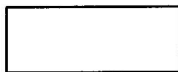


1

E & M 3. The rectangular wire loop shown above has length  $c$ , width  $(b - a)$ , and resistance  $R$ . It is placed in the plane of the page near a long straight wire, also in the plane of the page. The long wire carries a time-dependent current  $I = \alpha(1 - \beta t)$ , where  $\alpha$  and  $\beta$  are positive constants and  $t$  is time.

- What is the direction of the magnetic field inside the loop due to the current  $I$  in the long wire at  $t = 0$ ?
- In terms of  $a$ ,  $b$ ,  $c$ ,  $\alpha$ ,  $\beta$ , and fundamental constants, determine the following.
  - An expression for the magnitude of the magnetic flux through the loop as a function of  $t$ .
  - The magnitude of the induced emf in the loop.
- Show on the diagrams below the directions of the induced current in the loop for each of the following cases.

i.  $\beta t < 1$

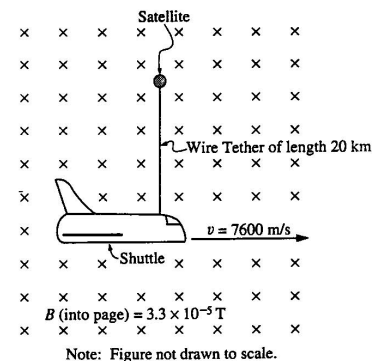


ii.  $\beta t > 1$



- What is the direction of the net force, if any, on the loop due to the induced current at  $t = 0$ ?

END OF SECTION II, ELECTRICITY AND MAGNETISM



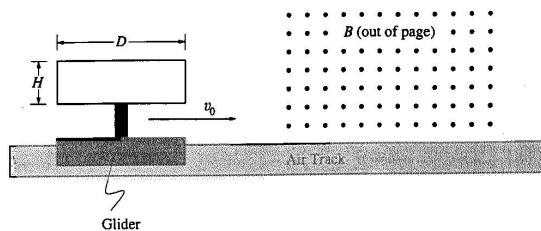
2

E & M 2. One of the space shuttle missions attempted to perform an experiment in orbit using a tethered satellite. The satellite was to be released and allowed to rise to a height of 20 kilometers above the shuttle. The tether was a 20-kilometer copper-core wire, thin and light, but extremely strong. The shuttle was in an orbit with speed 7,600 meters per second, which carried it through a region where the magnetic field of the Earth had a magnitude of  $3.3 \times 10^{-5}$  tesla. For your calculations, assume that the experiment was completed successfully, that the wire is perpendicular to the magnetic field, and that the field is uniform.

- An emf is generated in the tether.
  - Which end of the tether is negative?
  - Calculate the magnitude of the emf generated.

To complete the circuit, electrons are sprayed from the object at the negative end of the tether into the ionosphere and other electrons come from the ionosphere to the object at the positive end.

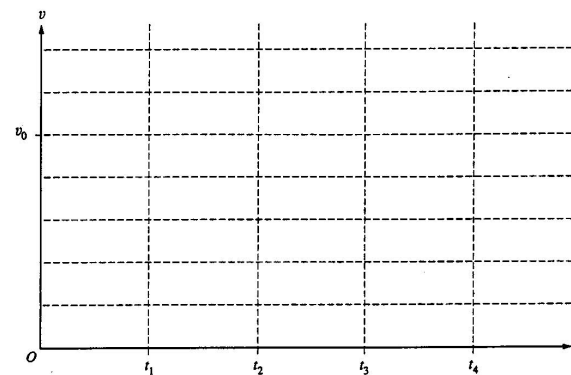
- If the resistance of the entire circuit is about 10,000 ohms, calculate the current that flows in the tether.
- A magnetic force acts on the wire as soon as the current begins to flow.
  - Calculate the magnitude of the force.
  - State the direction of the force.
- By how much would the shuttle's orbital energy change if the current remains constant at the value calculated in (b) for a period of 7 days in orbit?
- Imagine that the astronauts forced a current to flow the other way. What effect would that have, if any, on the orbit of the shuttle? Explain *briefly*.



E & M 3. The long, narrow rectangular loop of wire shown above has vertical height  $H$ , length  $D$ , and resistance  $R$ . The loop is mounted on an insulated stand attached to a glider, which moves on a frictionless horizontal air track with an initial speed of  $v_0$  to the right. The loop and glider have a combined mass  $m$ . The loop enters a long, narrow region of uniform magnetic field  $B$  directed out of the page toward the reader. Express your answers to the parts below in terms of  $B$ ,  $D$ ,  $H$ ,  $R$ ,  $m$ , and  $v_0$ .

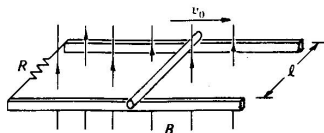
- What is the magnitude of the initial induced emf in the loop as the front end of the loop begins to enter the region containing the field?
- What is the magnitude of the initial induced current in the loop?
- State whether the initial induced current in the loop is clockwise or counterclockwise around the loop.
- Derive an expression for the velocity of the glider as a function of time  $t$  for the interval after the front edge of the loop has entered the magnetic field but before the rear edge has entered the field.

- (e) Using the axes below, sketch qualitatively a graph of speed  $v$  versus time  $t$  for the glider. The front end of the loop enters the field at  $t = 0$ . At  $t_1$  the back end has entered and the loop is completely inside the field. At  $t_2$  the loop begins to come out of the field. At  $t_3$  it is completely out of the field. Continue the graph until  $t_4$ , a short time after the loop is completely out of the field. These times may not be shown to scale on the  $t$ -axis below.



