Electromagnetic Induction

Electromagnetic Induction and Faraday's Law

- ***Michael Faraday*** **(1791-1867) discovered that a changing magnetic field could produce an electric current in a conductor placed in the magnetic field.**
- **Such a current is called an *induced current*.**
- **The phenomenon is called *electromagnetic induction*.**

Faraday's Law of Induction

- **Faraday found that the induced *emf* is proportional to the rate of change of the magnetic flux Φ_B passing through a loop of area A.**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- **In general:**

$$\boxed{\Phi_B = \int \vec{B}\cdot d\vec{A}}$$

- **For magnetic fields that are constant:**

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$

Faraday's Law of Induction

- **If the flux pass through N loops the *induced emf* is:**

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$$\boxed{\mathcal{E} = \oint \vec{E}\cdot d\vec{\ell} = -N\frac{d\Phi_B}{dt}}$$

- **The minus sign is necessary to give the correct direction the induced *emf* acts.**

Lenz's Law

An induced emf always gives rise to a current whose magnetic field opposes the original change in flux.

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$

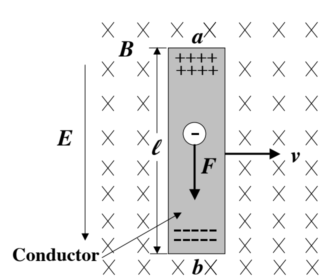
Note: An *emf* is induced whenever there is change in flux and can be induced in three ways:

1.) By changing the magnetic field B

2.) By changing the area A of the loop in the field

3.) By changing the orientation θ the magnetic field makes with the loop

Motional *emf*



$$\vec{F} = q\vec{v}\times\vec{B}$$

$$E = \frac{F}{q} = \frac{qvB}{q} = vB$$

(Electric Field in rod)

$$E = \frac{\Delta V}{\Delta d}$$

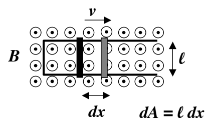
$$\mathcal{E} = vB\ell \quad \textbf{(Motional } emf\textbf{)}$$

For a conductor of any shape, moving in any magnetic field.

$$\mathcal{E} = \int_b^a \left(\vec{v}\times\vec{B}\right)\cdot d\vec{\ell}$$

emf Induced in a Moving Conductor

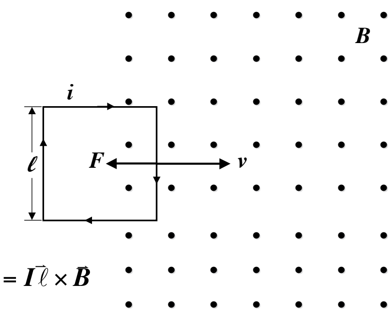
Assume that a uniform magnetic field *B* is perpendicular to the area bounded by a U-shaped conductor and a movable rod resting on it. The rod is made to move at a speed *v*.



The magnitude of the induced *emf* is given by

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(\vec{B}\cdot\vec{A})}{dt} = \frac{d(BA\cos\theta)}{dt} = B\frac{dA}{dt} = B\frac{\ell dx}{dt} = B\ell v$$

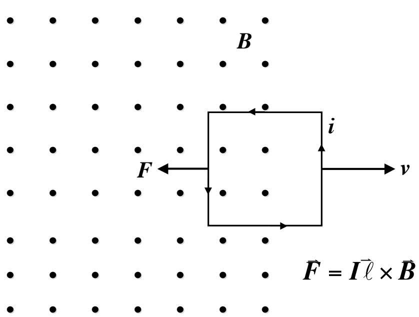
Same Kind of Problem



$$\vec{F} = I\vec{\ell}\times\vec{B}$$

Entering Magnetic Field

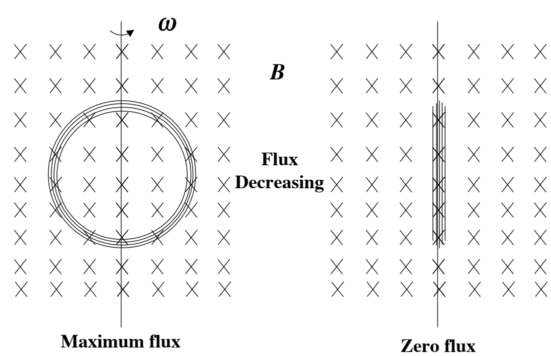
Same Kind of Problem



$$\vec{F} = I\vec{\ell}\times\vec{B}$$

Leaving Magnetic Field

Rotating Coils



Rotating Coils

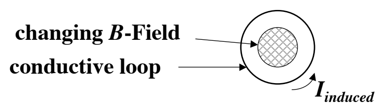
$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$

