1-Dimensional Motion of Objects (Uniform Acceleration)

Average Acceleration (m/s^2)

The *average acceleration* (a_{av}) of a particle is the change in velocity divided by the time interval.

$$\boxed{a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}}$$

Instantaneous Acceleration (m/s^2)

The *instantaneous acceleration* (a) is the acceleration of an object at a specific instant of time.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Instantaneous Acceleration (m/s^2)

The *instantaneous acceleration* can be found from *a graph of velocity versus time*. It is equal to the slope of the tangent to the curve at a particular instant of time.

Motion with Constant Acceleration

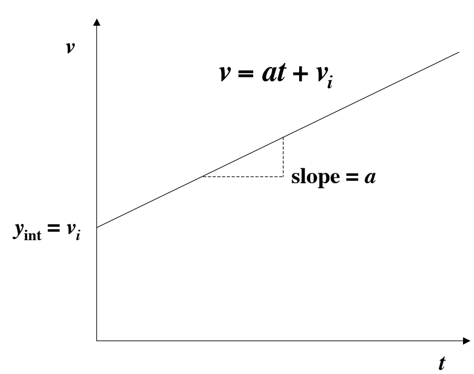
Constant acceleration is the special case in which the velocity changes at the same rate throughout the motion.

$$a = \frac{v - v_i}{t - t_i}$$

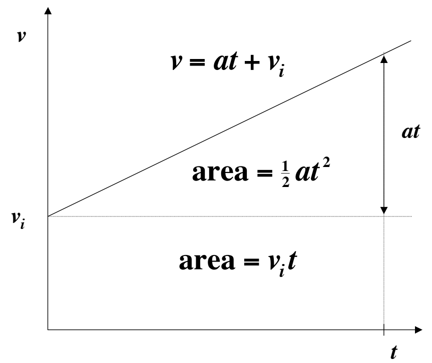
$$v = a(t - t_i) + v_i$$

$$\boxed{v = at + v_i}$$

Motion with Constant Acceleration



Motion with Constant Acceleration



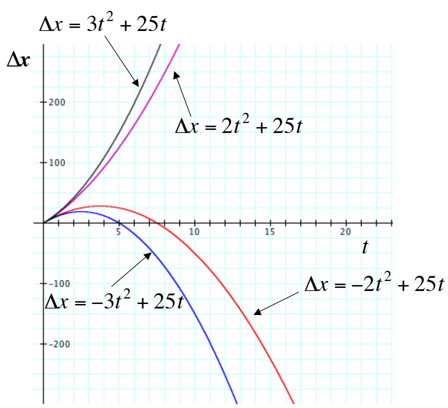
Motion with Constant Acceleration

The area under a velocity-time graph gives the displacement of the object.

$$\boxed{\Delta x = \tfrac{1}{2}at^2 + v_i t}$$

$$x_f - x_i = \tfrac{1}{2}at^2 + v_i t$$

$$\boxed{x_f = \tfrac{1}{2}at^2 + v_i t + x_i}$$



Motion with Constant Acceleration

$$x_f - x_i = \tfrac{1}{2}at^2 + v_i t$$

$$\Delta x = \tfrac{1}{2}a\Delta t^2 + v_i \Delta t$$

$$v_f = a\Delta t + v_i \text{ so } a\Delta t = v_f - v_i$$

$$v_{av} = \frac{\Delta x}{\Delta t}$$

$$v_{av} = \frac{\tfrac{1}{2}a\Delta t^2 + v_i \Delta t}{\Delta t} = \tfrac{1}{2}a\Delta t + v_i$$

$$v_{av} = \tfrac{1}{2}\left(v_f - v_i\right) + v_i = \tfrac{1}{2}v_f - \tfrac{1}{2}v_i + v_i$$

$$v_{av} = \tfrac{1}{2}v_f + \tfrac{1}{2}v_i$$

$$\boxed{v_{av} = \frac{v_f + v_i}{2}}$$

Motion with Constant Acceleration

$$\Delta x = \tfrac{1}{2}at^2 + v_i t$$

$$v_f = at + v_i \text{ so } t = \frac{v_f - v_i}{a}$$

$$\Delta x = \tfrac{1}{2}a\left(\frac{v_f - v_i}{a}\right)^2 + v_i\left(\frac{v_f - v_i}{a}\right)$$

$$2a\Delta x = \left(v_f - v_i\right)^2 + 2v_i\left(v_f - v_i\right)$$

$$2a\Delta x = \left(v_f^2 - 2v_f v_i + v_i^2\right) + 2v_i v_f - 2v_i^2$$

$$2a\Delta x = v_f^2 - v_i^2$$

$$\boxed{v_f^2 = v_i^2 + 2a\Delta x}$$

Motion with Constant Acceleration

$$\Delta x = \tfrac{1}{2}at^2 + v_i t$$

$$v_f = at + v_i \text{ so } a = \frac{v_f - v_i}{t}$$

$$\Delta x = \tfrac{1}{2}\left(\frac{v_f - v_i}{t}\right)t^2 + v_i t$$

$$\Delta x = \tfrac{1}{2}\left(v_f - v_i\right)t + v_i t$$

$$\Delta x = \tfrac{1}{2}v_f t - \tfrac{1}{2}v_i t + v_i t$$

$$\Delta x = \tfrac{1}{2}v_f t + \tfrac{1}{2}v_i t$$

$$\boxed{\Delta x = \left(\frac{v_f + v_i}{2}\right)t}$$

Motion with Constant Acceleration

Other useful relationships:

$$v_{av} = \frac{v_f + v_i}{2}$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = \left(\frac{v_f + v_i}{2}\right)t$$

Freely Falling Objects

An object falling under the influence of the earth's gravitational attraction is a situation in which there is constant acceleration towards the earth.

The ideal case in which there is no air resistance or decrease in acceleration with height is referred to as free fall. This includes rising as well as falling motion. For this idealized case the magnitude of the acceleration is:

$$a = g = 9.8\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

To show that a ball leaves and lands at the same speed.

$$\Delta y = 0$$

$$\Delta y = -\tfrac{1}{2}gt^2 + v_i t$$

$$0 = -\tfrac{1}{2}gt^2 + v_i t$$

$$0 = \left(-\tfrac{1}{2}gt + v_i\right)t$$

Solutions are: $t = 0$ **and** $t = \dfrac{2v_i}{g}$

$$v = -gt + v_i$$

$$v = -g\left(\frac{2v_i}{g}\right) + v_i$$

$$v = -2v_i + v_i = -v_i$$

To show that the time up is equal to the time down.

$v = 0$ **at the halfway point**

$$v = -gt_1 + v_i$$

$$0 = -gt_1 + v_i$$

Solution is: $t_1 = \dfrac{v_i}{g}$

Object returns to the starting position when $\Delta y = 0$

$$\Delta y = -\tfrac{1}{2}gt_2^2 + v_i t_2$$

$$0 = -\tfrac{1}{2}gt_2^2 + v_i t_2$$

$$0 = \left(-\tfrac{1}{2}gt_2 + v_i\right)t_2$$

Solutions are: $t_2 = 0$ **and** $t_2 = \dfrac{2v_i}{g} = 2t_1$