**STOP**

END OF SECTION II, ELECTRICITY AND MAGNETISM



E & M 3. Two horizontal conducting rails are separated by a distance  $\ell$  as shown above. The rails are connected at one end by a resistor of resistance  $R$ . A conducting rod of mass  $m$  can slide without friction along the rails. The rails and the rod have negligible resistance. A uniform magnetic field of magnitude  $B$  is perpendicular to the plane of the rails as shown. The rod is given a push to the right and then allowed to coast. At time  $t = 0$  (immediately after it is pushed) the rod has a speed  $v_0$  to the right.

- Indicate on the diagram above the direction of the induced current in the resistor.
- In terms of the quantities given, determine the magnitude of the induced current in the resistor at time  $t = 0$ .
- Indicate on the diagram above the direction of the force on the rod.
- In terms of the quantities given, determine the magnitude of the force acting on the rod at time  $t = 0$ .

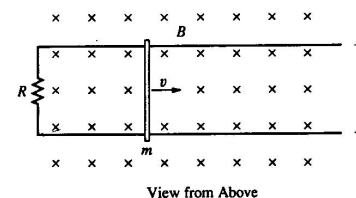
If the rod is allowed to continue to coast, its speed as a function of time will be as follows.

$$v = v_0 e^{-\left(\frac{B^2 \ell^2 t}{Rm}\right)}$$

- In terms of the quantities given, determine the power developed in the resistor as a function of time  $t$ .
- Show that the total energy produced in the resistor is equal to the initial kinetic energy of the bar.

END OF SECTION II, ELECTRICITY AND MAGNETISM

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, ELECTRICITY AND MAGNETISM, ONLY. DO NOT WORK ON ANY OTHER TEST MATERIALS.



E & M 3. A conducting rod is free to move on a pair of horizontal, frictionless conducting rails a distance  $\ell$  apart. The rails are connected at one end so a complete circuit is formed. The rod has a mass  $m$ , the resistance of the circuit is  $R$ , and there is a uniform magnetic field of magnitude  $B$  directed perpendicularly into the plane of the rails, as shown above. The rod and the rails have negligible resistance. At time  $t = 0$ , the rod has a speed  $v_0$  to the right. Determine each of the following in terms of  $\ell$ ,  $m$ ,  $R$ ,  $B$ , and  $v_0$ .

- The induced voltage in the rod at  $t = 0$
- The magnitude and the direction of the magnetic force on the rod at  $t = 0$
- The speed  $v$  of the rod as a function of time  $t$
- The total energy dissipated by the resistor beginning at  $t = 0$

END OF SECTION II, ELECTRICITY AND MAGNETISM

1987

PHYSICS C  
SECTION II, ELECTRICITY AND MAGNETISM  
Time—45 minutes  
3 Questions

ANSWER ALL OF THE QUESTIONS. EACH OF THE THREE QUESTIONS HAS EQUAL WEIGHT, BUT THE PARTS WITHIN A QUESTION MAY NOT HAVE EQUAL WEIGHT. SHOW YOUR WORK. CREDIT FOR YOUR ANSWERS DEPENDS ON THE QUALITY OF YOUR EXPLANATIONS.

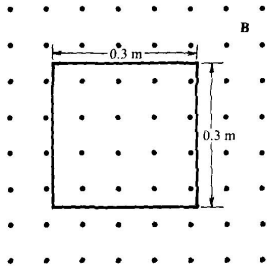
E&M 1. A total charge  $Q$  is distributed uniformly throughout a spherical volume of radius  $R$ . Let  $r$  denote the distance of a point from the center of the sphere of charge.

Use Gauss's law to derive an expression for the magnitude of the electric field at a point

- (a) outside the sphere,  $r > R$ ;
- (b) inside the sphere,  $r < R$ .

The electrostatic potential is assumed to be zero at an infinite distance from the sphere.

- (c) What is the potential at the surface of the sphere?
- (d) Determine the potential at the center of the sphere.

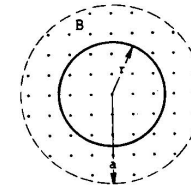


E&M 2. A square wire loop of resistance 6 ohms and side of length 0.3 meter lies in the plane of the page, as shown above. The loop is in a magnetic field  $\mathbf{B}$  that is directed out of the page. At time  $t = 0$ , the field has a strength of 2 teslas; it then decreases according to the equation  $B = 2e^{-4t}$ , where  $B$  is in teslas and  $t$  is in seconds.

- (a) Determine an expression for the flux through the loop as a function of time  $t$  for  $t > 0$ .
- (b) On the diagram above, indicate the direction of the current induced in the loop for time  $t > 0$ .
- (c) Determine an expression for the current induced in the loop for time  $t > 0$ .
- (d) Determine the total energy dissipated as heat during the time from zero to infinity.

6

1980



E&M 3. A spatially uniform magnetic field directed out of the page is confined to a cylindrical region of space of radius  $a$  as shown above. The strength of the magnetic field increases at a constant rate such that  $B = B_0 + Ct$ , where  $B_0$  and  $C$  are constants and  $t$  is time. A circular conducting loop of radius  $r$  and resistance  $R$  is placed perpendicular to the magnetic field.

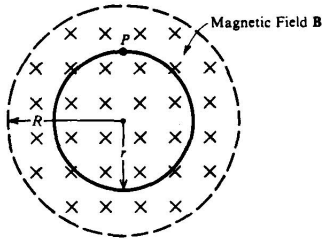
- (a) Indicate on the diagram above the direction of the induced current in the loop. Explain your choice.
- (b) Derive an expression for the induced current in the loop.
- (c) Derive an expression for the magnitude of the induced electric field at any radius  $r < a$ .
- (d) Derive an expression for the magnitude of the induced electric field at any radius  $r > a$ .

