Direct Current Circuits

Electric Current (Charges in Motion)

Electric Current (*I*)

The net amount of charge that passes through a conductor per unit time at any point. Current is defined as:

$$\boxed{I = \frac{dQ}{dt}}$$

Electric current is measured in *coulomb's per second* or *amperes*. (1A = 1 C/s)

In a single circuit, the current at any instant is the same at one point as any other point. (Charge is conserved.)

Drift Velocity

The free electrons in conductors are in constant random motion, so there is no net flow of charge in any one particular direction.

If a steady electric field $\vec{E}$ is established inside a conductor then charged particles are subjected to a steady force $\vec{F} = q\vec{E}$.

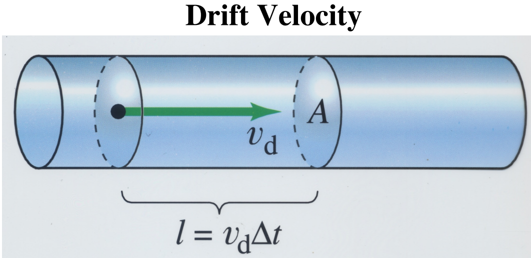
The charges moving in a conductor have frequent collisions with the massive nearly stationary ions of the material.

Drift Velocity

The net effect of the electric field is that in addition to the random motion of the charged particles within a conductor, there is also a very small net motion or *drift* of the moving charged particles as a group in the direction of the electric force.

This motion is described in terms of the *drift velocity* $\vec{v}_d$ of the particles.

Drift Velocity



$\Delta Q = Nev_d A\Delta t$ $N =$ concentration of charges $\left(m^{-3}\right)$

$$\boxed{I = \frac{\Delta Q}{\Delta t} = Nev_d A}$$

Current Density

The current *per unit cross-section area* is called the current density *J*.

$$J = \frac{I}{A} = \frac{Nev_d A}{A}$$

$$J = Nev_d$$

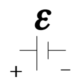
The electric field *E* in terms of current density *J* is:

$$\boxed{\vec{E} = \rho\vec{J}}$$

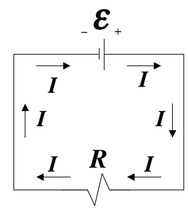
The constant ρ is called the *resistivity* of the material.

Batteries (*emf*)

In order to produce an electric current in a circuit, a potential difference is needed. Batteries are one way of providing a difference in potential (called *electromotive force* or *emf*).



Schematic Diagrams



The *direction of current* is by convention the *direction a positive charge moves* through the circuit, which is towards the negative terminal of the battery.

Ohm's Law

Georg Ohm (1787-1854)

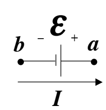
- **Current depends upon the conductivity of the material.**
- **It is more common to talk about *resistance R* (inverse of conductivity) and express this relationship as:**

$$\boxed{I = \frac{\Delta V}{R}} \text{ or } \boxed{\Delta V = I \cdot R}$$

- **The unit for resistance is called the *ohm* and is abbreviated Ω (omega)**

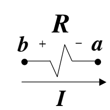
Voltage

For power sources:



$$\Delta V = V_a - V_b = V_{ab} > 0$$

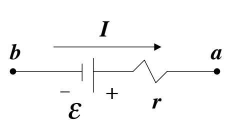
For resistive loads:



$$\Delta V_R = V_a - V_b = V_{ab} < 0$$

Terminal Voltage

The potential difference in a real battery is not equal to the emf due to *internal resistance* within the battery. This lowers the voltage available to the circuit.



$$\boxed{V_{ab} = \mathcal{E} - Ir}$$

V_{ab} is called the *terminal voltage* of the battery.

Resistance (*R*) and Resistivity (*ρ*)

It can be experimentally determined that the resistance of a wire is directly proportional to its length ℓ and inversely proportional to its cross-sectional area *A*.

$$\boxed{R = \frac{\rho \ell}{A}}$$

The proportionality constant *ρ* is called the *resistivity* and depends upon the material used for the wire.

$$\rho\ [=]\ \Omega \cdot \mathrm{m}$$

Electric Power

To find the power transformed by an electric device recall that energy is $Q\Delta V$. Power is the rate energy is transformed in the device or:

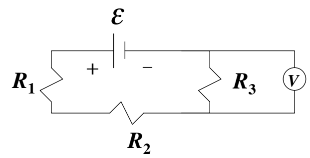
$$P = \frac{Q\Delta V}{t} = I\Delta V \qquad \boxed{P = I\Delta V}$$

