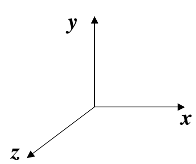
Magnetic Fields and Forces

Permanent Magnets

- **All magnets have both a north and a south pole.**
- **It is impossible to have a magnetic monopole.**
- **Opposite poles attract.**
- **Similar poles repel.**
- **The north pole of a compass points towards the magnetic south pole of a magnet.**
- **The direction of the magnetic field is defined as the direction in which the north pole of a compass needle points.**

Right-Handed Coordinate Systems



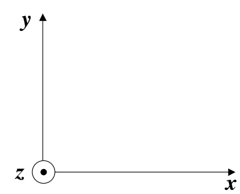
Out of the plane of the page.

↑ **In the plane of the page.**

⊙ **Out of the plane of the page.**

⊗ **Into the plane of the page.**

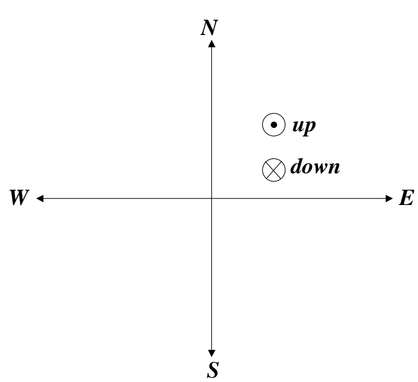
Right-Handed Coordinate Systems

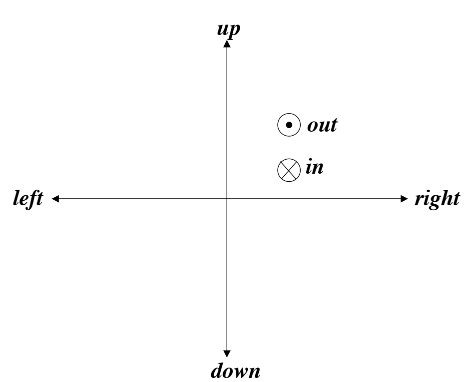


+*x* points to the right.

+*y* points upward.

+*z* points out of the plane of the page.





Magnetism

1.) A magnetic field exerts a force on a moving charge or an electric current.

2.) A moving charge or an electric current produces a magnetic field.

The SI unit of magnetic field (*B*) is a Tesla (T).

$$\mathrm{T}\,[=]\,\frac{\mathrm{N}}{\mathrm{A \cdot m}}\,[=]\,\frac{\mathrm{N \cdot s}}{\mathrm{C \cdot m}}$$

Force on an Electric Charge moving in a Magnetic Field

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \boxed{\vec{F}_M = q\vec{v} \times \vec{B}}$$

F **– Force due to magnetic field (N)**

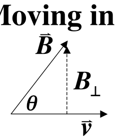
q **– charge of particle (C)**

v **– velocity of the charged particle (m/s)**

B **– magnetic field strength (T)**

Force on an Electric Charge Moving in a Magnetic Field

$$\vec{F} = q\vec{v} \times \vec{B}$$



- **The magnitude of the force depends upon the orientation of the velocity with respect to the magnetic field.**

$$F = qvB\sin\theta$$

θ **- the angle between the velocity vector of the charge and the direction of the magnetic field.**

- **The direction of the force is determined using the *right-hand rule*. (*v* is crossed into *B*)**

Right-Hand Rule #1

1.) Orient your right hand so that your outstretched fingers point along the direction of the motion of the particle (*v*).

2.) Orient the palm of your right hand so that it faces in the direction of the magnetic field *(B)* *or* curl the fingers of your right hand in the direction of the magnetic field.

3.) Your thumb points in the direction of the magnetic force (*F*) on a positive charge.

If the charge is negative the force is in the opposite direction *or* use your left hand.

Magnetic Flux

- ***Magnetic flux*** **through a surface is defined as:**

$$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$$

- **Magnetic flux is a *scalar* quantity.**
- **If *B* is uniform over a plane surface *A* then**

$$\Phi_B = BA\cos\theta$$

- **The SI unit of magnetic flux is called the *weber* (Wb).**

$$1\ \mathrm{Wb} = 1\ \mathrm{T \cdot m^2} = 1\ \mathrm{N \cdot m/A}$$

Gauss's Law for Magnetism

The total magnetic flux through a closed surface is always zero.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Motion of Charged Particles in a Magnetic Field

The motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

The force is always perpendicular to *v*, so cannot change the magnitude of the velocity, only its direction.

$$F = |q|vB = ma = m\frac{v^2}{R}$$

Force on an Electric Current in a Magnetic Field

$$\boxed{F = I\vec{\ell} \times B} \qquad \boxed{\vec{F} = \int Id\vec{\ell} \times \vec{B}}$$

F **– Force due to magnetic field (N)**

ℓ **– length of wire in magnetic field (m)**

I **– electric current in wire (A)**

B **– magnetic field strength (T)**

Force on an Electric Current in a Magnetic Field

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

- **The magnitude of the force depends upon the orientation of the current with respect to the magnetic field.**

$$F = I\ell B\sin\theta$$

θ **- the angle between direction of wire and the direction of the magnetic field.**

- **The direction of the force is determined using the *right-hand rule*. (ℓ is crossed into *B*)**

Right-Hand Rule #2

1.) **Orient your right hand so that your outstretched fingers point in the direction of the current (*I*) in the wire.**

2.) **Orient the palm of your right hand so that it faces in the direction of the magnetic field *(B)* or *curl* the fingers of your right hand in the direction of the magnetic field.**

3.) **Your thumb points in the direction of the magnetic force (*F*) on the wire.**

