΔV's = ?

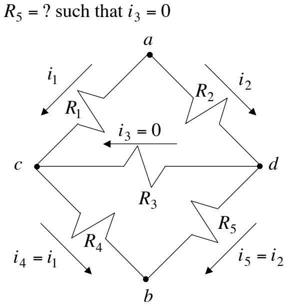
$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 2.97\text{ A} \\ 1.91\text{ A} \\ 1.57\text{ A} \end{pmatrix}$$



Resistor	Current	Voltage
$R_1 = 6\ \Omega$	$i_1 = I_1 - I_2 =$ 1.06 A	$\Delta V_1 = 6.36$ V
$R_2 = 3\ \Omega$	$i_2 = I_2 =$ 1.91 A	$\Delta V_2 = 5.73$ V
$R_3 = 2\ \Omega$	$i_3 = I_2 - I_3 =$ 0.34 A	$\Delta V_3 = 0.68$ V
$R_4 = 4\ \Omega$	$i_4 = I_1 - I_3 =$ 1.4 A	$\Delta V_4 = 5.6$ V
$R_5 = 4\ \Omega$	$i_5 = I_3 =$ 1.57 A	$\Delta V_5 = 6.28$ V

Example 4 :

c.) $R_5 = ?$ such that $i_3 = 0$



$R_1 = 6\ \Omega$

$R_2 = 3\ \Omega$

$R_3 = 2\ \Omega$

$R_4 = 4\ \Omega$

$R_5 = ?$

$$V_c = V_d$$

$$V_{ac} = V_{ad}$$

$$i_1 R_1 = i_2 R_2$$

$$i_1 = i_2 \frac{R_2}{R_1}$$

$$V_{cb} = V_{db}$$

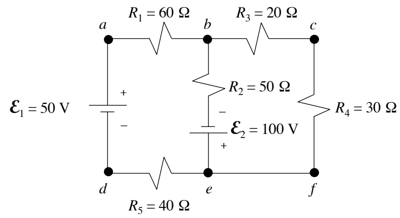
$$i_4 R_4 = i_5 R_5$$

$$i_1 R_4 = i_2 R_5$$

$$i_2 \frac{R_2}{R_1} R_4 = i_2 R_5$$

$$R_5 = \frac{R_2 R_4}{R_1} = \frac{(3\ \Omega)(4\ \Omega)}{6\ \Omega}$$

$$\boxed{R_5 = 2\ \Omega}$$



Example 5

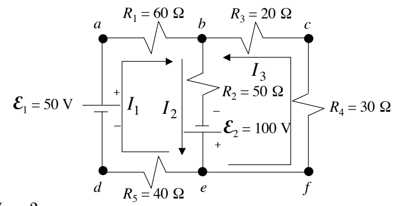
For the circuit shown above:

a.) Find the current in each battery.

b.) The amount of energy dissipated in the circuit in 15 minutes.

c.) The voltages V_{ae} and V_{fb}.

Example 5 :



a.) $I_1 = ?$ and $I_2 = ?$

$$@ \text{ junction } b: (1)\ I_1 + I_3 = I_2$$

$$\text{around loop } abed: (2)\ \mathcal{E}_1 + \mathcal{E}_2 = I_1(R_1 + R_5) + I_2 R_2$$

$$\text{around loop } bcfe: (3)\ \mathcal{E}_2 = I_2 R_2 + I_3(R_3 + R_4)$$

$$(1 \text{ and } 2)\ \mathcal{E}_1 + \mathcal{E}_2 = I_1(R_1 + R_5) + (I_1 + I_3)R_2 = I_1(R_1 + R_2 + R_5) + I_3 R_2$$

$$150\text{ V} = I_1(150\ \Omega) + I_3(50\ \Omega)$$

$$(1 \text{ and } 3)\ \mathcal{E}_2 = (I_1 + I_3)R_2 + I_3(R_3 + R_4) = I_1 R_2 + I_3(R_2 + R_3 + R_4)$$

$$100\text{ V} = I_1(50\ \Omega) + I_3(100\ \Omega)$$

Example 5 :

