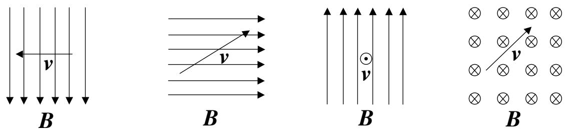
Example 1:
A positive charge with a velocity *v* is moving through a uniform magnetic field *B* as shown in the figures below. Use the right-hand rule to determine the direction of the magnetic force on the charge.

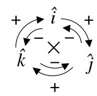


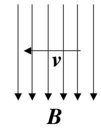
Example 1

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\hat{i} \times \hat{j} = \hat{k}$ $\quad \hat{j} \times \hat{i} = -\hat{k}$
$\hat{j} \times \hat{k} = \hat{i}$ $\quad \hat{k} \times \hat{j} = -\hat{i}$
$\hat{k} \times \hat{i} = \hat{j}$ $\quad \hat{i} \times \hat{k} = -\hat{j}$

$\hat{i} \times \hat{i} = 0$
$\hat{j} \times \hat{j} = 0$
$\hat{k} \times \hat{k} = 0$



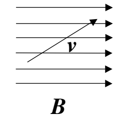


$\vec{F}_M = qv_x(-\hat{i}) \times B_y(-\hat{j})$

$\vec{F}_M = qv_x B_y(-\hat{i}) \times (-\hat{j})$

$\vec{F}_M = qv_x B_y \hat{k}$

+z or out ⊙



$\vec{F}_M = q\left(v_x\hat{i} + v_y\hat{j}\right) \times B_x(\hat{i})$

$\vec{F}_M = q\left(v_x B_x(\hat{i} \times \hat{i}) + v_y B_x(\hat{j} \times \hat{i})\right)$

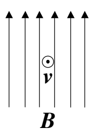
$\vec{F}_M = q\left(v_x B_x(0) + v_y B_x(-\hat{k})\right)$

$\vec{F}_M = qv_y B_x(-\hat{k})$

−z or in ⊗

Example 1

$\vec{F}_M = q\vec{v} \times \vec{B}$

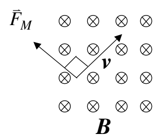


$\vec{F}_M = qv_z(\hat{k}) \times B_y(\hat{j})$

$\vec{F}_M = qv_z B_y(\hat{k} \times \hat{j})$

$\vec{F}_M = qv_z B_y(-\hat{i})$

−x or left ←



$\vec{F}_M = q\left(v_x\hat{i} + v_y\hat{j}\right) \times B_z(-\hat{k})$

$\vec{F}_M = q\left(v_x B_z\left(\hat{i} \times (-\hat{k})\right) + v_y B_z\left(\hat{j} \times (-\hat{k})\right)\right)$

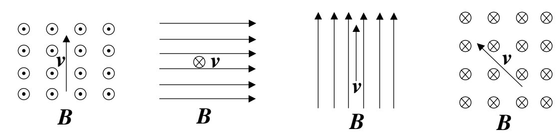
$\vec{F}_M = q\left(v_x B_z(\hat{j}) + v_y B_z(-\hat{i})\right)$

$\vec{F}_M = qv_y B_z(-\hat{i}) + qv_x B_z(\hat{j})$

Quadrant II ↖

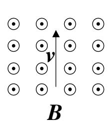
⊥ to $\vec{v}$ and $\vec{B}$

Example 2:
A negative charge with a velocity *v* is moving through a uniform magnetic field *B* as shown in the figures below. Use the right-hand rule to determine the direction of the magnetic force on the charge.



Example 2

$\vec{F}_M = q\vec{v} \times \vec{B}$



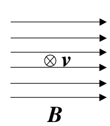
$\vec{F}_M = -qv_y(\hat{j}) \times B_z(\hat{k})$

$\vec{F}_M = -qv_y B_z(\hat{j} \times \hat{k})$

$\vec{F}_M = -qv_y B_z(\hat{i})$

$\vec{F}_M = qv_y B_z(-\hat{i})$

−x or left ←



$\vec{F}_M = -qv_z(-\hat{k}) \times B_x(\hat{i})$

$\vec{F}_M = -qv_z B_x(-\hat{k} \times \hat{i})$

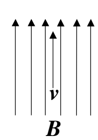
$\vec{F}_M = -qv_z B_x(-\hat{j})$

$\vec{F}_M = qv_z B_x(\hat{j})$

+y or up ↑

Example 2

$\vec{F}_M = q\vec{v} \times \vec{B}$



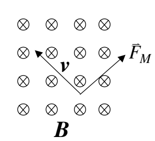
$\vec{F}_M = -qv_y(\hat{j}) \times B_y(\hat{j})$

$\vec{F}_M = -qv_y B_y(\hat{j} \times \hat{j})$

$\vec{F}_M = -qv_y B_y(0)$

$\vec{F}_M = 0$

No force



$\vec{F}_M = -q\left(v_x(-\hat{i}) + v_y\hat{j}\right) \times B_z(-\hat{k})$

$\vec{F}_M = -q\left(v_x B_z\left((-\hat{i}) \times (-\hat{k})\right) + v_y B_z\left(\hat{j} \times (-\hat{k})\right)\right)$

$\vec{F}_M = -q\left(v_x B_z(-\hat{j}) + v_y B_z(-\hat{i})\right)$

$\vec{F}_M = qv_y B_z(\hat{i}) + qv_x B_z(\hat{j})$

Quadrant I ↗

⊥ to $\vec{v}$ and $\vec{B}$

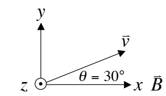
Example 3:

A beam of protons moves at 3 x 10^5 m/s through a uniform magnetic field with magnitude 2 T that is directed along the positive *x*-axis. The velocity of each proton lies in the *xy*-plane at an angle of 30° to the +*x*-axis. Find the force on the proton.