The SI unit for power is a J/s or watt (1 W = 1 J/s). For resistors, combining the above with Ohm's Law results in:

$$\boxed{P = I^2 R = \frac{\Delta V^2}{R}}$$

Measuring Voltage

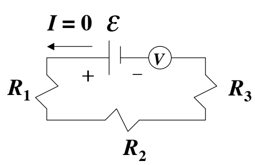
- ***Voltmeters are placed in parallel*** **with the points between which the voltage measurement is made**
- ***Voltmeters have a very high resistance*** **and do not affect the circuit (they draw a very small current)**

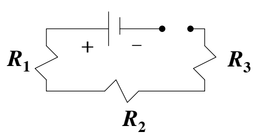


$$V_{meter} = V_{R_3}$$

Measuring Voltage

- ***Voltmeters placed in series*** **will block the current and create an** ***open circuit*****.**

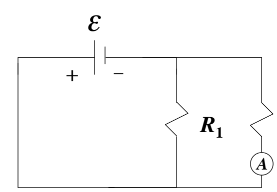




$$V_{meter} = \mathcal{E}$$

Measuring Current

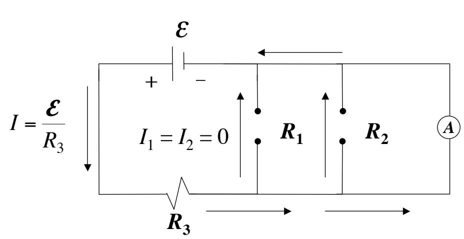
- ***Ammeters are placed in series*** **with the device through which the current measurement is made**
- ***Ammeters have a very low resistance*** **and do not affect the circuit (the voltage drop is very low)**



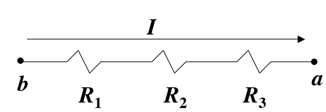
$$I_{meter} = I_{R_2}$$

Measuring Current

- ***Ammeters placed in parallel*** **will create a** ***short circuit*****. Anything parallel to the meter will have no voltage across them and therefore no current**



Resistors in Series

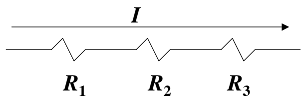


$$V_{ab} = \Delta V_1 + \Delta V_2 + \Delta V_3 = IR_1 + IR_2 + IR_3$$

$$V_{ab} = I\left(R_1 + R_2 + R_3\right)$$

$$I = \frac{V_{ab}}{R_1 + R_2 + R_3} = \frac{V_{ab}}{R_s}$$

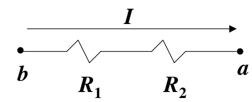
Resistors in Series



- *Current is the same* through each resistor and is the same as the current in the equivalent resistance
- Voltage drop across each resistor is different unless the resistance is the same:

$$\boxed{R_s = \sum_i R_i}$$

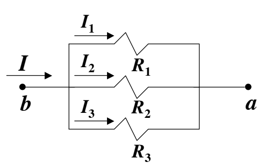
Resistors in Series (Voltage Divider)



$$I = \frac{V_{ab}}{R_1 + R_2}$$

$$\Delta V_1 = IR_1 = \frac{V_{ab} R_1}{R_1 + R_2} \qquad \Delta V_2 = IR_2 = \frac{V_{ab} R_2}{R_1 + R_2}$$

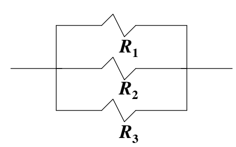
Resistors in Parallel



$$I = I_1 + I_2 + I_3 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3} = \frac{V_{ab}}{R_p}$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

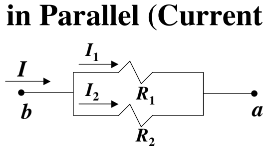
Resistors in Parallel



- *Voltage drop is the same* across each resistor and the same as the voltage drop across the equivalent resistance
- Current is different through each resistor, the higher the resistance the lower the current

$$\boxed{\frac{1}{R_p} = \sum_i \frac{1}{R_i}}$$

Resistors in Parallel (Current Divider)



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \quad \text{so} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

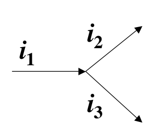
and $\quad V_{ab} = I \cdot \left(\frac{R_1 R_2}{R_1 + R_2} \right)$

$$I_1 = \frac{V_{ab}}{R_1} = I \cdot \left(\frac{R_2}{R_1 + R_2} \right) \quad \text{and} \quad I_2 = \frac{V_{ab}}{R_2} = I \cdot \left(\frac{R_1}{R_1 + R_2} \right)$$

Kirchhoff's Rules

1.) ***Junction Rule*** **(Conservation of charge)**

At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.