a.) $I_1 = ?$ and $I_2 = ?$

$$(150\text{ V} = I_1(150\ \Omega) + I_3(50\ \Omega))2$$

$$300\text{ V} = I_1(300\ \Omega) + I_3(100\ \Omega)$$

$$-(100\text{ V} = I_1(50\ \Omega) + I_3(100\ \Omega))$$

$$200\text{ V} = I_1(250\ \Omega)$$

$$I_1 = \frac{200\text{ V}}{250\ \Omega}$$

$$\boxed{I_1 = 0.8\text{ A}}$$

$$300\text{ V} = I_1(300\ \Omega) + I_3(100\ \Omega)$$

$$I_3 = \frac{300\text{ V} - I_1(300\ \Omega)}{100\ \Omega}$$

$$I_3 = \frac{300\text{ V} - (0.8\text{ A})(300\ \Omega)}{100\ \Omega}$$

$$I_3 = 0.6\text{ A}$$

$$I_2 = I_1 + I_3 = 0.8\text{ A} + 0.6\text{ A}$$

$$\boxed{I_2 = 1.4\text{ A}}$$

b.) $t = 15$ min, $E = ?$

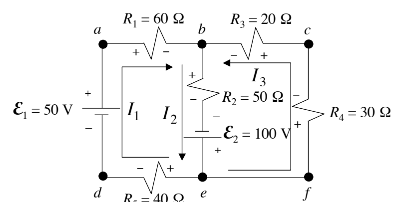
$$E = Pt = (P_1 + P_2)t$$

$$E = (\mathcal{E}_1 I_1 + \mathcal{E}_2 I_2)t$$

$$E = ((50\text{ V})(0.8\text{ A}) + (100\text{ V})(1.4\text{ A}))(3000\text{ s})$$

$$\boxed{E = 162{,}000\text{ J}}$$

Example 5 :



$I_1 = 0.8$ A

$I_2 = 1.4$ A

$I_3 = 0.6$ A

c.) $V_{ae} = ?$ and $V_{fb} = ?$

$$V_{ae} = V_a - V_e$$

Using point e as a reference $(V_e = 0)$

$$V_a = V_e - \Delta V_5 + \mathcal{E}_1 = V_e - I_1 R_5 + \mathcal{E}_1$$

$$V_a = 0 - (0.8\text{ A})(40\ \Omega) + 50\text{ V}$$

$$V_a = 18\text{ V}$$

$$V_{ae} = V_a - V_e = 18\text{ V} - 0$$

$$\boxed{V_{ae} = 18\text{ V}}$$

$$V_{fb} = V_f - V_b$$

Using point b as a reference $(V_b = 0)$

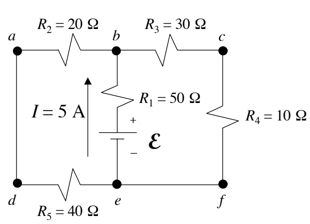
$$V_f = V_b + \Delta V_3 + \Delta V_4 = V_b + I_3 R_3 + I_3 R_4$$

$$V_f = 0 + (0.6\text{ A})(20\ \Omega) + (0.6\text{ A})(30\ \Omega)$$

$$V_f = 30\text{ V}$$

$$V_{fb} = V_f - V_b = 30\text{ V} - 0$$

$$\boxed{V_{fb} = 30\text{ V}}$$



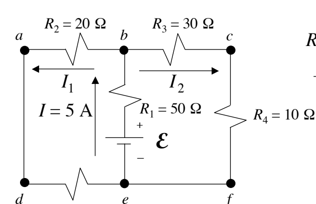
Example 6

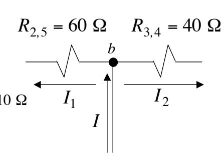
For the circuit shown above:

a.) Find the current through each resistor.

b.) Find the emf $\mathcal{E}$ of the battery.