Example 3 :

$$q = 1.6\times10^{-19}\ \text{C},\ v = 3\times10^{5}\ \frac{\text{m}}{\text{s}}\ (xy\text{-plane } 30° \text{ wrt } x\text{-axis})$$

$$\vec{B} = (2\ \text{T})\hat{i},\ \vec{F}_M = ?$$



$$\vec{v} = v_x\hat{i} + v_y\hat{j} = (v\cos\theta)\hat{i} + (v\sin\theta)\hat{j}$$

$$\vec{F}_M = q\vec{v}\times\vec{B}$$

$$\vec{F}_M = q\big((v\cos\theta)\hat{i} + (v\sin\theta)\hat{j}\big)\times B\hat{i}$$

$$\vec{F}_M = q\big((v\cos\theta)B(\hat{i}\times\hat{i}) + (v\sin\theta)B(\hat{j}\times\hat{i})\big)$$

$$\vec{F}_M = q\big((v\cos\theta)B(0) + (v\sin\theta)B(-\hat{k})\big)$$

$$\vec{F}_M = q(v\sin\theta)B(-\hat{k})$$

$$\vec{F}_M = (1.6\times10^{-19}\ \text{C})\left(3\times10^{5}\ \frac{\text{m}}{\text{s}}\sin30°\right)(2\ \text{T})(-\hat{k})$$

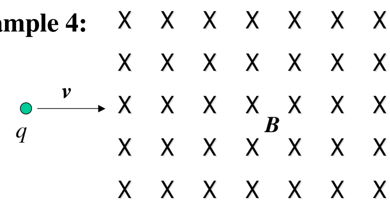
$$\boxed{\vec{F}_M = (4.8\times10^{-14}\ \text{N})(-\hat{k})\quad -z \text{ or in } \otimes}$$

$$F_M = q|\vec{v}\times\vec{B}| = qvB\sin\theta$$

$$F_M = (1.6\times10^{-19}\ \text{C})\left(3\times10^{5}\ \frac{\text{m}}{\text{s}}\right)(2\ \text{T})\sin30°$$

$$F_M = 4.8\times10^{-14}\ \text{N}$$

Example 4:



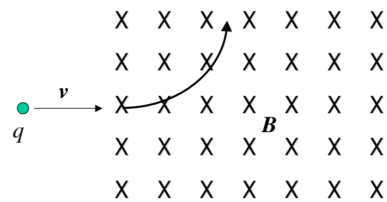
A particle with a charge of 2*e* and mass of 3 x 10^{-27} kg enters a magnitude field with a magnitude of 4 T as shown in the figure above. The speed of the particle is 4 x 10^5 m/s

a.) Draw the path of the particle as it moves through the field.

b.) Find the radius of the path.

Example 4

$$q = 3.2\times10^{-19}\ \text{C},\ m = 3\times10^{-27}\ \text{kg},\ \vec{v} = \left(4\times10^{5}\ \frac{\text{m}}{\text{s}}\right)\hat{i},\ \vec{B} = (4\ \text{T})(-\hat{k})$$



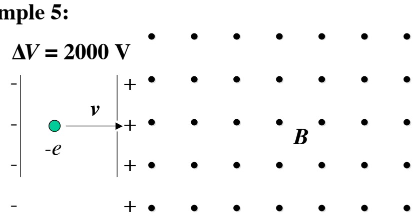
a.) Path of particle $\vec{F}_M = q\vec{v}\times\vec{B} = qv\hat{i}\times B(-\hat{k}) = qvB\big(\hat{i}\times(-\hat{k})\big) = qvB(\hat{j})$

The initial force is upwards and the particle continues in a circular path.

b.) $R = ?$ $\quad F_M = qvB = ma = m\dfrac{v^2}{R}$

$$R = \frac{mv}{qB} = \frac{(3\times10^{-27}\ \text{kg})\left(4\times10^{5}\ \frac{\text{m}}{\text{s}}\right)}{(3.2\times10^{-19}\ \text{C})(4\ \text{T})} = \boxed{9.38\times10^{-4}\ \text{m}}$$

Example 5:



An electron accelerates from rest through a potential difference of 2000 V and then enters a magnetic field with a strength of 0.5 T.

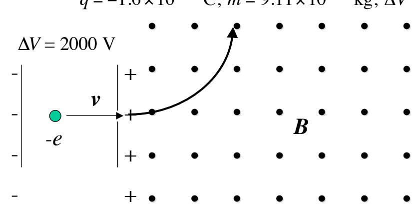
a.) What is the speed of the electron just before it enters the magnetic field?

b.) What is the force on the electron as it moves through the field?

c.) Describe the path of the electron as it moves through the field.

Example 5

$$q = -1.6\times10^{-19}\ \text{C},\ m = 9.11\times10^{-31}\ \text{kg},\ \Delta V = 2000\ \text{V},\ \vec{B} = (0.5\ \text{T})\hat{k}$$



$$\vec{F}_M = q\vec{v}\times\vec{B}$$

$$\vec{F}_M = -ev\hat{i}\times B\hat{k}$$

$$\vec{F}_M = -evB(\hat{i}\times\hat{k})$$

$$\vec{F}_M = -evB(-\hat{j})$$

$$\vec{F}_M = evB\hat{j}$$

a.) $v = ?$

$$\Delta K = -\Delta U = -q\Delta V$$

$$K_2 - \cancel{K_1} = -q\Delta V$$

$$\frac{1}{2}mv^2 = -q\Delta V$$

$$v = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2(-1.6\times10^{-19}\ \text{C})(2000\ \text{V})}{(9.11\times10^{-31}\ \text{kg})}}$$

$$\boxed{v = 2.65\times10^{7}\ \frac{\text{m}}{\text{s}}}$$

b.) $F_M = ?$

$$\vec{F}_M = q\vec{v}\times\vec{B}$$

$$F_M = qvB\sin\theta = qvB$$

$$F_M = \left(1.6\times10^{-19}\ \frac{\text{m}}{\text{s}}\right)\left(2.65\times10^{7}\ \frac{\text{m}}{\text{s}}\right)(0.5\ \text{T})$$

$$\boxed{F_M = 2.12\times10^{-12}\ \text{N}}$$

c.) Describe path

Circular path upward.

Example 6:

A straight horizontal wire carries a current of 50 A from west to east in a region where there is a horizontal magnetic field toward the northeast with a magnitude of 1.2 T and angle 45˚. Find the magnitude and direction of the force on a 1 m section of the wire.