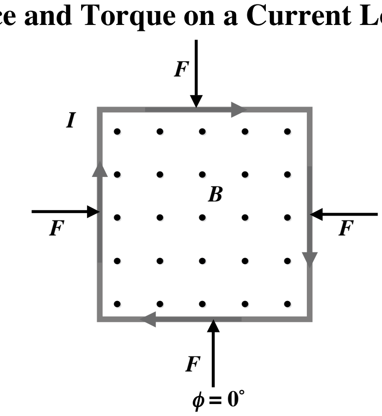
Magnetic Force on an Infinitesimal Wire Segment

If the conductor is not straight, it can be divided into infinitesimal segments $d\vec{\ell}$

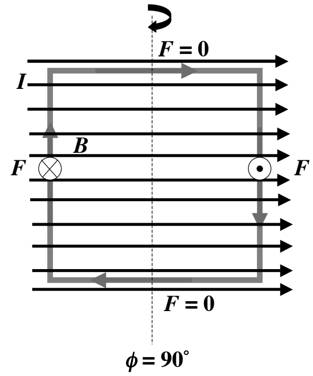
$$d\vec{F} = Id\vec{\ell} \times \vec{B}$$

The force is then found by integrating along the wire.

Force and Torque on a Current Loop



Force and Torque on a Current Loop



Force and Torque on a Current Loop

If a loop of current-carrying wire is in a magnetic field the total torque τ on the loop is

$$\tau = IBA\sin\phi$$

- **I – Current in loop (A)**
- **B – magnetic field strength (T)**
- **A – Area of loop (m^2)**
- **ϕ – Angle between field and a line perpendicular to the plane of the loop**

Torque on a Solenoid

A *solenoid* is a helical winding of wire, such as a coil wound on a circular cylinder. For a solenoid with N turns in a uniform field B the torque is

$$\tau = NIBA\sin\phi$$

ϕ – Angle between field and the axis of the solenoid.

Total Force on a Moving Particle

When a charged particle moves through a region in space where *both* electric and magnetic fields are present, both fields exert forces on the particle.

The *total force is the vector sum* of the electric and magnetic forces.

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Sources of Magnetic Fields

Magnetic Field of a Point Charge with Constant Velocity

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

- **r – radial distance from the *source point* to the *field point* (m)**
- **$\hat{r}$ – unit vector that points *from the source point to the field point***
- **μ_0 – permeability of free space ($4\pi \times 10^{-7}$ T · m/A)**

Magnetic Field of a Current Element
(Law of Biot and Savart)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell} \times \hat{r}}{r^2}$$

We can use this law to find the total magnetic field at any point in space due to the current in a complete circuit by integrating over all segments *dℓ* that carry current.

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{\ell} \times \hat{r}}{r^2}}$$

Magnetic Field of a Current Element
(Law of Biot and Savart)

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{\ell} \times \hat{r}}{r^2}$$

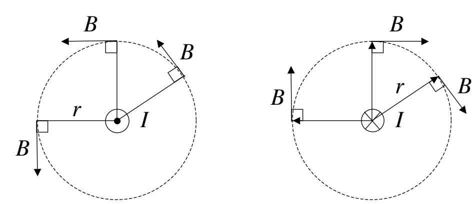
- **r – radial distance from the *source point* to the *field point* (m)**
- **$\hat{r}$ – unit vector that points *from the source point to the field point***
- **μ_0 – permeability of free space $(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})$**

Magnetic Field Due to an Electric Current in a Straight Wire

$$\boxed{B = \frac{\mu_0}{2\pi}\cdot\frac{I}{r}}$$

- **The magnetic field consists of concentric circles with the wire at their center.**
- **The direction of the magnetic field is determine using the *right-hand rule*. When the thumb points in the direction of the current, the fingers wrapped around the wire point in the direction of the magnetic field.**

Right-Hand Rule #3



Force Between Parallel Conductors

The lower conductor produces a magnetic field that, at the position of the upper conductor, has magnitude

$$B_2 = \frac{\mu_0}{2\pi}\cdot\frac{I_2}{r}$$

The force that this field exerts on a length ℓ of the upper conductor equal to

$$\vec{F} = I_1\vec{\ell} \times \vec{B}_2$$

Force Between Parallel Conductors

The force per unit length between two current-carrying conductors is therefore

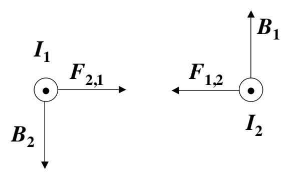
$$\boxed{\frac{F}{\ell} = \frac{\mu_0}{2\pi}\cdot\frac{I_1 I_2}{r}}$$

The direction of the force on the upper conductor is determined by applying the right-hand rule to

$$\vec{F} = I_1\vec{\ell} \times \vec{B}_2$$

Force Between Parallel Conductors

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r} \qquad \vec{F} = I_1 \vec{\ell} \times \vec{B}_2$$



$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Force Between Parallel Conductors

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r} \qquad \vec{F} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Ampere's Law

This is the magnetic equivalent of Gauss's Law

$$\boxed{\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}}$$

This is useful for symmetrical current distributions. (inside and outside of long wires, coaxial cylinders, and the center-axis of a solenoid.)

Current Density

The current *per unit cross-section area* is called the current density *J:*

$$J = \frac{I}{A} \quad \left(\frac{\text{A}}{\text{m}^2}\right)$$

This is useful for finding the magnetic field inside cylindrical conductors when using Ampere's Law.

$$I_{enclosed} = \int \vec{J} \cdot d\vec{A}$$