8

END OF SECTION II, ELECTRICITY AND MAGNETISM

7



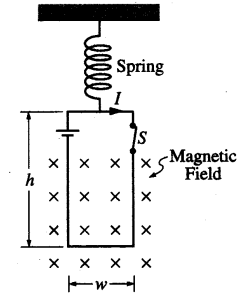


**E&M 3.** A spatially uniform magnetic field  $\mathbf{B}$ , perpendicular to the plane of the page, exists in a circular region of radius  $R = 0.75$  meter as shown above. A single wire loop of radius  $r = 0.5$  meter is placed concentrically in the magnetic field and in the plane of the page. The magnetic field increases into the page at a constant rate of 60 teslas per second.

- (a) Determine the induced emf in the loop.
- (b) Determine the magnitude and direction of the induced electric field at point  $P$  and indicate its direction on the diagram above.

The wire loop is replaced by an evacuated doughnut-shaped glass tube, within which a single electron orbits at a constant radius  $r = 0.5$  meter when the spatially uniform magnetic field is constant at  $10^{-4}$  tesla.

- (c) Determine the speed of the electron in this orbit.
- (d) The magnetic field is now made to increase at a constant rate of 60 teslas per second as in parts (a) and (b) above. Determine the tangential acceleration of the electron at the instant the field begins to increase.



**2006E3.** A loop of wire of width  $w$  and height  $h$  contains a switch and a battery and is connected to a spring of force constant  $k$ , as shown above. The loop carries a current  $I$  in a clockwise direction, and its bottom is in a constant, uniform magnetic field directed into the plane of the page.

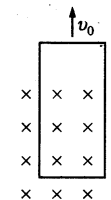
- a. On the diagram of the loop below, indicate the directions of the magnetic forces, if any, that act on each side of the loop.



- b. The switch  $S$  is opened, and the loop eventually comes to rest at a new equilibrium position that is a distance  $x$  from its former position. Derive an expression for the magnitude  $B_0$  of the uniform magnetic field in terms of the given quantities and fundamental constants.

The spring and loop are replaced with a loop of the same dimensions and resistance  $R$  but without the battery and switch. The new loop is pulled upward, out of the magnetic field, at constant speed  $v_0$ . Express algebraic answers to the following questions in terms of  $B_0$ ,  $v_0$ ,  $R$ , and the dimensions of the loop.

- c.i. On the diagram of the new loop below, indicate the direction of the induced current in the loop as the loop moves upward.



- ii. Derive an expression for the magnitude of this current.
- d. Derive an expression for the power dissipated in the loop as the loop is pulled at constant speed out of the field.
- e. Suppose the magnitude of the magnetic field is increased. Does the external force required to pull the loop at speed  $v_0$  increase, decrease, or remain the same? .

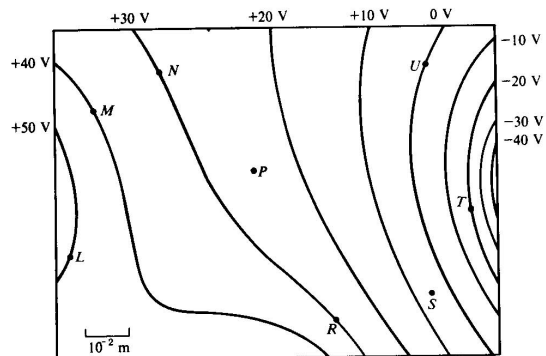
\_\_\_\_\_ Increases    \_\_\_\_\_Decreases    \_\_\_\_\_Remains the same

PHYSICS C  
SECTION II, ELECTRICITY AND MAGNETISM

Time—45 minutes

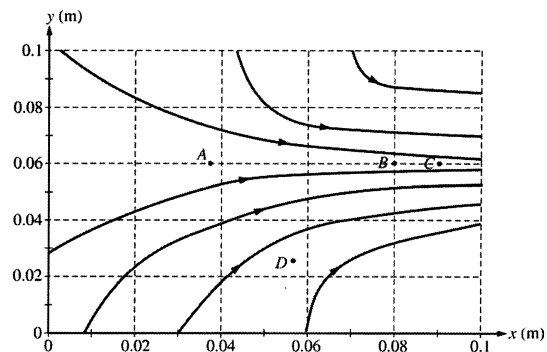
3 Questions

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**E&M 1.** Three point charges produce the electric equipotential lines shown on the diagram above.

- Draw arrows at points  $L$ ,  $N$ , and  $U$  on the diagram to indicate the direction of the electric field at these points.
- At which of the lettered points is the electric field  $E$  greatest in magnitude? Explain your reasoning.
- Compute an approximate value for the magnitude of the electric field  $E$  at point  $P$ .
- Compute an approximate value for the potential difference,  $V_M - V_S$ , between points  $M$  and  $S$ .
- Determine the work done by the field if a charge of  $+5 \times 10^{-12}$  coulomb is moved from point  $M$  to point  $R$ .
- If the charge of  $+5 \times 10^{-12}$  coulomb were moved from point  $M$  first to point  $S$ , and then to point  $R$ , would the answer to (e) be different, and if so, how?



Consider the electric field diagram above.

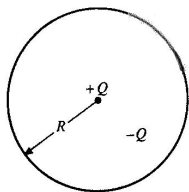
- Points  $A$ ,  $B$ , and  $C$  are all located at  $y = 0.06$  m.
  - At which of these three points is the magnitude of the electric field the greatest? Justify your answer.
  - At which of these three points is the electric potential the greatest? Justify your answer.
- An electron is released from rest at point  $B$ .
  - Qualitatively describe the electron's motion in terms of direction, speed, and acceleration.
  - Calculate the electron's speed after it has moved through a potential difference of 10 V.
- Points  $B$  and  $C$  are separated by a potential difference of 20 V. Estimate the magnitude of the electric field midway between them and state any assumptions that you make.
- On the diagram, draw an equipotential line that passes through point  $D$  and intersects at least three electric field lines.