Example 6

$I = 50\text{ A},\ \vec{\ell} = 1\text{ m}\angle 0,\ \vec{B} = 1.2\text{ T}\angle 45°,\ \vec{F} = ?$

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$

for uniform B - fields $\quad \vec{F} = I\vec{\ell} \times \vec{B}$

$$\vec{F} = I\ell\hat{i} \times \left(B_x\hat{i} + B_y\hat{j}\right)$$

$$\vec{F} = I\ell\left(B_x\left(\hat{i} \times \hat{i}\right) + B_y\left(\hat{i} \times \hat{j}\right)\right)$$

$$\vec{F} = I\ell\left(B_x(0) + B_y\left(\hat{k}\right)\right)$$

$$\vec{F} = I\ell B_y\hat{k} \ = I\ell B_y \sin\theta_B \hat{k}$$

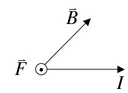
$$\vec{F} = (50\text{ A})(1\text{ m})(1.2\text{ T})(\sin 45°)\hat{k}$$

$$\boxed{\vec{F} = (42.4\text{ N})\hat{k}}$$

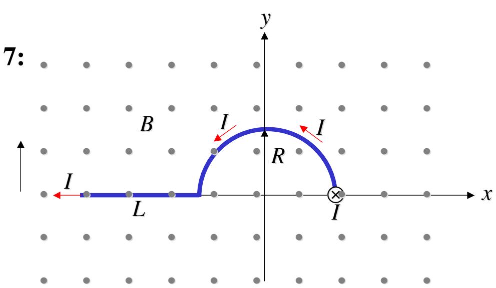
also $F = I\left|\vec{\ell} \times \vec{B}\right| = I\ell B \sin\theta$

$$F = (50\text{ A})(1\text{ m})(1.2\text{ T})(\sin 45°)$$

$$\boxed{F = (42.4\text{ N})}$$

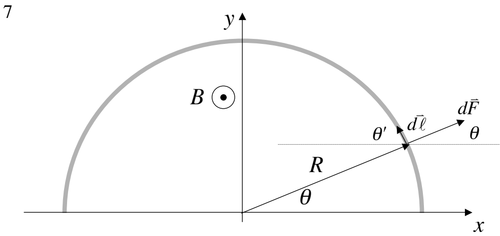


Example 7:



A conductor with straight segment of length L perpendicular to the xy-plane, followed by a semicircle section of radius R, and finally a straight segment with length L parallel to the x-axis carries a current I in a magnetic field B directed in the positive z direction. Find the total magnetic force on the conductor.

Example 7



$$d\vec{F} = Id\vec{\ell} \times \vec{B}$$

$d\vec{\ell} = (Rd\theta)\hat{\ell}$ and $\vec{B} = B\hat{k}$

$$\hat{\ell} = (-\cos\theta')\hat{i} + (\sin\theta')\hat{j}$$

$$\hat{\ell} = (-\sin\theta)\hat{i} + (\cos\theta)\hat{j}$$

$$d\vec{\ell} = (Rd\theta)\left((-\sin\theta)\hat{i} + (\cos\theta)\hat{j}\right)$$

$$d\vec{F} = I(Rd\theta)\left((-\sin\theta)\hat{i} + (\cos\theta)\hat{j}\right) \times B\hat{k}$$

$$d\vec{F} = I(Rd\theta)(-\sin\theta)\hat{i} \times B\hat{k} + I(Rd\theta)(\cos\theta)\hat{j} \times B\hat{k}$$

Example 7

$$d\vec{F} = I(Rd\theta)(-\sin\theta)\hat{i} \times B\hat{k} + I(Rd\theta)(\cos\theta)\hat{j} \times B\hat{k}$$

$$d\vec{F} = IRB\sin\theta d\theta\left(-\hat{i} \times \hat{k}\right) + IRB\cos\theta d\theta\left(\hat{j} \times \hat{k}\right)$$

$$d\vec{F} = IRB\sin\theta d\theta\left(\hat{j}\right) + IRB\cos\theta d\theta\left(\hat{i}\right)$$

$$\vec{F} = \int_0^\pi IRB\sin\theta d\theta\left(\hat{j}\right) + \int_0^\pi IRB\cos\theta d\theta\left(\hat{i}\right)$$

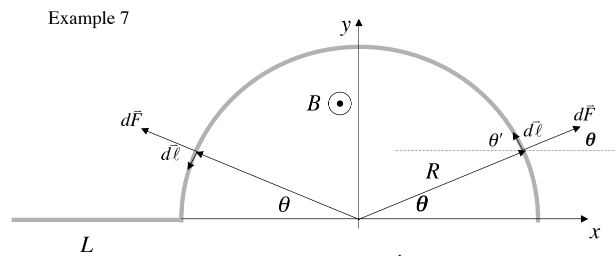
$$\vec{F} = -IRB\cos\theta\left(\hat{j}\right)\Big]_0^\pi + IRB\sin\theta\left(\hat{i}\right)\Big]_0^\pi$$

$$\vec{F} = -IRB(\cos\pi - \cos 0)\left(\hat{j}\right) + IRB(\sin\pi - \sin 0)\left(\hat{i}\right)$$

$$\vec{F} = -IRB(-1-1)\left(\hat{j}\right) + IRB(0-0)\left(\hat{i}\right)$$

$$\vec{F} = 2IRB\left(\hat{j}\right) + 0\left(\hat{i}\right) \ = \boxed{2IRB\left(\hat{j}\right)}$$

Example 7



$$\vec{F} = 2IRB\hat{j}$$

There is only a y - componet due to symmetry.

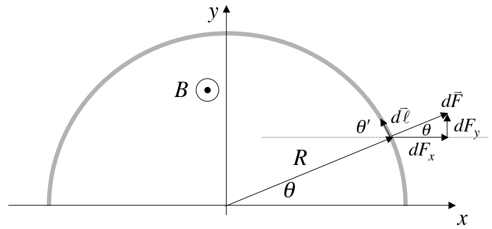
The force on the straight section of length L is :

$$\vec{F} = I\vec{L} \times \vec{B} \ = IL\hat{i} \times B\hat{k} \ = ILB\left(\hat{i} \times \hat{k}\right) \ = ILB\hat{j}$$

The total force on the wire is :

$$\boxed{\vec{F} = (ILB + 2IRB)\hat{j}}$$

Example 7



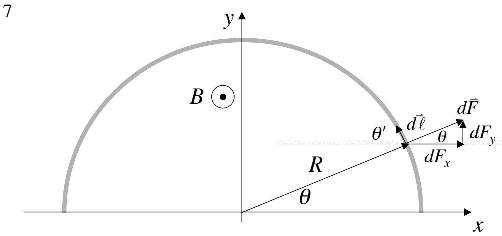
$$d\vec{F} = Id\vec{\ell} \times \vec{B}$$

$$dF = Id\ell B\sin\phi = Id\ell B\sin 90° = Id\ell B = IRd\theta B$$

$$dF_x = dF\cos\theta = (IRd\theta B)\cos\theta = IRB\cos\theta d\theta$$

$$dF_y = dF\sin\theta = (IRd\theta B)\sin\theta = IRB\sin\theta d\theta$$

Example 7



$$dF_x = IRB\cos\theta d\theta$$

$$F_x = \int_0^\pi IRB\cos\theta d\theta$$

$$F_x = IRB\sin\theta\Big]_0^\pi$$

$$F_x = IRB(\sin\pi - \sin 0)$$

$$F_x = 0$$

$$dF_y = IRB\sin\theta d\theta$$

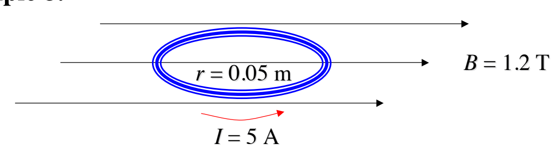
$$F_y = \int_0^\pi IRB\sin\theta d\theta$$

$$F_y = -IRB\cos\theta\Big]_0^\pi$$

$$F_y = -IRB(\cos\pi - \cos 0)$$

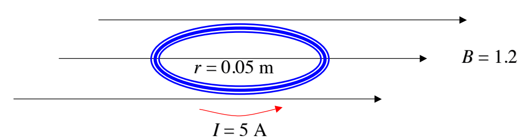
$$F_y = 2IRB$$

Example 8:



A circular coil 0.05 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a current of 5 A in the counterclockwise sense when viewed from above. The coil is in a uniform magnetic field directed toward the right, with magnitude 1.2 T. Find the torque on the coil.

Example 8



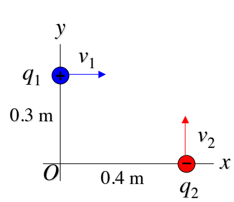
$$r = 0.05 \text{ m},\ N = 30 \text{ turns},\ I = 5 \text{ A}, \vec{B} = 1.2 \text{ T}\angle 0,\ \tau = ?$$

$$\tau = NIBA\sin\theta$$

$$\tau = (30)(5 \text{ A})(1.2 \text{ T})\pi(0.05 \text{ m})^2\sin 90°$$

$$\boxed{\tau = 1.41 \text{ N}\cdot\text{m}}$$

Example 9:

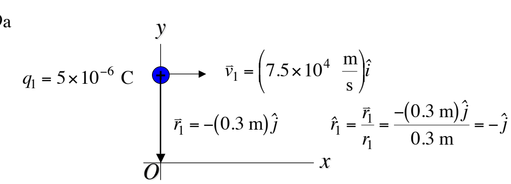


A pair of point charges, $q_1 = +5\ \mu$C and $q_2 = -3\ \mu$C, are moving as shown above with speed $v_1 = 7.5 \times 10^4$ m/s and $v_2 = 3.2 \times 10^4$ m/s . At this instant find

a.) the magnetic field produced at the origin

b.) the magnetic force that q_1 exerts on q_2

Example 9a

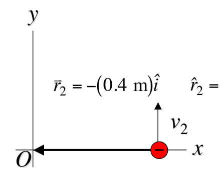


$$\vec{B}_1 = \frac{\mu_o}{4\pi}\frac{q_1\vec{v}_1 \times \hat{r}_1}{r_1^2}$$

$$\vec{B}_1 = \frac{\left(4\pi\times10^{-7} \ \frac{\text{T}}{\text{m}\cdot\text{A}}\right)}{4\pi}\frac{\left(5\times10^{-6} \text{ C}\right)\left(7.5\times10^4 \ \frac{\text{m}}{\text{s}}\right)\hat{i}\times\left(-\hat{j}\right)}{(0.3 \text{ m})^2}$$

$$\vec{B}_1 = \left(4.17\times10^{-7} \text{ T}\right)\left(\hat{i}\times-\hat{j}\right) = -\left(4.17\times10^{-7} \text{ T}\right)\hat{k}$$

Example 9a



$\vec{r}_2 = -(0.4 \text{ m})\hat{i}$ $\quad \hat{r}_2 = -\hat{i}$

$\vec{v}_2 = \left(3.2\times10^4 \ \frac{\text{m}}{\text{s}}\right)\hat{j}$

$q_2 = -3\times10^{-6}$ C

$$\vec{B}_2 = \frac{\mu_o}{4\pi}\frac{q_2\vec{v}_2\times\hat{r}_2}{r_2^{\,2}}$$

$$\vec{B}_2 = \frac{\left(4\pi\times10^{-7}\ \frac{\text{T}}{\text{m}\cdot\text{A}}\right)}{4\pi}\frac{\left(-3\times10^{-6}\text{ C}\right)\left(3.2\times10^4\ \frac{\text{m}}{\text{s}}\right)\hat{j}\times\left(-\hat{i}\right)}{(0.4\text{ m})^2}$$

$$\vec{B}_2 = \left(-6.0\times10^{-8}\text{ T}\right)\left(\hat{j}\times-\hat{i}\right) \quad = -\left(6.0\times10^{-8}\text{ T}\right)\hat{k}$$

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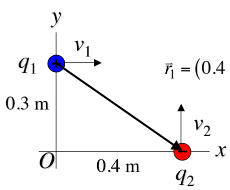
Example 9a

$$\vec{B}_o = \vec{B}_1 + \vec{B}_2 = -\left(4.17\times10^{-7}\text{ T}\right)\hat{k} - \left(6.0\times10^{-8}\text{ T}\right)\hat{k}$$

$$\boxed{\vec{B}_o = -\left(4.77\times10^{-7}\text{ T}\right)\hat{k}}$$

Example 9b



$\vec{r}_1 = (0.4\text{ m})\hat{i} - (0.3\text{ m})\hat{j}$

$r_1 = \sqrt{(0.4\text{ m})^2 + (-0.3\text{ m})^2} = 0.5\text{ m}$

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{(0.3\text{ m})\hat{i} - (0.3\text{ m})\hat{j}}{0.5\text{ m}} = 0.8\hat{i} - 0.6\hat{j}$$

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Example 9b

$$\vec{B}_1 = \frac{\mu_o}{4\pi}\frac{q_1\vec{v}_1\times\hat{r}_1}{r_1^{\,2}}$$

$$\vec{B}_1 = \frac{\left(4\pi\times10^{-7}\ \frac{\text{T}}{\text{m}\cdot\text{A}}\right)}{4\pi}\frac{\left(5\times10^{-6}\text{ C}\right)\left(7.5\times10^4\ \frac{\text{m}}{\text{s}}\right)\hat{i}\times\left(0.8\hat{i}-0.6\hat{j}\right)}{(0.5\text{ m})^2}$$

$$\vec{B}_1 = \left(-9.00\times10^{-8}\text{ T}\right)\hat{k}$$

$$\vec{F}_{2,1} = q_2\vec{v}_2\times\vec{B}_1$$

$$\vec{F}_{2,1} = \left(-3\times10^{-6}\text{ C}\right)\left(3.2\times10^4\ \frac{\text{m}}{\text{s}}\right)\hat{j}\times\left(-9.00\times10^{-8}\text{ T}\right)\hat{k}$$

$$\boxed{\vec{F}_{2,1} = \left(8.64\times10^{-9}\text{ N}\right)\hat{i}}$$

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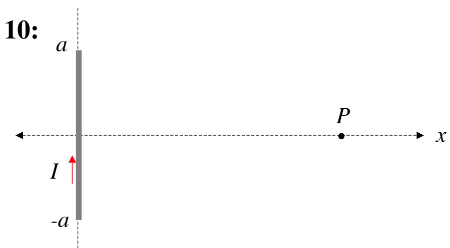
Example 10:



A straight vertical conductor lying on the *y*-axis carries a current *I* from *y* = -*a* to *y* = *a*. Find the strength and direction of the magnetic field for a point *P* which lies on the *x*-axis.