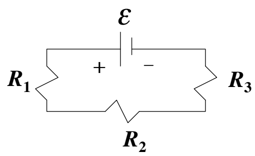


$$i_1 = i_2 + i_3$$

Kirchhoff’s Rules

2.) ***Loop Rule*** **(Conservation of energy)**

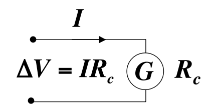
The sum of the changes in potential around any closed path of a circuit is zero.



$$\mathcal{E} + \Delta V_{R_1} + \Delta V_{R_2} + \Delta V_{R_3} = 0$$

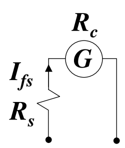
Galvanometers

- **A *galvanometer* is simply a meter that deflects in proportion to the current running through it.**
- **The maximum deflection is called the full-scale deflection.**
- **The key characteristics of a galvanometer are**
  - **The current I_{fs} required for full-scale deflection.**
  - **The resistance R_c of the coil of wire in the meter.**



Voltmeters

A *voltmeter* consists of a resistor put in series with a galvanometer. The value of this resistance R_s determines the full-scale reading of the meter ΔV_v.

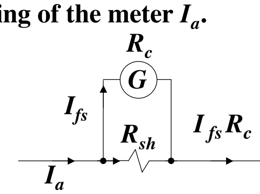


$$\Delta V_v = I_{fs}\left(R_s + R_c\right)$$

$$R_s = \frac{\left(\Delta V_v - I_{fs}R_c\right)}{I_{fs}}$$

Ammeters

An *ammeter* consists of a resistor put in parallel (called a *shunt resistor* or *shunt*) with a galvanometer. The value of this resistance R_{sh} determines the full-scale reading of the meter I_a.



$$I_{fs}R_c = \left(I_a - I_{fs}\right)R_{sh}$$

$$R_{sh} = \frac{I_{fs}R_c}{\left(I_a - I_{fs}\right)}$$

Some Tips

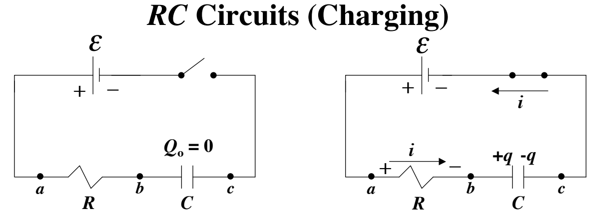
1.) The decrease in voltage between the two ends of a resistor is called a *voltage drop* ($V_R < 0$).

2.) Batteries provide an increase in potential difference.

3.) Label currents in each branch or loop with a symbol and arrow indicating the direction of the current. Let the algebra take care of the sign of the current. If you get a negative current it simply means it is in the opposite direction of what you assumed.

RC Circuits

RC Circuits (Charging)



$$\mathcal{E} - v_{ab} - v_{bc} = 0$$

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

$$\mathcal{E} - R\frac{dq}{dt} - \frac{q}{C} = 0$$

RC Circuits (Charging)

$$\mathcal{E} - R\frac{dq}{dt} - \frac{q}{C} = 0$$

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{C\mathcal{E} - q}{RC}$$

$$\frac{dq}{dt} = -\frac{q - C\mathcal{E}}{RC}$$

$$\frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC}$$

$$-\frac{dt}{RC} = \frac{dq}{q - C\mathcal{E}}$$

$$\int_0^t -\frac{dt}{RC} = \int_0^q \frac{dq}{q - C\mathcal{E}}$$

$$-\frac{t}{RC}\bigg|_0^t = \ln(q - C\mathcal{E})\Big|_0^q$$

$$-\frac{t}{RC} - 0 = \ln(q - C\mathcal{E}) - \ln(-C\mathcal{E})$$

$$-\frac{t}{RC} = \ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right)$$

$$e^{-t/RC} = \frac{q - C\mathcal{E}}{-C\mathcal{E}}$$

$$-C\mathcal{E}e^{-t/RC} = q - C\mathcal{E}$$

$$q = C\mathcal{E} - C\mathcal{E}e^{-t/RC}$$

$$\boxed{q(t) = Q_f\left(1 - e^{-t/RC}\right) \quad \text{where} \quad Q_f = C \cdot \mathcal{E}}$$