PHYSICS C  
SECTION II, ELECTRICITY AND MAGNETISM

Time—45 minutes

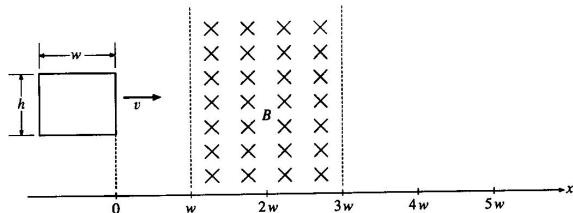
3 Questions

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E & M 1. A negative charge  $-Q$  is uniformly distributed throughout the spherical volume of radius  $R$  shown above. A positive point charge  $+Q$  is at the center of the sphere. Determine each of the following in terms of the quantities given and fundamental constants.

- (a) The electric field  $E$  outside the sphere at a distance  $r > R$  from the center
- (b) The electric potential  $V$  outside the sphere at a distance  $r > R$  from the center
- (c) The electric field inside the sphere at a distance  $r < R$  from the center
- (d) The electric potential inside the sphere at a distance  $r < R$  from the center



E & M 2. A rectangular loop of wire of resistance  $R$  and dimensions  $h$  and  $w$  moves with a constant speed  $v$  toward and through a region containing a uniform magnetic field of strength  $B$  directed into the plane of the page. The region has a width of  $2w$  as shown above.

- (a) For the interval after the right-hand edge of the loop has entered the field but before the left-hand side of the loop has reached the field, determine each of the following in terms of  $B$ ,  $w$ ,  $h$ ,  $v$ , and  $R$ .
  - i) The magnitude of the induced current in the loop
  - ii) The magnitude of the applied force required to move the loop at constant speed

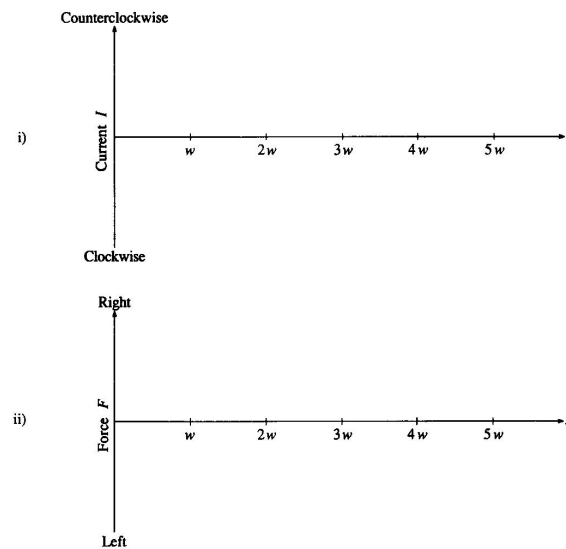
13

14

(b) On the axes below, plot the following as functions of position  $x$  of the right edge of the loop shown above.

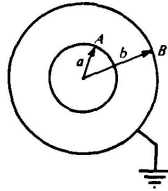
- i) The induced current  $I$  in the loop
- ii) The applied force  $F$  required to keep the loop moving at constant speed

Let counterclockwise current be positive, clockwise current be negative, forces to the right be positive, and forces to the left be negative. The graphs should begin with the loop in the position shown ( $x = 0$ ) and continue until the right edge of the loop is a distance  $2w$  to the right of the region containing the field ( $x = 5w$ ).



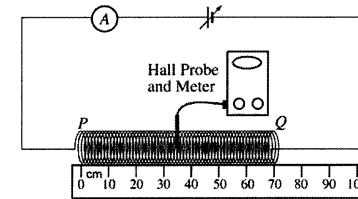
PHYSICS C  
SECTION II, ELECTRICITY AND MAGNETISM  
Time—45 minutes  
3 Questions

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E&M 1. Two concentric, conducting spherical shells,  $A$  and  $B$ , have radii  $a$  and  $b$ , respectively, ( $a < b$ ). Shell  $B$  is grounded, whereas shell  $A$  is maintained at a positive potential  $V_0$ .

- Using Gauss's law, develop an expression for the magnitude  $E$  of the electric field at a distance  $r$  from the center of the shells in the region between the shells. Express your answer in terms of the charge  $Q$  on the inner shell.
- By evaluating an appropriate integral, develop an expression for the potential  $V_0$  in terms of  $Q$ ,  $a$ , and  $b$ .
- Develop an expression for the capacitance of the system in terms of  $a$  and  $b$ .



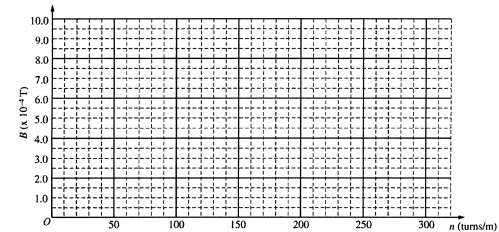
A student performs an experiment to obtain the value of  $\mu_0$ , the magnetic permeability of vacuum. She measures the magnetic field along the axis of the long, 100-turn solenoid  $PQ$  shown above. She connects ends  $P$  and  $Q$  of the solenoid to a variable power supply and an ammeter as shown. End  $P$  of the solenoid is taped at the 0 cm mark of a meterstick. The solenoid can be stretched so that the position of end  $Q$  can be varied. The student then positions a Hall probe\* in the center of the solenoid to measure the magnetic field along its axis. She measures the field for a fixed current of 3.0 A and various positions of the end  $Q$ . The data she obtains are shown below.