Example 6 :





a.) $I_1 = ?$ and $I_2 = ?$

@ junction b: (current divider)

$$I_1 = I\frac{R_{3,4}}{R_{2,5} + R_{3,4}} = 5\text{ A}\frac{40\ \Omega}{60\ \Omega + 40\ \Omega}$$

$$\boxed{I_1 = 2\text{ A}}$$

$$I_2 = I\frac{R_{2,5}}{R_{2,5} + R_{3,4}} = 5\text{ A}\frac{60\ \Omega}{60\ \Omega + 40\ \Omega}$$

$$\boxed{I_2 = 3\text{ A}}$$

b.) $\mathcal{E} = ?$

around loop $ebad$: $\mathcal{E} = IR_1 + I_1(R_2 + R_5)$

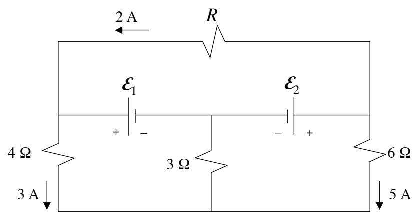
$$\mathcal{E} = (5\text{ A})(50\ \Omega) + (2\text{ A})(20\ \Omega + 40\ \Omega)$$

$$\boxed{\mathcal{E} = 370\text{ V}}$$

around loop $ebcf$: $\mathcal{E} = IR_1 + I_2(R_3 + R_4)$

$$\mathcal{E} = (5\text{ A})(50\ \Omega) + (3\text{ A})(30\ \Omega + 10\ \Omega)$$

$$\boxed{\mathcal{E} = 370\text{ V}}$$



Example 7

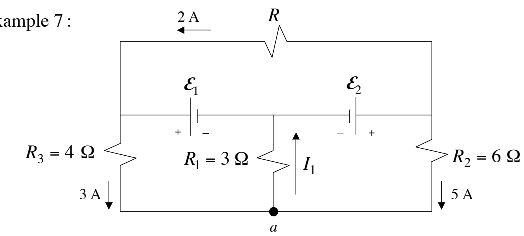
In the circuit shown above find

a.) the current in the 3 Ω resistor

b.) the unknown emf's $\mathcal{E}_1$ and $\mathcal{E}_2$

c.) the resistance R

Example 7 :



a.) current in R_1 = ?

@ junction a: $I_1 = 3\text{ A} + 5\text{ A}$

$\boxed{I_1 = 8\text{ A}}$

b.) $\mathcal{E}_1$ = ? and $\mathcal{E}_2$ = ?

$\mathcal{E}_1 = \Delta V_3 + \Delta V_1 = (3\text{ A})(4\ \Omega) + (8\text{ A})(3\ \Omega)$

$\boxed{\mathcal{E}_1 = 36\text{ V}}$

$\mathcal{E}_2 = \Delta V_2 + \Delta V_1 = (5\text{ A})(6\ \Omega) + (8\text{ A})(3\ \Omega)$

$\boxed{\mathcal{E}_2 = 54\text{ V}}$

c.) R = ?

$\Delta V_R = \mathcal{E}_2 - \mathcal{E}_1$

$I_R R = \mathcal{E}_2 - \mathcal{E}_1$

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{I_R} = \frac{54\text{ V} - 36\text{ V}}{2\text{A}}$$

$\boxed{R = 9\ \Omega}$

Example 8

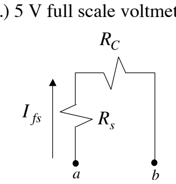
A galvanometer has a coil resistance of 500 Ω and a full scale current of 10 μA. Show how to make

a.) A voltmeter with a full scale reading of 5 V.

b.) An ammeter with a full scale reading of 5 mA.