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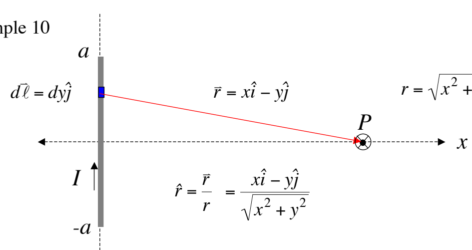
Example 10



$d\vec{\ell} = dy\hat{j}$ $\quad \vec{r} = x\hat{i} - y\hat{j}$ $\quad r = \sqrt{x^2+y^2}$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i}-y\hat{j}}{\sqrt{x^2+y^2}}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{\ell}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{\ell}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\int_{-a}^{a}\frac{I\left(dy\hat{j}\right)\times\left(x\hat{i}-y\hat{j}\right)}{\left(\sqrt{x^2+y^2}\right)^3}$$

$$= \frac{\mu_0}{4\pi}\int_{-a}^{a}\frac{I\left(dy\hat{j}\right)\times\left(x\hat{i}-y\hat{j}\right)}{\left(x^2+y^2\right)^{3/2}} = \frac{\mu_0}{4\pi}\int_{-a}^{a}\frac{Ixdy}{\left(x^2+y^2\right)^{3/2}}\left(\hat{j}\times\hat{i}\right)$$

$\left(-\hat{k}\right)$

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Example 10

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{-a}^{a}\frac{Ixdy}{\left(x^2+y^2\right)^{3/2}}\left(-\hat{k}\right) = \frac{\mu_0 Ix}{4\pi}\int_{-a}^{a}\frac{dy}{\left(x^2+y^2\right)^{3/2}}\left(-\hat{k}\right)$$

From integral tables: $\displaystyle\int\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{y}{x^2\left(x^2+y^2\right)^{1/2}} + C$

$$\vec{B} = \frac{\mu_0 Ix}{4\pi}\left.\frac{y}{x^2\left(x^2+y^2\right)^{1/2}}\right]_{-a}^{a}\left(-\hat{k}\right) = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}\left(-\hat{k}\right)$$

$$\vec{B} = \frac{-\mu_0 I}{2\pi}\frac{a}{x\sqrt{x^2+a^2}}\hat{k}$$

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Example 10

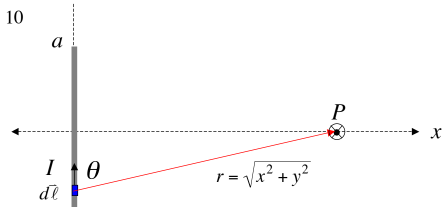
$$B = \frac{\mu_0 I}{2\pi} \frac{a}{x\sqrt{x^2 + a^2}}$$

for $a >> x$, $\sqrt{x^2 + a^2} \approx \sqrt{a^2} = a$ and

$$B = \frac{\mu_0 I}{2\pi} \frac{a}{xa}$$

$$B = \frac{\mu_0 I}{2\pi x} \text{ (infinitely long wire)}$$

Example 10

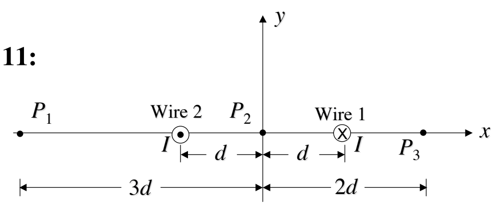


$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \int \frac{I |d\vec{\ell}| |\hat{r}| \sin\theta}{r^2} = \frac{\mu_0}{4\pi} \int_{-a}^{a} \frac{I dy(1)}{(x^2 + y^2)} \frac{x}{\sqrt{x^2 + y^2}}$$

$$B = \frac{\mu_0}{4\pi} \int_{-a}^{a} \frac{Ixdy}{(x^2 + y^2)^{3/2}}$$

Example 11:

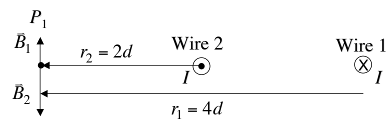


Two long, straight, parallel wires perpendicular to the *xy*-plane, each carry a current *I* but in opposite directions.

a.) Find the magnitude and direction of the magnetic field at points P_1, P_2, and P_3.

b.) Find the magnitude and direction of the magnetic field at any point on the *x*-axis to the right of wire 2 in terms of the *x*-coordinate of the point.

Example 11a



$$\vec{B}_{P_1} = \vec{B}_1 + \vec{B}_2$$

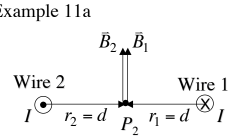
$$\vec{B}_{P_1} = \frac{\mu_0 I}{2\pi r_1} \hat{j} + \frac{\mu_0 I}{2\pi r_2} (-\hat{j})$$

$$\vec{B}_{P_1} = \frac{\mu_0 I}{2\pi (4d)} \hat{j} + \frac{\mu_0 I}{2\pi (2d)} (-\hat{j})$$

$$\vec{B}_{P_1} = \frac{\mu_0 I}{8\pi d} \hat{j} - \frac{\mu_0 I}{4\pi d} \hat{j}$$

$$\boxed{\vec{B}_{P_1} = \frac{\mu_0 I}{8\pi d} (-\hat{j})}$$

Example 11a

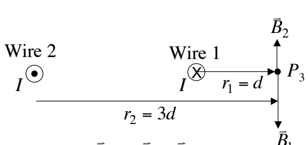


$$\vec{B}_{P_2} = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_{P_2} = \frac{\mu_0 I}{2\pi r_1} \hat{j} + \frac{\mu_0 I}{2\pi r_2} \hat{j}$$

$$\vec{B}_{P_2} = \frac{\mu_0 I}{2\pi d} \hat{j} + \frac{\mu_0 I}{2\pi d} \hat{j}$$

