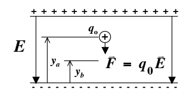
Electric Potential

Electric Potential Energy (Uniform Field)

Electric potential energy **(U) is analogous to gravitational potential energy.**



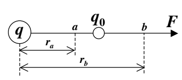
$$W_{a\to b} = Fd = q_0 E d$$

$$U = q_0 E y \qquad (U = mgy)$$

$$W_{a\to b} = -\Delta U = -(U_b - U_a)$$

$$W_{a\to b} = U_a - U_b = q_0 E (y_a - y_b)$$

Electric Potential Energy of Two Point Charges



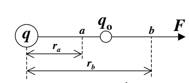
$$F_r = k\frac{qq_0}{r^2}$$

The work done by the electric force of q when moving a test charge q_0 from a to b is:

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b} k\frac{qq_0}{r^2}dr$$

$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{-1}{r}\right)\Bigg]_{r_a}^{r_b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Electric Potential Energy of Two Point Charges



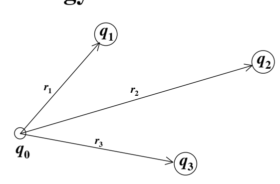
$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = U_a - U_b$$

The *electric potential energy* of two point charges is:

$$\boxed{U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}} \qquad U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

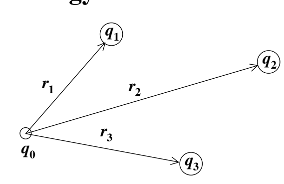
Electric Potential Energy of Several Point Charges



The potential energy associated with a test charge q_0 due to several charges (q_1, q_2, q_3 , ...q_i) is:

$$U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric Potential Energy of Several Point Charges



The potential energy associated with a test charge q_0 due to charges q_1, q_2, and q_3 is:

$$U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_i}{r_i} + \frac{q_i}{r_i} + \frac{q_3}{r_3}\right)$$

Potential Energy to Assemble Charges

If we start with charges q_1, q_2, q_3, ... all separated from each other by infinite distances and then bring them together so that the distance between q_i and q_j is r_{ij}, the total potential energy U is the sum of the potential energies of interaction for each pair of charges.

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

This sum extends over all pairs of charges, and only terms with $i < j$ are considered in order to avoid counting each pair more than once.

Electric Potential Energy

A *positive potential energy* means work must be done to assemble the charge arrangement, starting with the charges at infinity.

A *negative potential energy* means work must be done to disassemble the charge arrangement, ending with the charges at infinity.

Electric Potential

***Potential (V)* is potential energy per unit charge. The *potential* at any point in an electric field is the potential energy *U per unit charge* associated with a test charge q_0 at that point.**

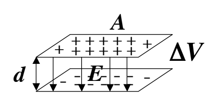
$$V = \frac{U}{q_0}$$

$$U_E = qV = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

The SI unit of potential is the *volt*.

$$1\ \text{V} = 1\ \text{volt} = 1\ \frac{\text{J}}{\text{C}}$$

Uniform Electric Fields



$$W_{field} = \int \vec{F}\cdot d\vec{r}$$

$$W_{field} = Fd\cos\theta$$

$$W_{field} = q_0 Ed\cos\theta$$

$$W_{field} = q_0 Ed\cos\theta \quad = -\Delta U$$

$$Ed\cos\theta = -\frac{\Delta U}{q_0} \quad = -\Delta V$$

$$E = \frac{-\Delta V}{d}$$

$$\Delta V = -Ed$$

Electric Potential of Point Charges

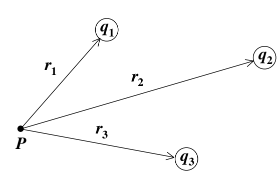
The *potential* of a single point charge is:

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

The *potential* due to a collection of point charges is:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric Potential of Several Point Charges



The potential at point P due to charges q_1, q_2, and q_3 is:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

Electric Potential and Potential Difference

The *potential difference* between any two points *a* and *b* numerically equals the work done *against* the field in moving a unit positive charge from *a* to *b* with no acceleration.

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -\left(V_b - V_a\right) = V_a - V_b$$

$$\frac{W_{a\to b}}{q_0} = V_a - V_b = V_{ab}$$

V_{ab} is called the potential of *a* with respect to *b*. In an electric circuit the potential difference between two points is called the *voltage*.

Determining Potential Difference from the Electric Field

$$W_{a\to b} = \int_a^b F \cdot d\vec{l} = \int_a^b q_0 \vec{E} \cdot d\vec{l}$$

$$W_{a\to b} = -\Delta U = -q_0 \Delta V$$

$$\frac{W_{a\to b}}{q_0} = -\left(V_b - V_a\right)$$

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} \qquad \boxed{\Delta V = -\int \vec{E} \cdot d\vec{r}}$$

The value of $V_b - V_a$ is independent of the path taken from *a* to *b*.

Determining the Electric Field from Potential

If the electric field is radial with respect to a point or an axis and *r* is the distance from the point or the axis then:

$$E_r = -\frac{dV}{dr}$$

For uniform electric fields:

$$E_x = -\frac{\Delta V}{\Delta x} \qquad \boxed{E_x = -\frac{dV}{dx}}$$

Note that an equivalent unit for electric field is:

$$\frac{\text{V}}{\text{m}} = \frac{\text{N}}{\text{C}}$$

Potential Tidbits

It is only meaningful to talk about differences in potential energy.

A *positive charge* moves naturally from a *high potential* to a *low potential*.

A *negative charge* moves naturally from a *low potential* to a *high potential*.

Some More Tidbits

The points on an *equipotential surface* all have the same potential. The electric field *E* is always *directed perpendicularly* to *equipotential surfaces*.

The surface of a *conductor* is an *equipotential surface*.

***Electric Potential V* and *Potential Energy U* are SCALARS!**

An *electron volt eV* is the *energy* of a particle with the *charge equal to that of an electron* moving through a potential difference of *one volt*.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

Potential of a Continuous Charge Distribution

When there is a continuous distribution along a *line*, over a *surface*, or throughout a *volume*; we divide the charge into elements *dq* and integrate:

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

where *r* is the distance from the charge element *dq* to the field point where we are finding *V*.