Example 8 : $R_C = 500\ \Omega$ and $I_{fs} = 10\ \mu\text{A}$

a.) 5 V full scale voltmeter



$V_{ab} = 5\text{ V}$

$V_{ab} = \Delta V_{R_s} + \Delta V_{R_c}$

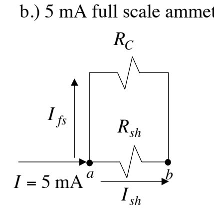
$V_{ab} = I_{fs}R_s + I_{fs}R_c$

$$R_s = \frac{V_{ab}}{I_{fs}} - R_c$$

$$R_s = \frac{5\text{ V}}{10\times10^{-6}\text{ A}} - 500\ \Omega$$

$\boxed{R_s = 499{,}500\ \Omega}$

b.) 5 mA full scale ammeter



$V_{ab} = \Delta V_{R_{sh}} = \Delta V_{R_c}$

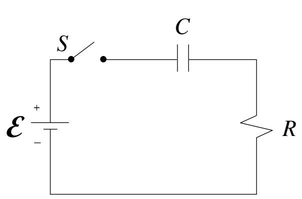
$I_{sh}R_{sh} = I_{fs}R_c$

$(I - I_{fs})R_{sh} = I_{fs}R_c$

$$R_{sh} = \frac{I_{fs}R_c}{(I - I_{fs})} = \frac{(10\times10^{-6}\text{ A})(500\ \Omega)}{(5\times10^{-3}\text{ A} - 10\times10^{-6}\text{ A})}$$

$\boxed{R_{sh} = 1.002\ \Omega}$

Example 9:



An ideal battery with an emf $\mathcal{E}$ = 100 V is connected to a resistor with resistance R = 100 Ω and an initially uncharged capacitor with capacitance C = 1 μF. The circuit is completed when switch S is closed at time t = 0.

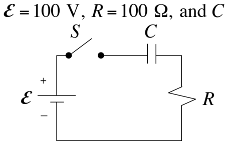
a.) Find the current through and voltage across each device immediately after the switch is closed.

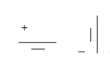
b.) A long time after the switch is closed, find the charge on the capacitor and the voltage across the resistor.

c.) What is the charge on the capacitor after 0.2 ms?

d.) Find the total energy dissipated in the resistor.

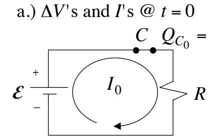
Example 9 : $\mathcal{E} = 100\text{ V}$, $R = 100\ \Omega$, and $C = 1\ \mu\text{F}$





a.) ΔV's and I's @ $t = 0$

$Q_{C_0} = 0$ and $\Delta V_{C_0} = 0$

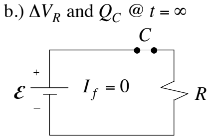


$\mathcal{E} = I_0 R$

$$I_0 = \frac{\mathcal{E}}{R} = \frac{100\text{ V}}{100\ \Omega} = 1\text{ A}$$

$\boxed{I_{R_0} = I_{C_0} = 1\text{ A},\ \Delta V_{R_0} = 100\text{ V},\ \Delta V_{C_0} = 0}$

b.) ΔV_R and Q_C @ $t = \infty$



$\Delta V_{R_f} = I_f R$

$\boxed{\Delta V_{R_f} = 0}$

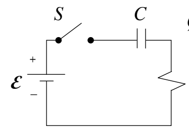
$\Delta V_{C_f} = \mathcal{E}$

$\Delta V_{C_f} = 100\text{ V}$

$Q_{C_f} = C\Delta V_{C_f} = (1\times10^{-6}\text{ F})(100\text{ V})$

$\boxed{Q_{C_f} = 100\ \mu\text{C}}$

Example 9 $\mathcal{E} = 100\ \text{V},\ R = 100\ \Omega,\ \text{and}\ C = 1\ \mu\text{F}$



$Q_{C_f} = \mathcal{E}C = 100\ \mu\text{C}$

c.) q_C @ $t = 0.2$ ms

$$q_C = Q_{C_f}\left(1 - e^{-t/\tau}\right)$$

$$\tau = RC = (100\ \Omega)\left((1\times10^{-6}\ \text{F})\right)$$

$$\tau = 0.1\ \text{ms}$$

$$q_C = (100\ \mu\text{C})\left(1 - e^{-(0.2\ \text{ms}/0.1\ \text{ms})}\right)$$

$$\boxed{q_C = 86.5\ \mu\text{C}}$$

d.) E_R from $t = 0$ to ∞

$$P = \frac{dE}{dt}\ \text{so}\ E = \int P dt$$

$$P = i^2R$$

$$q_C = Q_{C_f}\left(1 - e^{-t/\tau}\right) = \mathcal{E}C\left(1 - e^{-t/RC}\right)$$