Trial	Position of End $Q$ (cm)	Measured Magnetic Field (T) (directed from $P$ to $Q$ )	$n$ (turns/m)
1	40	$9.70 \times 10^{-4}$	
2	50	$7.70 \times 10^{-4}$	
3	60	$6.80 \times 10^{-4}$	
4	80	$4.90 \times 10^{-4}$	
5	100	$4.00 \times 10^{-4}$	

- Complete the last column of the table above by calculating the number of turns per meter.

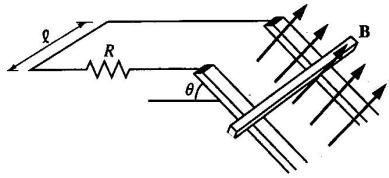
\* A Hall Probe is a device used to measure the magnetic field at a point.

- On the axes below, plot the measured magnetic field  $B$  versus  $n$ . Draw a best-fit straight line for the data points.



- From the graph, obtain the value of  $\mu_0$ , the magnetic permeability of vacuum.
- Using the theoretical value of  $\mu_0 = 4\pi \times 10^{-7} \text{ TM/A}$ , determine the percent error in the experimental value of  $\mu_0$  computed in part (c).

1998 PHYSICS C—E & M



17

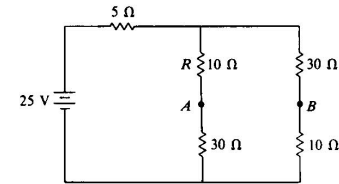
E & M 3. A conducting bar of mass  $m$  is placed on two long conducting rails a distance  $l$  apart. The rails are inclined at an angle  $\theta$  with respect to the horizontal, as shown above, and the bar is able to slide on the rails with negligible friction. The bar and rails are in a uniform and constant magnetic field of magnitude  $B$  oriented perpendicular to the incline. A resistor of resistance  $R$  connects the upper ends of the rails and completes the circuit as shown. The bar is released from rest at the top of the incline. Express your answers to parts (a) through (d) in terms of  $m$ ,  $l$ ,  $\theta$ ,  $B$ ,  $R$ , and  $g$ .

- Determine the current in the circuit when the bar has reached a constant final speed.
- Determine the constant final speed of the bar.
- Determine the rate at which energy is being dissipated in the circuit when the bar has reached its constant final speed.
- Express the speed of the bar as a function of time  $t$  from the time it is released at  $t = 0$ .
- Suppose that the experiment is performed again, this time with a second identical resistor connecting the rails at the bottom of the incline. Will this affect the final speed attained by the bar, and if so, how? Justify your answer.

**STOP**

END OF SECTION II, ELECTRICITY AND MAGNETISM

1986



18

E & M 2. Five resistors are connected as shown above to a 25-volt source of emf with zero internal resistance.

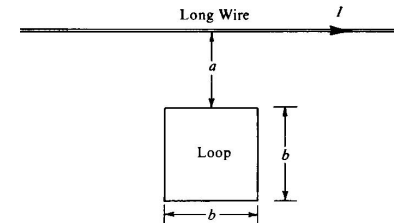
- Determine the current in the resistor labeled  $R$ .

A 10-microfarad capacitor is connected between points  $A$  and  $B$ . The currents in the circuit and the charge on the capacitor soon reach constant values. Determine the constant value for each of the following.

- The current in the resistor  $R$
- The charge on the capacitor

The capacitor is now replaced by a 2.0-henry inductor with zero resistance. The currents in the circuit again reach constant values. Determine the constant value for each of the following.

- The current in the resistor  $R$
- The current in the inductor



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E & M 3. A long wire carries a current in the direction shown above. The current  $I$  varies linearly with time  $t$  as follows.

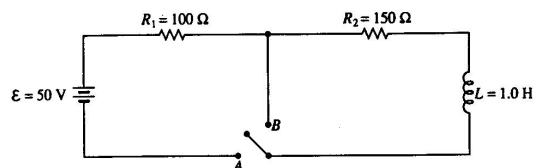
$$I = ct, \text{ where } c \text{ is a positive constant}$$

The long wire is in the same plane as a square loop of wire of side  $b$ , as shown in the diagram. The side of the loop nearest the long wire is parallel to it and a distance  $a$  from it. The loop has a resistance  $R$  and is fixed in space.

- Determine the magnetic field  $B$  at a distance  $r$  from the long wire as a function of time.
- Indicate on the diagram the direction of the induced current in the loop.
- Determine the induced current in the loop.
- State whether the magnetic force on the loop is toward or away from the wire.
- Determine the magnitude of the magnetic force on the loop as a function of time.

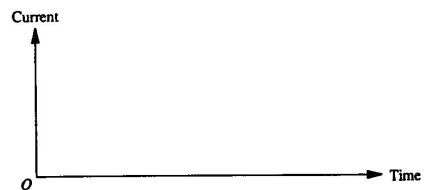
END OF SECTION II, ELECTRICITY AND MAGNETISM

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E & M 2. In the circuit above, the switch is initially open as shown. At time  $t = 0$ , the switch is closed to position  $A$ .

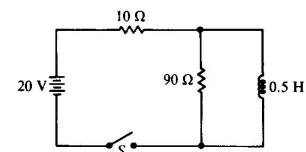
- Determine the current immediately after the switch is closed.
- Determine the current after a long time when a steady state situation has been reached.
- On the axes below, sketch a graph of the current *versus* time after the switch is closed.



- Determine the energy stored in the inductor  $L$  when the steady state has been reached.

Some time after the steady state situation has been reached, the switch is moved almost instantaneously from position  $A$  to position  $B$ .

- Determine the current in the inductor immediately after the position of the switch is changed.
- Determine the potential difference across the inductor immediately after the position of the switch is changed.
- What happens to the energy stored in the inductor as calculated in part (d) above?