$$\frac{d\Phi_B}{dt} = \frac{dBA\cos\theta}{dt} = BA\frac{d\cos\theta}{dt}$$

Since $\theta = \omega t$ (assuming constant angular speed)

$$\frac{d\Phi_B}{dt} = BA\frac{d\cos\omega t}{dt} = -\omega BA\sin\omega t$$

Induced Electric Fields



There must be a force that make the charges move around the loop.

The changing magnetic flux causes an *induced electric field* in the loop.

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

Maxwell's Equations

1.) Gauss's Law for Electric Fields

$$\oint E \cdot d\bar{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

2.) Gauss's Law for Magnetic Fields

$$\oint \bar{B} \cdot d\bar{A} = 0$$

3.) Ampere's Law

$$\oint B \cdot d\bar{\ell} = \mu_o \left(i_c + \varepsilon_o \frac{d\Phi_E}{dt} \right)$$

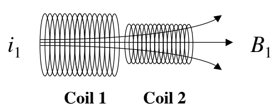
4.) Faraday's Law

$$\oint E \cdot d\bar{\ell} = -\frac{d\Phi_B}{dt}$$

Inductance

Mutual Inductance

Consider two neighboring coils of wire. A current flowing in coil 1 produces a magnetic flux through coil 2.



If the current i_1 changes then it will induce a current in coil 2.

If Φ_{B2} is the flux in coil 2 due to the current in coil 1 then the induced *emf* will be

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B_2}}{dt}$$

Mutual Inductance

If we introduce a proportionality constant M_{21} called the *mutual inductance* of the two coils

$$N_2 \Phi_{B_2} = M_{21} i_1$$

where Φ_{B2} is the flux in a single turn of coil 2. Then

$$N_2 \frac{d\Phi_{B_2}}{dt} = M_{21} \frac{di_1}{dt}$$

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B_2}}{dt} = -M_{21} \frac{di_1}{dt}$$

Mutual Inductance

This can be repeated for the case where a current i_2 in coil 2 induces an *emf* in coil 1 resulting in

$$\mathcal{E}_1 = -N_1 \frac{d\Phi_{B_1}}{dt} = -M_{12} \frac{di_2}{dt}$$

where Φ_{B1} is the flux in a single turn of coil 1.

It turns out that M_{21} is always equal to M_{12} and is written as M, and it completely characterizes the induced-*emf* interaction between the two coils. The mutually induced emf's are then

$$\mathcal{E}_1 = -M \frac{di_2}{dt} \quad \text{and} \quad \mathcal{E}_2 = -M \frac{di_1}{dt}$$

Mutual Inductance

Mutual inductance (M) describes the coupling between two coils in which a changing current in one coil induces an emf in an adjacent coil.

$$M = \frac{N_2 \Phi_{B_2}}{i_1} = \frac{N_1 \Phi_{B_1}}{i_2}$$

The SI unit of mutual inductance is called the Henry (1 H).

$$1\ \text{H} = 1\ \frac{\text{Wb}}{\text{A}} = 1\ \frac{\text{V} \cdot \text{s}}{\text{A}} = 1\ \Omega \cdot \text{s}$$

Self Inductance

Any circuit that carries a varying current will have an emf induced in it by the variation in *its own* magnetic field. Such an *emf* is a called a self-induced emf.