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(\mathcal{E}C\left(1 - e^{-t/RC}\right)\right)$$

$$i = -\mathcal{E}Ce^{-t/RC}\left(\frac{-1}{RC}\right)$$

$$i = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0e^{-t/RC}$$

Example 9 : d.) E_R from $t = 0$ to ∞

$$E = \int P dt = \int_0^\infty i^2 R dt$$

$$E = \int_0^\infty \left(\frac{\mathcal{E}}{R}e^{-t/RC}\right)^2 R dt$$

$$E = \int_0^\infty \frac{\mathcal{E}^2}{R^2}e^{-2t/RC} R dt = \int_0^\infty \frac{\mathcal{E}^2}{R}e^{-2t/RC} dt$$

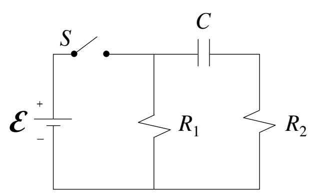
$$E = \frac{-\mathcal{E}^2C}{2}\int_0^\infty e^{-2t/RC}\left(\frac{-2}{RC}\right)dt$$

$$E = \frac{-\mathcal{E}^2C}{2}e^{-2t/RC}\Big]_0^\infty = \frac{-\mathcal{E}^2C}{2}\left(e^{-\infty} - e^{-0}\right)$$

$$E = \frac{-\mathcal{E}^2C}{2}(0 - 1) = \frac{\mathcal{E}^2C}{2} = \frac{1}{2}C\mathcal{E}^2$$

$$E = \frac{1}{2}(1\times10^{-6}\ \text{F})(100\ \text{V})^2$$

$$\boxed{E = 5\ \text{mJ}}$$



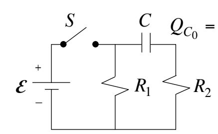
Example 10:

An ideal battery with an emf $\mathcal{E} = 12$ V, two resistors with resistances $R_1 = 4\ \Omega$ and $R_2 = 6\ \Omega$, and an initially uncharged capacitor with capacitance $C = 6\ \mu$F. The circuit is completed when switch S is closed at time $t = 0$.

a.) At time $t = 2\tau$, what is the potential difference across the capacitor?

b.) At time $t = 2\tau$, what are the potential differences across the two resistors? Do those potential differences increase, decrease, or remain the same while the capacitor is being charged?

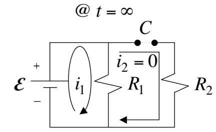
Example 10 : $\mathcal{E} = 12\ \text{V},\ R_1 = 4\ \Omega,\ R_2 = 6\ \Omega, \text{and}\ C = 6\ \mu\text{F}$



$Q_{C_0} = 0$ and $\Delta V_{C_0} = 0$

a.) @ $t = 2\tau$, $\Delta v_C = ?$

@ $t = \infty$



$$i_1 = \frac{\mathcal{E}}{R_1} = \frac{12\ \text{V}}{4\ \Omega} = 3\ \text{A}$$

$$\Delta V_{R_{1f}} = i_1R_1 = (3\ \text{A})(4\ \Omega) = 12\ \text{V}$$

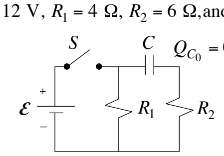
$$\Delta V_{R_{1f}} = \Delta V_{C_f} = 12\ \text{V}$$

$$\Delta v_C(t) = \Delta V_{C_f}\left(1 - e^{-t/\tau}\right)$$

$$\Delta v_C(2\tau) = (12\ \text{V})\left(1 - e^{-2}\right)$$

$$\boxed{\Delta v_C(2\tau) = 10.38\ \text{V}}$$

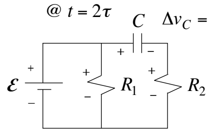
Example 10 : $\mathcal{E} = 12\ \text{V},\ R_1 = 4\ \Omega,\ R_2 = 6\ \Omega, \text{and}\ C = 6\ \mu\text{F}$