E&M 3. In the circuit shown above, the switch  $S$  is initially open and all currents are zero. For the instant immediately after the switch is closed, determine each of the following.

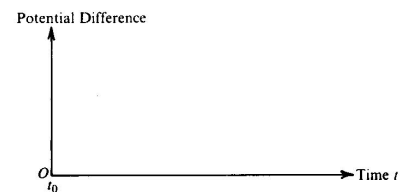
- The potential difference across the 90-ohm resistor
- The rate of change of current in the inductor

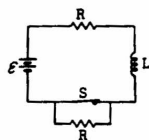
The switch has remained closed for a long time. Determine each of the following.

- The current in the inductor
- The energy stored in the inductor

Later, at time  $t_0$ , the switch is reopened.

- For the instant immediately after the switch is reopened, determine the potential difference across the 90-ohm resistor.
- On the axes below, sketch a graph of the potential difference across the 90-ohm resistor for  $t > t_0$ .





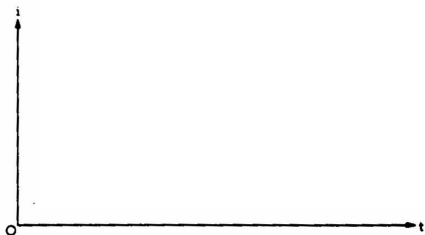
E&M 3. When the switch  $S$  in the circuit shown above is closed, an inductance  $L$  is in series with a resistance  $R$  and battery of emf  $\mathcal{E}$ .

(a) Determine the current  $i_A$  in the circuit after the switch  $S$  has been closed for a very long time.

After being closed for a long time, the switch  $S$  is opened at time  $t = 0$ .

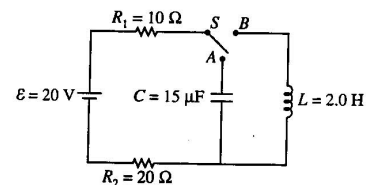
(b) Determine the current  $i_B$  in the circuit after the switch has been open for a very long time.

(c) On the axes below, sketch a graph of the current as a function of time  $t$  for  $t \geq 0$  and indicate the values of the currents  $i_A$  and  $i_B$  on the vertical axis.



(d) By relating potential differences and emf's around the circuit, write the differential equation that can be used to determine the current as a function of time.

(e) Write the equation for the current as a function of time for all time  $t \geq 0$ .



E & M. 2. In the circuit shown above, the switch  $S$  is initially in the open position shown, and the capacitor is uncharged. A voltmeter (not shown) is used to measure the correct potential difference across resistor  $R_1$ .

(a) On the circuit diagram above, draw the voltmeter with the proper connections for correctly measuring the potential difference across resistor  $R_1$ .

(b) At time  $t = 0$ , the switch is moved to position  $A$ . Determine the voltmeter reading for the time immediately after  $t = 0$ .

(c) After a long time, a measurement of potential difference across  $R_1$  is again taken. Determine for this later time each of the following.

i. The voltmeter reading

ii. The charge on the capacitor

(d) At a still later time  $t = T$ , the switch  $S$  is moved to position  $B$ . Determine the voltmeter reading for the time immediately after  $t = T$ .

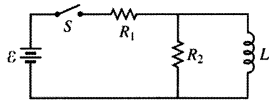
(e) A long time after  $t = T$ , the current in  $R_1$  reaches a constant final value  $I_f$ .

i. Determine  $I_f$ .

ii. Determine the final energy stored in the inductor.

(f) Write, but do not solve, a differential equation for the current in resistor  $R_1$  as a function of time  $t$  after the switch is moved to position  $B$ .

2005

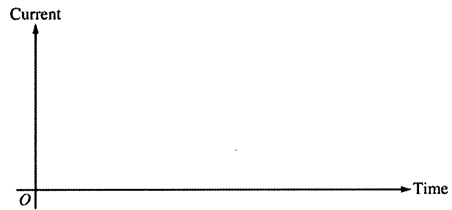


24

E&M 2

In the circuit shown above, resistors 1 and 2 of resistance  $R_1$  and  $R_2$ , respectively, and an inductor of inductance  $L$  are connected to a battery of emf  $\mathcal{E}$  and a switch  $S$ . The switch is closed at time  $t = 0$ . Express all algebraic answers in terms of the given quantities and fundamental constants.

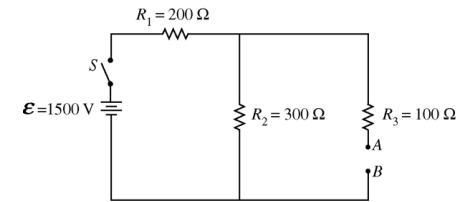
- Determine the current through resistor 1 immediately after the switch is closed.
- Determine the magnitude of the initial rate of change of current,  $dI/dt$ , in the inductor.
- Determine the current through the battery a long time after the switch has been closed.
- On the axes below, sketch a graph of the current through the battery as a function of time.



Some time after steady-state has been reached, the switch is opened.

- Determine the voltage across resistor 2 just after the switch has been opened.

2008



25

E&M. 2.

In the circuit shown above,  $A$  and  $B$  are terminals to which different circuit components can be connected.

- Calculate the potential difference across  $R_2$  immediately after the switch  $S$  is closed in each of the following cases.
  - A  $50 \Omega$  resistor connects  $A$  and  $B$ .
  - A  $40 \text{ mH}$  inductor connects  $A$  and  $B$ .
  - An initially uncharged  $0.80 \mu\text{F}$  capacitor connects  $A$  and  $B$ .
- The switch gets closed at time  $t = 0$ . On the axes below, sketch the graphs of the current in the  $100 \Omega$  resistor  $R_3$  versus time  $t$  for the three cases. Label the graphs  $R$  for the resistor,  $L$  for the inductor, and  $C$  for the capacitor.