Self-inductance **(*L*) of a circuit is given by:**

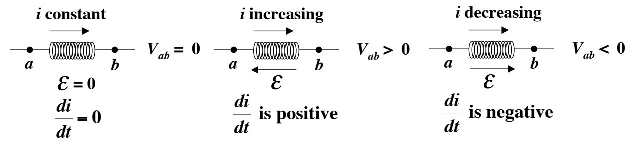
$$L = \frac{N\Phi_B}{i}$$

$$Li = N\Phi_B \quad \text{or} \quad L\frac{di}{dt} = N\frac{d\Phi_B}{dt}$$

Since $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ $\quad \boxed{\mathcal{E} = -L\frac{di}{dt}}$ **(self-induced *emf*)**

Inductors

A circuit element designed to have a particular inductance is called an *inductor* (*L*). The potential difference V_{ab} between the terminals of the inductor is equal in magnitude to the self-induced *emf*.



$$\Delta V_L = -\mathcal{E} = +L\frac{di}{dt}$$

Inductors and Energy

The total energy *U* needed to establish a final current *I* in an inductor with inductance *L* can also be determined.

$$P = \Delta V_L i = Li\frac{di}{dt}$$

The energy *dU* supplied to the inductor during time interval *dt* is:

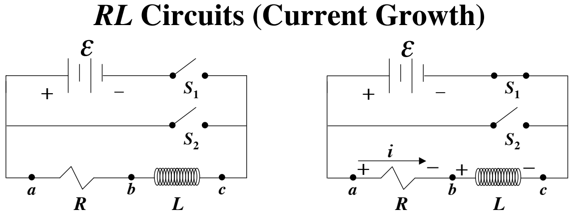
$$dU = Pdt$$

$$U_L = L\int_0^I idi = \frac{1}{2}LI^2 \qquad \boxed{U_L = \frac{1}{2}LI^2}$$

Capacitors versus Inductors

Capacitors	Inductors
$\Delta v_C = \frac{q_C}{C}$ so $\Delta v_C = \frac{1}{C}\int idt$	$\Delta v_L = L\frac{di}{dt}$
Stores energy in the form of an electric field.	**Stores energy in the form of a magnetic field.**
$U_C = \frac{1}{2}C\Delta V^2$	$U_L = \frac{1}{2}LI^2$
Resist changes in voltage.	**Resist changes in current.**

RL Circuits (Current Growth)



$$\mathcal{E} - v_{ab} - v_{bc} = 0$$

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L}$$

RL Circuits (Current Growth)

$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L}$$

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} - \frac{R}{L}i = -\frac{R}{L}\left(i - \frac{\mathcal{E}}{R}\right)$$

$$\frac{di}{i - \frac{\mathcal{E}}{R}} = -\frac{R}{L}dt$$

$$\int_0^i \frac{di}{i - \frac{\mathcal{E}}{R}} = \int_0^t -\frac{R}{L}dt$$

$$\ln\left(i - \frac{\mathcal{E}}{R}\right)\Bigg]_0^i = -\frac{R}{L}t$$

$$\ln\left(i - \frac{\mathcal{E}}{R}\right) - \ln\left(-\frac{\mathcal{E}}{R}\right) = -\frac{R}{L}t$$

$$\ln\frac{\left(i - \frac{\mathcal{E}}{R}\right)}{-\frac{\mathcal{E}}{R}} = -\frac{R}{L}t$$

$$\frac{\left(i - \frac{\mathcal{E}}{R}\right)}{-\frac{\mathcal{E}}{R}} = e^{-\frac{R}{L}t}$$

$$i - \frac{\mathcal{E}}{R} = -\frac{\mathcal{E}}{R}e^{-\frac{R}{L}t}$$

$$i = \frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R}e^{-\frac{R}{L}t}$$

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

RL Circuits (Current Growth)

The current in an *RL* circuit varies according to:

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

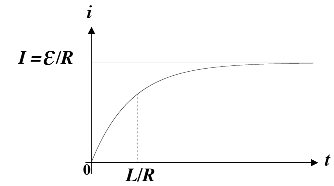
L/R **is called the *time constant* (τ) and is the time it takes the current to become 63.2% of its final value.**

The voltage across *L* varies according to:

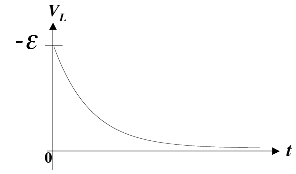
$$\Delta V_L = -L\frac{di}{dt} = -\mathcal{E}e^{-\frac{R}{L}t}$$

RL Circuits (Current Growth)

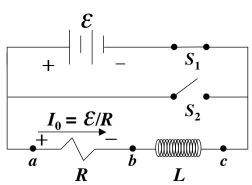
$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$