$Q_{C_0} = 0$ and $\Delta V_{C_0} = 0$

b.) @ $t = 2\tau$, $\Delta v_{R_1} = ?$ and $\Delta v_{R_2} = ?$

@ $t = 2\tau$



$\Delta v_C = 10.38$ V

$$\boxed{\Delta v_{R_1} = \mathcal{E} = 12\ \text{V}}$$

$$\Delta v_{R_1} - \Delta v_C - \Delta v_{R_2} = 0$$

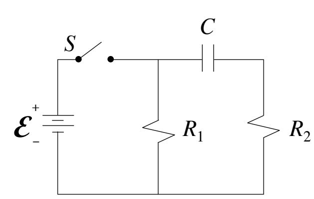
$$\Delta v_{R_2} = \Delta v_{R_1} - \Delta v_C$$

$$\Delta v_{R_2} = 12\ \text{V} - 10.38\ \text{V}$$

$$\boxed{\Delta v_{R_2} = 1.62\ \text{V}}$$

Δv_{R_1} is always 12 V

Δv_{R_2} is decreasing



Example 10: (continued)

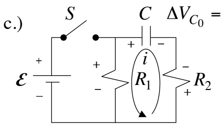
c.) A long time after the switch has been closed the switch is opened.

i.) Write an equation for the charge on the capacitor after the switch is opened.

ii.) Write an equation for the voltage on each resistor after the switch is opened.

Example 10 : $\mathcal{E} = 12\ \text{V},\ R_1 = 4\ \Omega,\ R_2 = 6\ \Omega, \text{and } C = 6\ \mu\text{F}$

c.)



$$\tau = (R_1 + R_2)C = (4\ \Omega + 6\ \Omega)(6 \times 10^{-6}\ \text{F})$$

$$\tau = 60\ \mu\text{s}$$

i.) $q_C(t) = ?$

$$q_C(t) = Q_{C_0} e^{-t/\tau}$$

$$Q_{C_0} = C\Delta V_{C_0} = (6 \times 10^{-6}\ \text{F})(12\ \text{V})$$

$$Q_{C_0} = 72\ \mu\text{C}$$

$$\boxed{q_C(t) = (72\ \mu\text{C})e^{-t/60\ \mu\text{s}}}$$

ii.) $\Delta v_{R_1}(t) = ?$ and $\Delta v_{R_2}(t) = ?$

$$i(t) = I_0 e^{-t/\tau}$$

$$I_0 = \frac{\Delta V_{C_0}}{R_1 + R_2} = \frac{12\ \text{V}}{4\ \Omega + 6\ \Omega}$$

$$I_0 = 1.2\ \text{A}$$

$$\Delta v_{R_1}(t) = i(t)R_1 = I_0 R_1 e^{-t/\tau}$$

$$\Delta v_{R_2}(t) = i(t)R_2 = I_0 R_2 e^{-t/\tau}$$

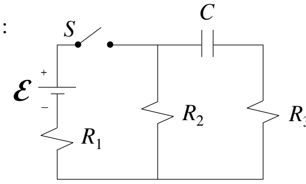
$$\Delta v_{R_1}(t) = (1.2\ \text{A})(4\ \Omega)e^{-t/60\ \mu\text{s}}$$

$$\boxed{\Delta v_{R_1}(t) = (4.8\ \text{V})e^{-t/60\ \mu\text{s}}}$$

$$\Delta v_{R_2}(t) = (1.2\ \text{A})(6\ \Omega)e^{-t/60\ \mu\text{s}}$$

$$\boxed{\Delta v_{R_2}(t) = (7.2\ \text{V})e^{-t/60\ \mu\text{s}}}$$

Example 11:

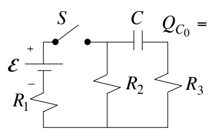


In the circuit above $\mathcal{E} = 1000$ V, $C = 10\ \mu$F, and $R_1 = R_2 = R_3 = 1$ MΩ. The capacitor is completely uncharged when switch S is closed.

a.) Determine the current through each resistor at $t = 0$ and $t = \infty$.