$$\boxed{\vec{B}_{P_2} = \frac{\mu_0 I}{\pi d} \hat{j}}$$



$$\vec{B}_{P_3} = \vec{B}_1 + \vec{B}_2$$

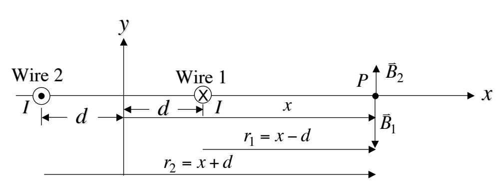
$$\vec{B}_{P_3} = \frac{\mu_0 I}{2\pi r_1} (-\hat{j}) + \frac{\mu_0 I}{2\pi r_2} \hat{j}$$

$$\vec{B}_{P_3} = \frac{\mu_0 I}{2\pi d} (-\hat{j}) + \frac{\mu_0 I}{2\pi (3d)} \hat{j}$$

$$\vec{B}_{P_3} = -\frac{\mu_0 I}{2\pi d} \hat{j} + \frac{\mu_0 I}{6\pi d} \hat{j}$$

$$\boxed{\vec{B}_{P_3} = \frac{\mu_0 I}{3\pi d} (-\hat{j})}$$

Example 11b



$$\vec{B}_P = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_P = \frac{\mu_0 I}{2\pi r_1} (-\hat{j}) + \frac{\mu_0 I}{2\pi r_2} \hat{j}$$

$$\vec{B}_P = \frac{\mu_0 I}{2\pi (x - d)} (-\hat{j}) + \frac{\mu_0 I}{2\pi (x + d)} \hat{j}$$

$$\vec{B}_P = \frac{\mu_0 I}{2\pi} \left(\frac{-1}{(x - d)} + \frac{1}{(x + d)} \right) \hat{j}$$

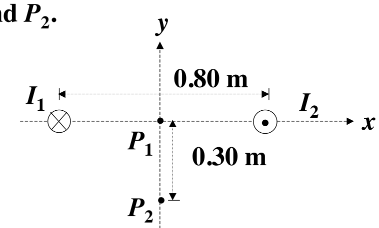
$$\vec{B}_P = \frac{\mu_0 I}{2\pi} \left(\frac{-(x + d) + x - d}{(x^2 - d^2)} \right) \hat{j}$$

$$\vec{B}_P = \frac{\mu_0 I}{2\pi} \left(\frac{-2d}{(x^2 - d^2)} \right) \hat{j}$$

$$\boxed{\vec{B}_P = \frac{\mu_0 I}{2\pi} \left(\frac{2d}{(x^2 - d^2)} \right) (-\hat{j})}$$

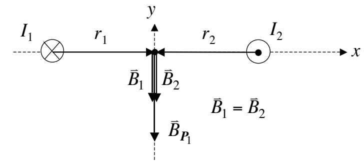
Example 12:

Two long wires perpendicular to the *x*-axis carry a currents of 0.4 A as shown below. Point P_1 is located at the origin which is halfway between the wires and point P_2 is on the *y*-axis 0.30 m below the origin. Find the magnitude and direction of the magnetic field at points P_1 and P_2.



Example 12

$r_1 = r_2 = 0.4\text{ m},\ I_1 = I_2 = 0.4\text{ A},\ \vec{B}_{P_1} = ?$



$$\vec{B}_{P_1} = \vec{B}_1 + \vec{B}_2 = 2\vec{B}_1$$

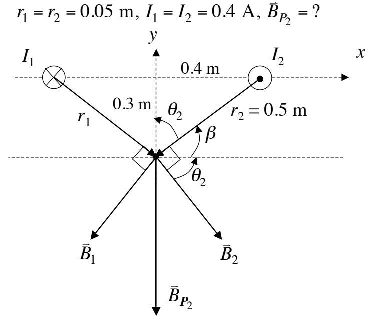
$$\vec{B}_{P_1} = 2\frac{\mu_0 I_1}{2\pi r_1}\left(-\hat{j}\right)$$

$$\vec{B}_{P_1} = 2\left(\frac{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.4\text{ A})}{2\pi(0.4\text{ m})}\right)\left(-\hat{j}\right)$$

$$\boxed{\vec{B}_{P_1} = \left(4.0\times10^{-7}\text{ T}\right)\left(-\hat{j}\right) \text{ or } \vec{B}_{P_1} = 4.0\times10^{-7}\text{ T}\ \angle 270^\circ}$$

Example 12

$r_1 = r_2 = 0.05\text{ m},\ I_1 = I_2 = 0.4\text{ A},\ \vec{B}_{P_2} = ?$

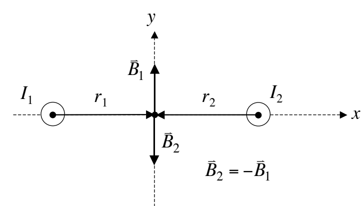


$$B_{P_2} = B_{1_y} + B_{2_y} = 2B_{2_y} = 2B_2\sin\theta_2$$

$$B_{P_2} = 2\left(\frac{\mu_0 I_2}{2\pi r_2}\right)\sin\theta_2 = 2\left(\frac{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.4\text{ A})}{2\pi(0.5\text{ m})}\right)\left(\frac{0.4\text{ m}}{0.5\text{ m}}\right) \boxed{\begin{array}{l} = 2.56\times10^{-7}\text{ T} \\ \theta_{P_2} = 270^\circ \end{array}}$$

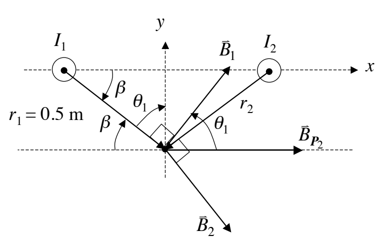
Example 12 (Both currents out of the page)

$\vec{B}_{P_1} = ?$



$$\vec{B}_{P_1} = \vec{B}_1 + \vec{B}_2 = 0$$

Example 12 (Both currents out of the page)



$$B_{P_2} = B_{1_x} + B_{2_x} = 2B_{1_x} = 2B_1\cos\theta_1$$

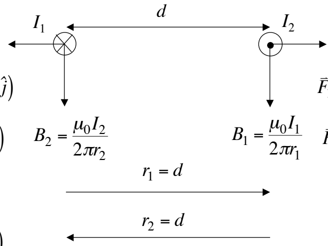
$$B_{P_2} = 2\left(\frac{\mu_0 I_1}{2\pi r_1}\right)\cos\theta_1 = 2\left(\frac{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.4\text{ A})}{2\pi(0.5\text{ m})}\right)\left(\frac{0.3\text{ m}}{0.5\text{ m}}\right) \boxed{\begin{array}{l} = 1.92\times10^{-7}\text{ T} \\ \theta_{P_2} = 0 \end{array}}$$

Example 13:

Two straight, parallel, conducting cables are 4.5 mm apart and each carry a current of 15,000 A in opposite directions. Find the force between the cables.

