Example 1:

a.) What is the acceleration due to gravity at the surface of Venus?

(M_V = 4.88 x 10^{24} kg and R_V = 6.07 x 10^6 m)

b.) What is the escape velocity for an object on the surface of Venus?

c.) What is the total energy of a satellite with a mass of 15,000 kg orbiting in a circular orbit 200 km above the surface of Venus?

Example 1: $M_V = 4.88\times10^{24}\ \text{kg}$ and $R_V = 6.07\times10^{6}\ \text{m}$

a.) $g_V = ?$

$$F_g = G\frac{M_V\,m}{R_V^{\,2}} = mg_V$$

$$g_V = G\frac{M_V}{R_V^{\,2}} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(4.88\times10^{24}\ \text{kg}\right)}{\left(6.07\times10^{6}\ \text{m}\right)^2}$$

$$\boxed{g_V = 8.83\ \frac{\text{m}}{\text{s}^2}}$$

b.) $v_e = ?$

$$K = -U \quad \text{so} \quad \frac{1}{2}mv_e^{\,2} = G\frac{M_V m}{R_V}$$

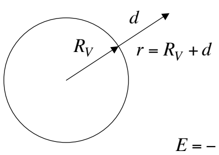
$$v_e = \sqrt{2G\frac{M_V}{R_V}} = \sqrt{2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(4.88\times10^{24}\ \text{kg}\right)}{\left(6.07\times10^{6}\ \text{m}\right)}}$$

$$\boxed{v_e = 10{,}356\ \frac{\text{m}}{\text{s}}}$$

Example 1:

c.) $E = ?$

$M_V = 4.88\times10^{24}\ \text{kg}$, $R_V = 6.07\times10^{6}\ \text{m}$, $m = 15{,}000\ \text{kg}$, and $d = 200\ \text{km}$



$$E = K + U = \frac{1}{2}mv^2 - G\frac{M_V m}{r}$$

$$F_g = G\frac{M_V\,m}{r^2} = ma_c = m\frac{v^2}{r} \quad \text{so } v = \sqrt{G\frac{M_V}{r}}$$

$$E = \frac{1}{2}m\left(\sqrt{G\frac{M_V}{r}}\right)^2 - G\frac{M_V m}{r} = \frac{1}{2}G\frac{M_V m}{r} - G\frac{M_V m}{r}$$

$$E = -\frac{1}{2}G\frac{M_V m}{r} = -\frac{1}{2}G\frac{M_V m}{R_V + d}$$

$$E = -\frac{1}{2}\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(4.88\times10^{24}\ \text{kg}\right)\left(15{,}000\ \text{kg}\right)}{\left(6.07\times10^{6}\ \text{m} + 2.0\times10^{5}\ \text{m}\right)}$$

$$\boxed{E = -3.9\times10^{11}\,\text{J}}$$

Example 2:

What is the weight of 85 kg person on the surface of Pluto?

(M_P = 1.2 x 10^{22} kg and R_P = 1.15 x 10^6 m)

Example 2: $M_P = 1.2\times10^{22}\ \text{kg}$, $R_P = 1.15\times10^{6}\ \text{m}$, and $m = 85\ \text{kg}$

$F_g = ?$

$$F_g = G\frac{M_P\,m}{R_P^{\,2}}$$

$$F_g = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(1.2\times10^{22}\ \text{kg}\right)\left(85\ \text{kg}\right)}{\left(1.15\times10^{6}\ \text{m}\right)^2}$$

$$\boxed{F_g = 51.4\ \text{N}}$$

$$g_P = G\frac{M_P}{R_P^{\,2}} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(1.2\times10^{22}\ \text{kg}\right)}{\left(1.15\times10^{6}\ \text{m}\right)^2}$$

$$g_P = 0.61\ \frac{\text{m}}{\text{s}^2}$$

$$F_g = mg_P = \left(85\ \text{kg}\right)\left(0.61\ \frac{\text{m}}{\text{s}^2}\right)$$

$$\boxed{F_g = 51.4\ \text{N}}$$

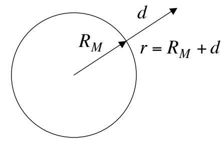
Example 3:

Find the speed of a satellite that would orbit Mars 200 km above its surface.

($M_M = 6.42 \text{ x } 10^{23}$ kg and $R_M = 3.38 \text{ x } 10^{6}$ m)

Example 3: $M_M = 6.42\times10^{23}$ kg , $R_M = 3.38\times10^{6}$ m, and $d = 200$ km

$v = ?$



$$F_g = G\frac{M_M\, m}{r^2} = ma_c = m\frac{v^2}{r}$$

$$v = \sqrt{G\frac{M_M}{r}}$$

$$v = \sqrt{G\frac{M_M}{R_M + d}}$$

$$v = \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(6.42\times10^{23}\ \text{kg}\right)}{\left(3.38\times10^{6}\ \text{m} + 2.0\times10^{5}\ \text{m}\right)}}$$

$$\boxed{v = 3459\ \frac{\text{m}}{\text{s}}}$$

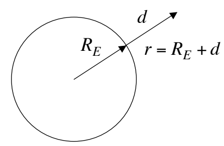
Example 4:

How many minutes would it take a satellite to orbit Earth 150 km above its surface?

($M_E = 5.98 \text{ x } 10^{24}$ kg and $R_E = 6.37 \text{ x } 10^{6}$ m)

Example 4: $M_E = 5.98\times10^{24}$ kg , $R_E = 6.37\times10^{6}$ m, and $d = 150$ km

$T = ?$



$$F_g = G\frac{M_E\, m}{r^2} = ma_c = m\frac{v^2}{r}$$

$$v = \sqrt{G\frac{M_E}{r}} = \frac{2\pi r}{T}$$

$$T = 2\pi r\sqrt{\frac{r}{GM_E}} = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

$$T = 2\pi\sqrt{\frac{(R_E + d)^3}{GM_E}}$$

$$T = 2\pi\sqrt{\frac{\left(6.37\times10^{6}\ \text{m} + 1.5\times10^{5}\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(5.98\times10^{24}\ \text{kg}\right)}}$$

$$\boxed{T = 5238\ \text{s} = 87.3\ \text{min}}$$

Example 5:

A spaceship is fired from the Earth's surface with an initial speed of $2.00 \text{ x } 10^{4}$ m/s. What will be its speed when it is very far away from the Earth? (Neglect friction)

($M_E = 5.98 \text{ x } 10^{24}$ kg and $R_E = 6.37 \text{ x } 10^{6}$ m)

Example 5:

$M = M_E = 5.98\times10^{24}$ kg, $v_1 = 2.00\times10^{4}\ \frac{\text{m}}{\text{s}}$, $r_1 = R_E = 6.37\times10^{6}$ m, $r_2 = \infty$, $v_2 = ?$

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 - G\frac{Mm}{r_1} = \frac{1}{2}mv_2^2 - \cancel{G\frac{Mm}{r_2}} \quad r_2 = \infty$$

$$v_2 = \sqrt{v_1^2 - 2G\frac{M}{r_1}}$$

$$v_2 = \sqrt{\left(2.00\times10^{4}\ \frac{\text{m}}{\text{s}}\right)^2 - 2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)}}$$

$$\boxed{v_2 = 1.66\times10^{4}\ \frac{\text{m}}{\text{s}}}$$