Electricity and Magnetism Free Response Answers

1.) 1992 E&M 3

- a.) Out of the page.    b.) i.)  $\Phi_m = \frac{\mu_0 \alpha C}{2\pi} (1 - \beta t) \ln\left(\frac{b}{a}\right)$     ii.)  $\mathcal{E} = -\frac{\mu_0 \alpha C \beta}{2\pi} \ln\left(\frac{b}{a}\right)$     c.) i.) counterclockwise  
ii.) counterclockwise  
d.) The force is directed toward the wire.

2.) 1994 E&M 2

- a.) i.) shuttle end    ii.)  $V = 5016 \text{ V}$     b.)  $I = 0.5016 \text{ A}$     c.) i.)  $F = 0.331 \text{ N}$     ii.) to the left  
d.)  $\Delta U = Pt = I^2 R t = 1.52 \times 10^9 \text{ J}$     e.) If the current was forced to flow the other way, the direction of the magnetic force on the current would be reversed. This force would do work on the shuttle, and the resulting gain in energy would cause an increase in radius of the orbit.

3.) 1995 E&M 3

- a.)  $\mathcal{E} = BHv_0$     b.)  $I = \frac{BHv_0}{R}$     d.)  $v = v_0 \exp\left(-\frac{B^2 H^2 t}{mR}\right)$   
c.) The current must be clockwise to create a magnetic field that opposes the applied field (Lenz's law).  
e.) Upon entering the field ( $t = 0$ ) the glider will slow down exponentially until the entire loop is in the field ( $t = t_1$ ). At this time the glider will maintain a constant speed until the front edge of the loop begins to come out of the field ( $t = t_2$ ). The glider will again start to slow down exponentially until the loop is completely out of the field ( $t = t_3$ ) where the glider will once again move at a constant speed.

4.) 1984 E&M 3

- a.) clockwise current    b.)  $I = \frac{B\ell v_0}{R}$     c.) left    d.)  $F = \frac{B^2 \ell^2 v_0}{R}$     e.)  $P = \frac{B^2 \ell^2 v_0^2}{R} \exp\left(-\frac{2B^2 \ell^2 t}{mR}\right)$   
f.) show that  $E = \int_0^{\infty} P dt = \int_0^{\infty} I^2 R dt = \frac{1}{2} m v_0^2$

5.) 1991 E&M 3

- a.)  $\mathcal{E} = B\ell v_0$     b.)  $F_M = \frac{B^2 \ell^2 v_0}{R}$ , to the left    c.)  $v = v_0 \exp\left(-\frac{B^2 \ell^2 t}{mR}\right)$     d.)  $E = \frac{1}{2} m v_0^2$

6.) 1987 E&M 1

- a.)  $E = \frac{Q}{4\pi\epsilon_0 r^2}$     b.)  $E = \frac{Qr}{4\pi\epsilon_0 R^3}$     c.)  $V = \frac{Q}{4\pi\epsilon_0 R}$     d.)  $V = \frac{3Q}{8\pi\epsilon_0 R}$

7.) 1987 E&M 2

- a.)  $\Phi = 0.18e^{-4t}$     b.) counterclockwise    c.)  $i = 0.12e^{-4t}$     d.)  $E = 0.108 \text{ J}$

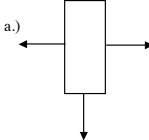
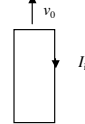
8.) 1980 E&M 3

- a.) clockwise    b.)  $I = -\frac{\pi r^2 C}{R}$     c.)  $E = \frac{rC}{2}$     d.)  $E = \frac{a^2 C}{2r}$

9.) 1985 E&M 3

- a.)  $\mathcal{E} = 47 \text{ V}$     b.)  $E = 15 \frac{\text{V}}{\text{m}}$     c.)  $v = 8.8 \times 10^6 \frac{\text{m}}{\text{s}}$     d.)  $a = 2.6 \times 10^{12} \frac{\text{m}}{\text{s}^2}$

10.) 2006 E&M 3

- a.)     b.)  $B_0 = \frac{kx}{Iw}$     c.) i.)     ii.)  $I_{\text{ind}} = \frac{B_0 w v_0}{R}$   
d.)  $P = \frac{B_0^2 w^3 v_0^2}{R}$     e.) Increases due to more induced current resulting in more force

11.) 1986 E&M 1

- a.) All vectors drawn with correct sense (higher to lower potential) and all vectors drawn perpendicular to equipotential lines.  
b.) Magnitude of electric field is greatest at point T because equipotential lines are closest together near T.  
c.)  $E = \frac{\Delta V}{\Delta x} = 500 \frac{\text{V}}{\text{m}}$     d.)  $V_M - V_S = 35 \text{ V}$     e.)  $W = q\Delta V = 5 \times 10^{-11} \text{ J}$   
f.) No, work does not depend upon path.