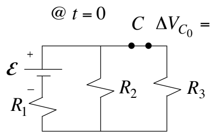
b.) Draw a graph of the potential difference for each resistor for $t = 0$ to $t = \infty$.

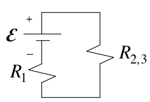
Example 11 : $\mathcal{E} = 1000\ \text{V},\ R_1 = R_2 = R_3 = 1\ \text{M}\Omega, \text{and } C = 10\ \mu\text{F}$

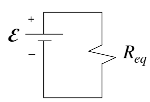


a.) i's @ $t = 0$ and $t = \infty$

@ $t = 0$







$$R_{2,3} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$R_{2,3} = \left(\frac{1}{1\ \text{M}\Omega} + \frac{1}{1\ \text{M}\Omega}\right)^{-1}$$

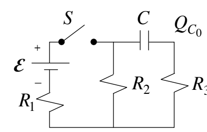
$$R_{2,3} = 0.5\ \text{M}\Omega$$

$$R_{eq} = R_{2,3} + R_1$$

$$R_{eq} = 0.5\ \text{M}\Omega + 1\ \text{M}\Omega$$

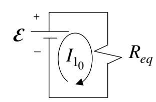
$$R_{eq} = 1.5\ \text{M}\Omega$$

Example 11 : $\mathcal{E} = 1000\ \text{V},\ R_1 = R_2 = R_3 = 1\ \text{M}\Omega, \text{and } C = 10\ \mu\text{F}$



a.) i's @ $t = 0$ and $t = \infty$

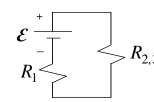
@ $t = 0$



$$I_{1_0} = \frac{\mathcal{E}}{R_{eq}}$$

$$I_{1_0} = \frac{1000\ \text{V}}{1.5\ \text{M}\Omega}$$

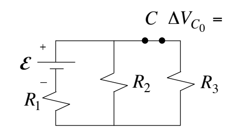
$$\boxed{I_{1_0} = 0.667\ \text{mA}}$$



$$\Delta V_{2,3} = I_{1_0} R_{2,3}$$

$$\Delta V_{2,3} = (0.667\ \text{mA})(0.5\ \text{M}\Omega)$$

$$\Delta V_{2,3} = 333.3\ \text{V}$$



$$\Delta V_2 = \Delta V_3 = \Delta V_{2,3} = 333.3\ \text{V}$$

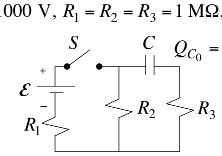
$$I_{2_0} = \frac{\Delta V_2}{R_2} = \frac{333.3\ \text{V}}{1.5\ \text{M}\Omega}$$

$$\boxed{I_{2_0} = 0.333\ \text{mA}}$$

$$I_{3_0} = \frac{\Delta V_3}{R_3} = \frac{333.3\ \text{V}}{1.5\ \text{M}\Omega}$$

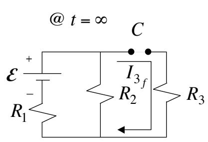
$$\boxed{I_{3_0} = 0.333\ \text{mA}}$$

Example 11 : $\mathcal{E} = 1000\ \text{V},\ R_1 = R_2 = R_3 = 1\ \text{M}\Omega, \text{and } C = 10\ \mu\text{F}$