Example 13

$d = 4.5\text{ mm},\ I_1 = I_2 = 15{,}000\text{ A in opposite directions},\ \frac{F}{\ell} = ?$



$\vec{F}_{2,1} = I_1\vec{\ell}\times\vec{B}_2$

$\vec{F}_{2,1} = I_1\ell\left(-\hat{k}\right)\times\vec{B}_2\left(-\hat{j}\right)$

$\vec{F}_{2,1} = I_1\ell\vec{B}_2\left(\hat{k}\times\hat{j}\right)$

$\vec{F}_{2,1} = I_1\ell\vec{B}_2\left(-\hat{i}\right)$

$\frac{\vec{F}_{2,1}}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}\left(-\hat{i}\right)$

$\vec{F}_{1,2} = I_2\vec{\ell}\times\vec{B}_1$

$\vec{F}_{1,2} = I_2\ell\hat{k}\times\vec{B}_1\left(-\hat{j}\right)$

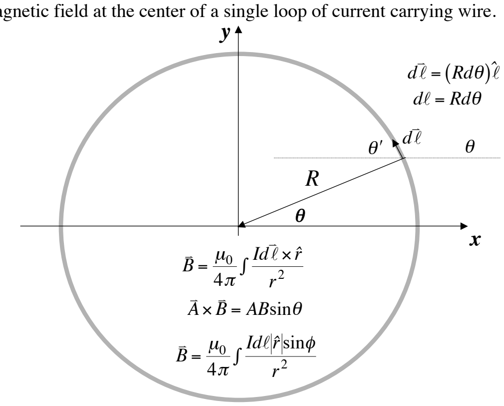
$\vec{F}_{1,2} = I_2\ell\vec{B}_1\left(\hat{k}\times\left(-\hat{j}\right)\right)$

$\vec{F}_{1,2} = I_2\ell\vec{B}_1\hat{i}$

$\frac{\vec{F}_{1,2}}{\ell} = \frac{\mu_0 I_2 I_1}{2\pi d}\hat{i}$

$$F_{1,2} = F_{2,1} = \frac{\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(15{,}000\text{ A})(15{,}000\text{ A})}{2\pi\left(4.5\times10^{-3}\text{ m}\right)} = \boxed{10{,}000\ \frac{\text{N}}{\text{m}}}$$

Magnetic field at the center of a single loop of current carrying wire.



$d\vec{\ell} = \left(Rd\theta\right)\hat{\ell}$

$d\ell = Rd\theta$

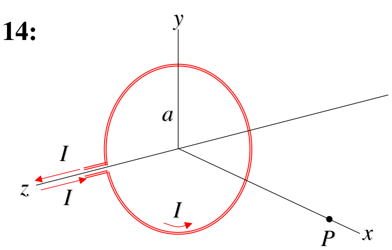
$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{\ell}\times\hat{r}}{r^2}$

$\vec{A}\times\vec{B} = AB\sin\theta$

$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\ell|\hat{r}|\sin\phi}{r^2}$

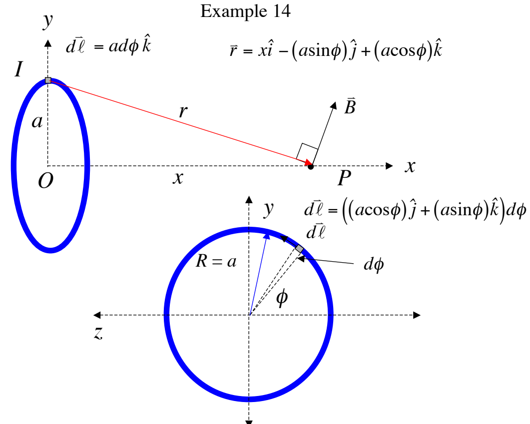
$$\vec{B} = \frac{\mu_0}{4\pi}\int_0^{2\pi}\frac{IRd\theta(1)\sin90^\circ}{R^2} = \frac{\mu_0 I}{4\pi R}\int_0^{2\pi}d\theta = \frac{\mu_0 I}{4\pi R}\theta\Big|_0^{2\pi} = \frac{\mu_0 I}{4\pi R}2\pi = \frac{\mu_0 I}{2R}$$

Example 14:

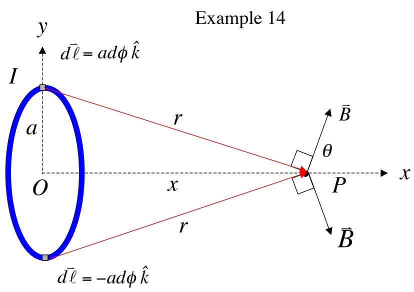


A circular conducting loop with radius *a* carries a current *I*. The loop lies in the *yz*-plane at the origin of the *x*-axis. Find the magnetic field at a point *P* that lies along the *x*-axis.

Example 14

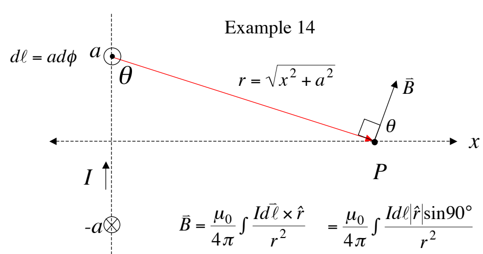


Example 14



We have symmetry and *y*-components cancel and *x*-components add.