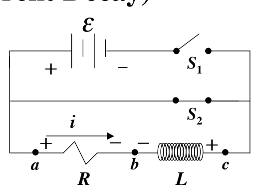
$$\Delta V_L = -\mathcal{E}e^{-\frac{R}{L}t}$$



RL Circuits (Current Decay)



$\Delta v_L = 0$



$\Delta v_L = \Delta v_R$

$$v_{bc} - v_{ab} = 0$$

$$v_{bc} = v_{ab}$$

$$-L\frac{di}{dt} = iR$$

$$\frac{di}{i} = \frac{-R}{L}dt$$

RL Circuits (Current Decay)

$$\frac{di}{i} = \frac{-R}{L}dt$$

$$\int_{I_0}^{i}\frac{di}{i} = \int_0^t \frac{-R}{L}dt$$

$$\ln(i)\Big]_{I_0}^{i} = \frac{-R}{L}t$$

$$\ln(i) - \ln(I_0) = \frac{-R}{L}t$$

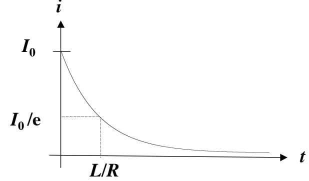
$$\ln\left(\frac{i}{I_0}\right) = \frac{-R}{L}t$$

$$\frac{i}{I_0} = e^{-\frac{R}{L}t}$$

$$i(t) = I_0 e^{-\frac{R}{L}t} \quad \left(I_0 = \frac{\mathcal{E}}{R}\right)$$

RL Circuits (Current Decay)

$$i(t) = I_0 e^{-\frac{R}{L}t} \quad \left(I_0 = \frac{\mathcal{E}}{R}\right)$$



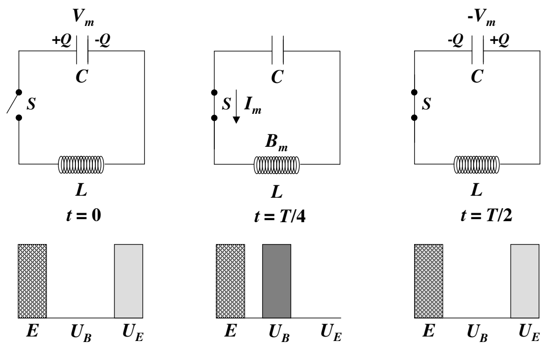
RC versus *RL* Circuits

RC		***RL***	
Uncharged		**Unmagnetized**	
$\Delta v_C = 0$ $i_C \neq 0$	short circuit	$i_L = 0$ $\Delta v_L \neq 0$	open circuit
Charged		**Magnetized**	
$\Delta v_C \neq 0$ $i_C = 0$	open circuit	$i_L \neq 0$ $\Delta v_L = 0$	short circuit

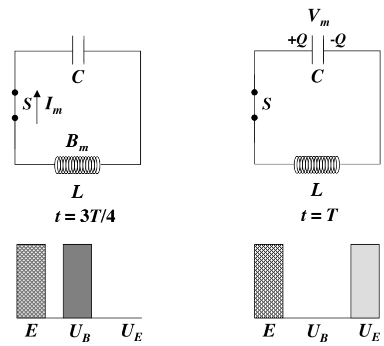
Charged capacitor has same voltage as the device that is electrically parallel to it.

Magnetized inductor has same current as the device that is electrically in series with it.

LC Circuits (Electrical Oscillation)

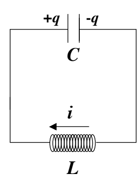


LC Circuits (Electrical Oscillation)



LC Circuits

In an *LC* circuit with no energy losses the charge on the capacitor oscillates back and forth.



$$-L\frac{di}{dt} - \frac{q}{C} = 0$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

LC Circuits

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

This equation has exactly the same form as that for simple harmonic motion.

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

whose solution was $x = A\cos(\omega t + \phi)$

where $\omega = \sqrt{\frac{k}{m}}$

LC Circuits

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

In the analogous electrical situation the capacitor charge *q* is given by

$$q = Q\cos(\omega t + \phi)$$

and the angular frequency ω of the oscillation is given by

$$\omega = \sqrt{\frac{1}{LC}}$$