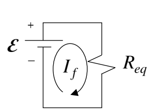


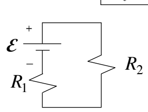
a.) i's @ $t = 0$ and $t = \infty$

@ $t = \infty$



$$\boxed{I_{3_f} = 0}$$





$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 1\ \text{M}\Omega + 1\ \text{M}\Omega$$

$$R_{eq} = 2\ \text{M}\Omega$$

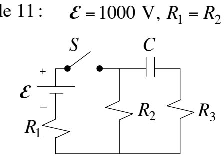
$$I_f = \frac{\mathcal{E}}{R_{eq}}$$

$$I_f = \frac{1000\ \text{V}}{2\ \text{M}\Omega} = 0.500\ \text{mA}$$

$$I_{1_f} = I_{2_f} = I_f$$

$$\boxed{I_{1_f} = 0.500\ \text{mA} \quad I_{2_f} = 0.500\ \text{mA}}$$

Example 11 : $\mathcal{E} = 1000\ \text{V},\ R_1 = R_2 = R_3 = 1\ \text{M}\Omega, \text{and } C = 10\ \mu\text{F}$

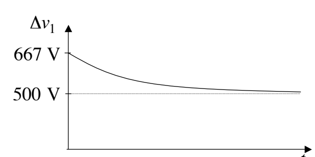


@ $t = 0$	@ $t = \infty$
$I_{1_0} = 0.667$ mA	$I_{1_f} = 0.500$ mA
$I_{2_0} = 0.333$ mA	$I_{2_f} = 0.500$ mA
$I_{3_0} = 0.333$ mA	$I_{3_f} = 0$

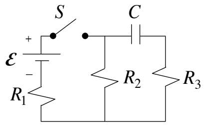
b.) graphs of voltage across each resistor for $t = 0$ to $t = \infty$

$$\Delta V_{1_0} = I_{1_0} R_1 = (0.667 \times 10^{-3}\ \text{A})(1 \times 10^{6}\ \Omega) = 667\ \text{V}$$

$$\Delta V_{1_f} = I_{1_f} R_1 = (0.500 \times 10^{-3}\ \text{A})(1 \times 10^{6}\ \Omega) = 500\ \text{V}$$



Example 11 : $\mathcal{E} = 1000$ V, $R_1 = R_2 = R_3 = 1$ MΩ, and $C = 10\ \mu$F

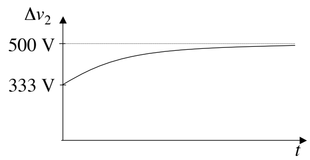


	@ $t = 0$	@ $t = \infty$
	$I_{1_0} = 0.667$ mA	$I_{1_f} = 0.500$ mA
	$I_{2_0} = 0.333$ mA	$I_{2_f} = 0.500$ mA
	$I_{3_0} = 0.333$ mA	$I_{3_f} = 0$

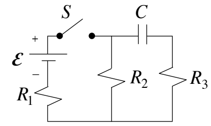
b.) graphs of voltage across each resistor for $t = 0$ to $t = \infty$

$$\Delta V_{2_0} = I_{2_0} R_2 = \left(0.333\times10^{-3}\ \text{A}\right)\left(1\times10^{6}\ \Omega\right) = 333\ \text{V}$$

$$\Delta V_{2_f} = I_{2_f} R_2 = \left(0.500\times10^{-3}\ \text{A}\right)\left(1\times10^{6}\ \Omega\right) = 500\ \text{V}$$



Example 11 : $\mathcal{E} = 1000$ V, $R_1 = R_2 = R_3 = 1$ MΩ, and $C = 10\ \mu$F



	@ $t = 0$	@ $t = \infty$
	$I_{1_0} = 0.667$ mA	$I_{1_f} = 0.500$ mA
	$I_{2_0} = 0.333$ mA	$I_{2_f} = 0.500$ mA
	$I_{3_0} = 0.333$ mA	$I_{3_f} = 0$

b.) graphs of voltage across each resistor for $t = 0$ to $t = \infty$

$$\Delta V_{3_0} = I_{3_0} R_3 = \left(0.333\times10^{-3}\ \text{A}\right)\left(1\times10^{6}\ \Omega\right) = 333\ \text{V}$$

$$\Delta V_{3_f} = I_{3_f} R_3 = (0)\left(1\times10^{6}\ \Omega\right) = 0$$