RC Circuits (Charging)

The charge on the capacitor varies according to:

$$q(t) = Q_f\left(1 - \mathrm{e}^{-t/RC}\right) = C\mathcal{E}\left(1 - \mathrm{e}^{-t/RC}\right)$$

The current at any time is given by:

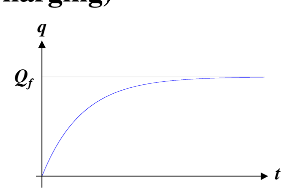
$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}\mathrm{e}^{-t/RC}$$

$$\boxed{i = I_o\mathrm{e}^{-t/RC}}$$

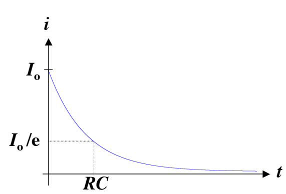
RC **is called the** ***time constant*** **(*τ*) and is the time it takes the capacitor to become 63.2% charged.**

RC Circuits (Charging)

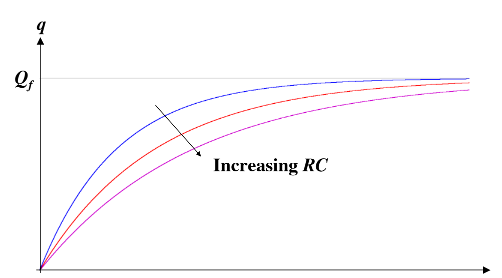
$$q(t) = Q_f\left(1 - \mathrm{e}^{-t/RC}\right)$$



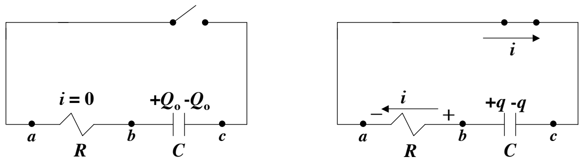
$$i = I_o\mathrm{e}^{-t/RC}$$



RC Circuits (Charging)



RC Circuits (Discharging)



$$\Delta v_C - \Delta v_R = 0$$

$$\frac{q}{C} - iR = 0$$

$$\frac{q}{C} + R\frac{dq}{dt} = 0 \quad \left(i = -\frac{dq}{dt}\right)$$

RC Circuits (Discharging)

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

$$\frac{dq}{dt} = -\frac{q}{RC}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

$$\int_{Q_o}^{q} \frac{dq}{q} = \int_0^t -\frac{dt}{RC}$$

$$\ln(q) - \ln(Q_0) = -\frac{t}{RC} - 0$$

$$\ln\left(\frac{q}{Q_0}\right) = -\frac{t}{RC}$$

$$\frac{q}{Q_0} = e^{-t/RC}$$

$$\boxed{q = Q_0 e^{-t/RC}}$$

RC Circuits (Discharging)

The charge on the capacitor varies according to:

$$q = Q_o e^{-t/RC}$$

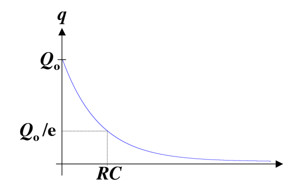
The current at any time is given by:

$$i = -\frac{dq}{dt} = \frac{Q_o}{RC} e^{-t/RC}$$

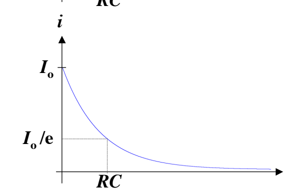
$$\boxed{i = I_o e^{-t/RC}}$$

RC Circuits (Discharging)

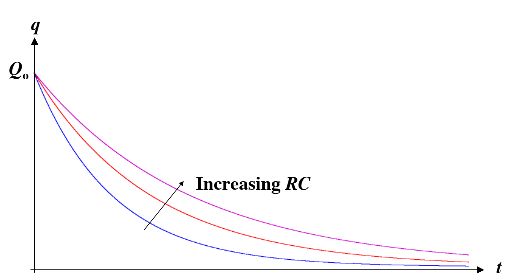
$$q = Q_o e^{-t/RC}$$



$$i = I_o e^{-t/RC}$$



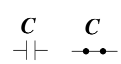
RC Circuits (Discharging)



RC Circuits

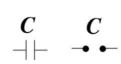
Uncharged

$\Delta v_C = 0$ **short**
$i_C \neq 0$ **circuit**



Fully Charged

$\Delta v_C \neq 0$ **open**
$i_C = 0$ **circuit**



Charged capacitor has same voltage as the device that is electrically parallel to it.