Example 14



$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{\ell}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\int\frac{Id\ell|\hat{r}|\sin90^\circ}{r^2}$$

$$B_x = \frac{\mu_0}{4\pi}\int_0^{2\pi}\frac{I(ad\phi)(1)(1)}{\left(x^2+a^2\right)}\cos\theta = \frac{\mu_0}{4\pi}\int_0^{2\pi}\frac{I(ad\phi)}{\left(x^2+a^2\right)}\frac{a}{\sqrt{x^2+a^2}}$$

$$= \frac{\mu_0}{4\pi}\int_0^{2\pi}\frac{Ia^2d\phi}{\left(x^2+a^2\right)^{3/2}} = \frac{\mu_0}{4\pi}\frac{Ia^2}{\left(x^2+a^2\right)^{3/2}}\int_0^{2\pi}d\phi$$

Example 14

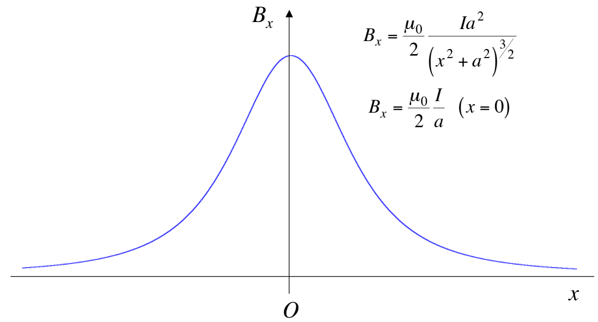
$$B_x = \frac{\mu_0}{4\pi}\frac{Ia^2}{\left(x^2+a^2\right)^{3/2}}\int_0^{2\pi} d\phi$$

$$B_x = \frac{\mu_0}{4\pi}\frac{Ia^2}{\left(x^2+a^2\right)^{3/2}}2\pi$$

$$B_x = \frac{\mu_0}{2}\frac{Ia^2}{\left(x^2+a^2\right)^{3/2}}$$

$$\vec{B} = \frac{\mu_0}{2}\frac{Ia^2}{\left(x^2+a^2\right)^{3/2}}\hat{\imath} \text{ (on the axis of a circular loop)}$$

Magnetic Field of a Circular Loop



$$B_x = \frac{\mu_0}{2}\frac{Ia^2}{\left(x^2+a^2\right)^{3/2}}$$

$$B_x = \frac{\mu_0}{2}\frac{I}{a} \quad (x=0)$$

Example 15:

Use Ampere's Law to derive the magnetic field

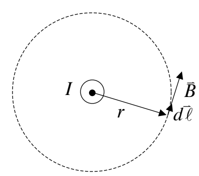
a.) A distance *r* from a long straight wire carrying a current *I*.

b.) At the center of a solenoid with a length *L* with *N* turns carrying a current *I*.

c.) A long cylindrical conductor of radius *R* carrying a uniform current *I*.

d.) A long cylindrical conductor of radius *R* carrying a current *I* whose current density a *J* is nonuniform and varies as $J = br^2$, where *b* is a constant and *r* is measured from the central axis of the cylinder.

Example 15a (Long wire carrying current *I*)



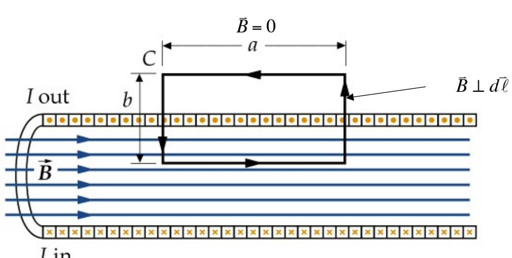
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B\oint d\ell = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Example 15b (Solenoid of length *L* with *N* turns carrying current *I*)

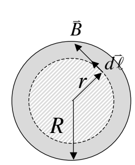


$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$Ba = \mu_0 naI \qquad \left(n = \frac{N}{L} = \frac{\text{number of turns}}{\text{total length}}\right)$$

$$\boxed{B_s = \mu_0 nI}$$

Example 15c (Long cylindrical conductor of radius *R* carrying a uniform current *I*)



$$r < R$$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

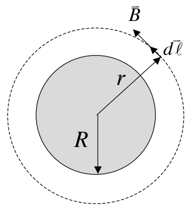
$$B\oint d\ell = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 J A(r)$$

$$B(2\pi r) = \mu_0 \frac{I}{\pi R^2}\pi r^2$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R^2}r \quad (r < R)}$$

Example 15c (Long cylindrical conductor of radius R carrying a uniform current I)



$r > R$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

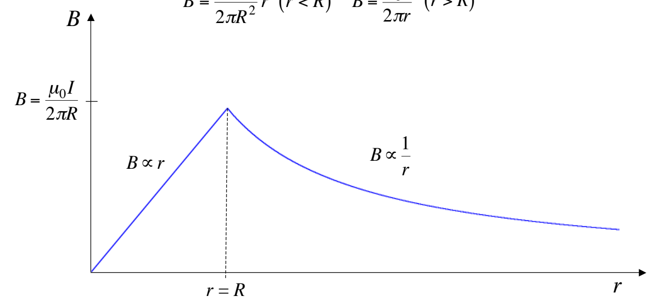
$$B\oint d\ell = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I$$

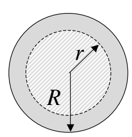
$$\boxed{B = \frac{\mu_0 I}{2\pi r} \quad (r > R)}$$

Example 15c (Long cylindrical conductor of radius R carrying a uniform current I)

$$B = \frac{\mu_0 I}{2\pi R^2} r \;\; (r < R) \qquad B = \frac{\mu_0 I}{2\pi r} \;\; (r > R)$$



Example 15d Long cylindrical conductor of radius R carrying a current I that varies as $J = br^2$



$J = br^2 \qquad A = \pi r^2$

$I_{enc} = \int \vec{J}\cdot d\vec{A} \qquad dA = 2\pi r dr$

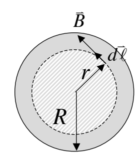
$$I_{enc} = \int_0^r br^2 2\pi r dr = 2\pi b \int_0^r r^3 dr$$

$$I_{enc} = \frac{\pi b r^4}{2} \quad (r < R)$$

$$(r = R) \quad I = \frac{\pi b R^4}{2} \quad \text{so} \quad b = \frac{2I}{\pi R^4}$$

$$\boxed{I_{enc} = I\frac{r^4}{R^4} \quad (r < R)}$$

Example 15d Long cylindrical conductor of radius R carrying a current I that varies as $J = br^2$



$$I_{enc} = I\frac{r^4}{R^4} \qquad (r < R)$$

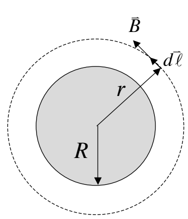
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B\oint d\ell = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I\frac{r^4}{R^4}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi}\frac{r^3}{R^4} \qquad (r < R)}$$

Example 15d Long cylindrical conductor of radius R carrying a current I that varies as $J = br^2$



$$I_{enc} = I \quad (r > R)$$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B\oint d\ell = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r} \qquad (r > R)}$$

Example 15d Long cylindrical conductor of radius R carrying a current I that varies as $J = br^2$

$$B = \frac{\mu_0 I}{2\pi}\frac{r^3}{R^4} \;\; (r < R) \qquad B = \frac{\mu_0 I}{2\pi r} \;\; (r > R)$$