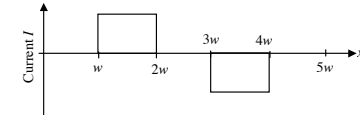
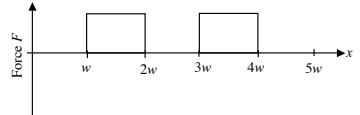
12.) 2005 E&M 1

- a.) i.) point C where lines are closest together    ii.) point A The field points in the direction of decreasing potential.    b.) i.) The electron moves to the left with increasing speed. Magnitude of acceleration is decreasing.  
b.) ii.)  $v = 1.9 \times 10^6 \text{ m/s}$     c.)  $E = 2000 \text{ V/m}$     d.) curved line concave up must be perpendicular to the electric field lines

13.) 1989 E&M 1

- a.)  $E = 0$     b.)  $V = 0$     c.)  $E = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r^2} - \frac{r}{R^3}\right)$     d.)  $V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} + \frac{r^2}{2R^2} - \frac{3}{2R}\right)$

14.) 1989 E&M 2

- a.) i.)  $I = \frac{Bhv}{R}$     ii.)  $F_A = \frac{B^2 h^2 v}{R}$   
b.) i.) Counterclockwise    ii.) Right  
    

15.) 1983 E&M 1

a.)  $E = \frac{Q}{4\pi\epsilon_0 r^2}$     b.)  $V_0 = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab}$     c.)  $C = \frac{4\pi\epsilon_0 ab}{b-a}$

16.) 2005 E&M 3

a.) 

n
250
200
160
125
100

 graph  $B$  vs  $n$  and draw a line of best fit using a straight edge  
 percent error =  $\frac{|\mu_0 - \mu_{0ex}|}{\mu_0}(100)$

c.) find slope of the line of best fit and set equal to  $\mu_0 I$ . Solve for  $\mu_0$  ( $I = 3.0$  A).

17.) 1998 E&M 3

a.)  $I = \frac{mgR\sin\theta}{B}$     b.)  $v = \frac{mgR\sin\theta}{B^2\ell^2}$     c.)  $P = \frac{(m^2 g^2 \sin^2 \theta) R}{B^2 \ell^2}$     d.)  $v(t) = \frac{mgR\sin\theta}{B^2\ell^2} \left( 1 - \exp\left(-\frac{B^2\ell^2 t}{mR}\right) \right)$

e.) Yes, the final speed of the bar decreases because the two resistors are in parallel across the  $emf$  in the bar and the new effective resistance is  $R/2$ . This results in a speed reduction using the expression from (b).

18.) 1986 E&M 2

a.)  $I_R = 0.5$  A    b.)  $I_R = 0.5$  A    c.)  $Q = 100$   $\mu$ C    d.)  $I_R = 0.9375$  A    e.)  $I_L = 0.624$  A

19.) 1986 E&M 3

a.)  $B = \frac{\mu_0 c t}{2\pi r}$     b.) Counterclockwise    c.)  $i = \frac{\mu_0 c b}{2\pi R} \ln\left(\frac{a+b}{a}\right)$     d.) Away from the wire.

e.)  $F_{net} = \left( \frac{\mu_0^2 c^2 b^2 t}{4\pi^2 R} \ln\left(\frac{a+b}{a}\right) \right) \left( \frac{1}{a} - \frac{1}{a+b} \right)$

20.) 1991 E&M 2

a.)  $I_t = 0$     b.)  $I = 0.2$  A    c.) Current starts at 0 and approaches 0.2 A modified exponential growth  
 d.)  $U_L = 0.02$  J    e.)  $I = 0.2$  A    f.)  $V_L = 30$  V    g.) The stored energy is dissipated in the resistor as thermal energy.

21.) 1987 E&M 3

a.)  $V_{90\Omega} = 18$  V    b.)  $\left| \frac{di}{dt} \right| = 36$   $\frac{A}{s}$     c.)  $i = 2$  A    d.)  $U_L = 1.0$  J    e.)  $V_{90\Omega} = 180$  V

f.) Voltage starts at 180 V and exponentially decays towards zero as time approaches infinity.

22.) 1982 E&M 3

a.)  $i_A = \frac{\mathcal{E}}{R}$     b.)  $i_B = \frac{\mathcal{E}}{2R}$

c.) Current starts at  $\frac{\mathcal{E}}{R}$  and exponentially decays towards  $\frac{\mathcal{E}}{2R}$  as time approaches infinity.

d.)  $\mathcal{E} - 2Ri - L \frac{di}{dt} = 0$     e.)  $i(t) = \frac{\mathcal{E}}{2R} \left( 1 + \exp\left(-\frac{2Rt}{L}\right) \right)$

23.) 1998 E&M 2

a.) Draw meter connected parallel to  $R_1$ .    b.)  $V = 6.67$  V    c.) i.)  $V = 0$     ii.)  $Q = 300$   $\mu$ C

d.)  $V = 0$     e.) i.)  $I = \frac{\mathcal{E}}{R_1 + R_2} = 0.67$  A    ii.)  $U_L = 0.444$  J    f.)  $\mathcal{E} - I(R_1 + R_2) - L \frac{dI}{dt} = 0$

24.) 2005 E&M 1

a.)  $I_{int} = \frac{\mathcal{E}}{R_1 + R_2}$     b.)  $\frac{dI}{dt} = \frac{R_2 \mathcal{E}}{(R_1 + R_2)L}$     c.)  $I_{batt} = \frac{\mathcal{E}}{R_1}$

d.) graph starts at initial  $I$  found in (a) and asymptotically approaches current final  $I$  found in (c) concave down

e.)  $V_{R_2} = \frac{\mathcal{E} R_2}{R_1}$

25.) 2008 E&M 2

a.) i.)  $V_{R_2} = 500$  V    ii.)  $V_{R_2} = 900$  V    iii.)  $V_{R_2} = 410$  V

b.) Resistor graph:

The current is constant with the value less than the initial value of the capacitor graph or the steady state value of the inductor graph.

Inductor graph:

The inductor initially opposes the flow of current, so the initial current in that branch is zero. Eventually, the inductor acts like a wire and does not impede the flow of charge, as the rate of change of current decreases to zero. The graph starts at  $I = 0$  at time  $t = 0$  and is concave down and asymptotic to the initial current in the capacitor case.

Capacitor graph:

Initially, the capacitor is uncharged and current is a maximum in the branch containing  $R_3$ . As the capacitor charges the current in the branch decreases exponentially to zero concave up.