Linear Momentum and its Conservation

Newton's 2nd Law and Momentum (*p*)

Newton's 2nd Law

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

Newton's 2nd Law says that the net force on an object is the time rate of change of the product of the object's mass and velocity. This combination is called the *momentum* or *linear momentum*, of the object.

$$\boxed{\vec{p} = m\vec{v}} \quad \textbf{(definition of momentum)}$$

Newton's 2nd Law and Momentum

Newton's 2nd Law can therefore be expressed in terms of momentum.

$$\sum F = \frac{d\vec{p}}{dt} \qquad \boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

The net force acting on an object equals the time rate of change of momentum of the object.

Impulse (J)

Consider an object acted on by a *constant net force* during a time interval Δt from t_1 to t_2. The *impulse J* of the net force is defined to be the product of the net force and the time interval.

$$\vec{J} = \sum \vec{F} dt = \int \vec{F} dt$$

The units of impulse are N · s. Since 1 N = kg · m/s^2, the impulse has the same units as momentum (kg · m/s).

Impulse-Momentum Theorem

If the net force is constant then:

$$\sum F = \frac{d\vec{p}}{dt} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1}$$

$$\sum F(t_2 - t_1) = \vec{p}_2 - \vec{p}_1$$

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

The change in momentum of a object during a time interval equals the impulse of the net force that acts on the object during that interval.

Impulse-Momentum Theorem

If the forces are not constant then:

$$\boxed{\vec{J} = \int \vec{F} dt = \Delta\vec{p}}$$

This is the area under a force versus time graph over the specified time interval.

Conservation of Momentum

The momentum of any closed, isolated system does not change. ***This is true only if the vector sum of the external forces acting on the system is zero.***

$$\sum_n \vec{p}_{n_i} = \sum_n \vec{p}_{n_f}$$

$$\vec{p}_{1_i} + \vec{p}_{2_i} + \cdots + \vec{p}_{n_i} = \vec{p}_{1_f} + \vec{p}_{2_f} + \cdots + \vec{p}_{n_f}$$

Where:

$\vec{p}_{n_i}$ = **initial momentum vector of object *n***

$\vec{p}_{n_f}$ = **final momentum vector of object *n***

Conservation of Momentum

For 2-D problems, the *x* and *y* components of momentum must be treated separately.

$$\sum_n p_{x_{n_i}} = \sum_n p_{x_{n_f}} \quad \text{and} \quad \sum_n p_{y_{n_i}} = \sum_n p_{y_{n_f}}$$

Inelastic Collisions Between Objects

- **Momentum is *conserved.***
- **Kinetic Energy is *not conserved.***
- **When objects stick together the collision is called a *completely inelastic collision.***

Elastic Collisions Between Objects

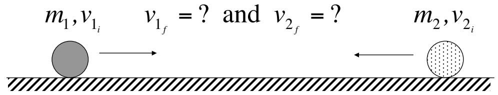
- **Momentum is *conserved.***
- **Kinetic Energy is *also conserved.***

Collisions Between Objects

In any collision, momentum is conserved and the total momentum before equals the total momentum after; in elastic collisions only, the total kinetic energy before equals the total kinetic energy after.

1D Elastic Collisions Between Objects

m_1, v_{1_i} $\quad v_{1_f} = ?$ and $v_{2_f} = ?$ $\quad m_2, v_{2_i}$



$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$(1)\quad m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}$$

$$\sum K_i = \sum K_f$$

$$(2)\quad \frac{1}{2} m_1 v_{1_i}^{\ 2} + \frac{1}{2} m_2 v_{2_i}^{\ 2} = \frac{1}{2} m_1 v_{1_f}^{\ 2} + \frac{1}{2} m_2 v_{2_f}^{\ 2}$$

1D Elastic Collisions Between Objects

$$(2)\ \tfrac{1}{2}m_1 v_{1_i}^2 + \tfrac{1}{2}m_2 v_{2_i}^2 = \tfrac{1}{2}m_1 v_{1_f}^2 + \tfrac{1}{2}m_2 v_{2_f}^2$$

$$m_1 v_{1_i}^2 + m_2 v_{2_i}^2 = m_1 v_{1_f}^2 + m_2 v_{2_f}^2$$

$$m_1\left(v_{1_i}^2 - v_{1_f}^2\right) = m_2\left(v_{2_f}^2 - v_{2_i}^2\right)$$

$$(3)\ m_1\left(v_{1_i} - v_{1_f}\right)\left(v_{1_i} + v_{1_f}\right) = m_2\left(v_{2_f} - v_{2_i}\right)\left(v_{2_f} + v_{2_i}\right)$$

$$(1)\ m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}$$

$$m_1 v_{1_i} - m_1 v_{1_f} = m_2 v_{2_f} - m_2 v_{2_i}$$

$$(4)\ m_1\left(v_{1_i} - v_{1_f}\right) = m_2\left(v_{2_f} - v_{2_i}\right)$$

1D Elastic Collisions Between Objects

$$(4)\ m_1\left(v_{1_i} - v_{1_f}\right) = m_2\left(v_{2_f} - v_{2_i}\right)$$

$$(3)\ \cancel{m_1\left(v_{1_i} - v_{1_f}\right)}\left(v_{1_i} + v_{1_f}\right) = \cancel{m_2\left(v_{2_f} - v_{2_i}\right)}\left(v_{2_f} + v_{2_i}\right)$$

$$(5)\ \left(v_{1_i} + v_{1_f}\right) = \left(v_{2_f} + v_{2_i}\right)$$

$$\boxed{(6)\ \left(v_{1_i} - v_{2_i}\right) = -\left(v_{1_f} - v_{2_f}\right)}$$

The relative speed of the objects before the collision equals the negative of the relative speed after the collision.

$$\boxed{(1)\ m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}}$$

Center of Mass

Center of Mass

The center of mass of a system of particles is given by:

$$\boxed{x_{\text{cm}} = \frac{\sum_i m_i x_i}{\sum_i m_i}} \quad \text{and} \quad y_{\text{cm}} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

The center of mass is the mass-weighted average position of the particles.

Motion of the Center of Mass

The velocity of the center of mass of a system of particles is given by (time derivatives of x_{cm} and y_{cm})

$$v_{\text{cm}_x} = \frac{\sum_i m_i v_{i_x}}{\sum_i m_i} \quad \text{and} \quad v_{\text{cm}_y} = \frac{\sum_i m_i v_{i_y}}{\sum_i m_i}$$

These are equivalent to the vector equation

$$\vec{v}_{cm} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i}$$

Motion of the Center of Mass

$$\vec{v}_{cm} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i} \quad \text{so} \quad \sum_i m_i \vec{v}_{cm} = \sum_i m_i \vec{v}_i$$

If we denote the total mass $m_1 + m_2 + \cdots$ by M then

$$M\vec{v}_{\text{cm}} = \sum_i m_i \vec{v}_i = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = \vec{P}$$

***P* is the total momentum of the system of particles.**
The total momentum is equal to the total mass times the velocity of the center of mass.