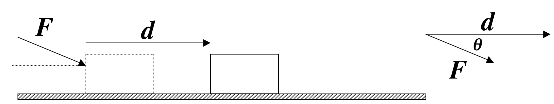
Work and Kinetic Energy

Work

Work **(W) the product of the force exerted on an object and the distance the object moves in the direction of the force (constant force only).**

$$W = \vec{F}\cdot\vec{d} \quad = Fd\cos\theta \quad (\mathrm{N\cdot m = J})$$

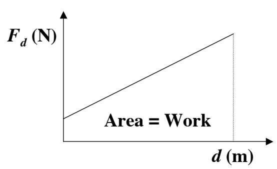
θ is the angle between the force and the direction of motion.



Work

Work is done on an object only if the object moves and is accelerating or changing height.

If the force is not constant then a force-displacement graph can be used to determine the work done. The *area under a force-displacement graph is the work done*.



Work

When the force varies during a straight-line displacement, and the force is in the same direction as the displacement, the work done by the force is given by:

$$W = \int_{x_1}^{x_2} F\cdot dx$$

If the force makes an angle θ with the displacement, the work done by the force is:

$$W = \int_{P_1}^{P_2} F\cos\theta\cdot dl = \int_{P_1}^{P_2} F_{\parallel}\cdot dl = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l}$$

Energy

Energy **(E) the ability to do work.**

Kinetic Energy **(KE)- the ability of an object to do work because of its motion.**

On the Equation Sheet:

$$\Delta E = W = \int \vec{F}\cdot d\vec{r}$$

Kinetic Energy

If a particle is accelerate from rest to a velocity v, the kinetic energy gained by the particle is equal to the work done by the force causing the acceleration.

From mechanics:

$$v^2 = v_i^2 + 2ad$$

$$v_i = 0$$

$$v^2 = 2ad \quad \text{or} \quad a = \frac{v^2}{2d}$$

$$F = ma = m\frac{v^2}{2d} \quad \text{or} \quad Fd = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2}mv^2 \qquad K = \frac{1}{2}mv^2$$

Work-Energy Theorem

When a force does work on an object, it must increase the energy of the object by a like amount.

$$W_{net} = \Delta KE = KE_f - KE_i$$

Power

Power **(*P*) - the rate of doing work. (Power is the rate at which energy is transferred and is a scalar quantity.)**

The *average power* is:

$$P_{av} = \frac{\Delta W}{\Delta t} \qquad \left(\frac{\text{J}}{\text{s}} = \text{Watt (W) and 1 hp} = 746\text{ W}\right)$$

The *instantaneous power* is:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad P = \frac{dE}{dt}$$

Power

When a force *F* acts on a particle moving with velocity *v*, the instantaneous power or rate at which the force does work is:

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

Work and Springs

The force required to stretch an *ideal spring* beyond its unstretched length by an amount *x* is:

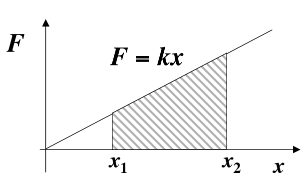
$$F = kx \quad \text{(Hooke's Law)}$$

where *k* is a constant called the *force constant* (or *spring constant*) of the spring.

Hooke's Law holds for compression as well as stretching, but the force and displacement are in opposite directions.

Work and Springs

To stretch a spring requires work



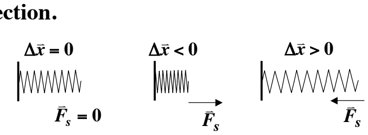
$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$

Work and Springs

If a spring is compressed or stretched, it then has the ability to do work by applying a force in that direction. The force exerted by the spring is:

$$\vec{F}_s = -k\Delta\vec{x}$$

The negative sign is necessary so that the force is in the correct direction.



Potential Energy and Energy Conservation

Potential Energy

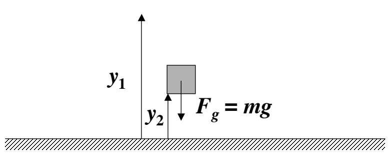
Potential Energy **is energy associated with position and is the measure of potential or possibility to do work.**

1.) ***Gravitational Potential Energy*** **(U_g)**

2.) ***Elastic Potential Energy*** **(U_e)**

Gravitational Potential Energy

Gravitational Potential Energy is associated with a body's weight and height above the ground.



$$W_g = F \cdot d = F_g(y_1 - y_2) = mgy_1 - mgy_2$$

Gravitational Potential Energy

The product of the weight *mg* and the height above the origin is called the *gravitational potential energy*.

$$U_g = mgy$$

$$\boxed{\Delta U_g = mg\Delta h}$$

Gravitational Potential Energy

$$W_g = U_{g_1} - U_{g_2} = -(U_{g_2} - U_{g_1}) = -\Delta U_g$$

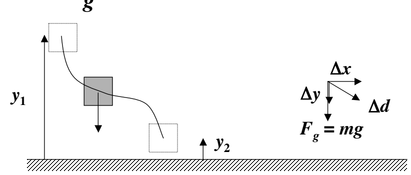
***The negative sign is essential*. When the body moves up the work done by the gravitational force is negative, and U_g increases.**

When the body moves down the work done by the gravitational force is positive, and U_g decreases.

In general, when a conservative force does work the potential energy is lowered.

$$W_{force} = -\Delta U$$

U_g on Curved Paths



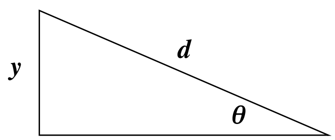
$$W_g = F_g \cdot \Delta\vec{d} = -mg\hat{\mathbf{j}} \cdot \left(\Delta x\hat{\mathbf{i}} + \Delta y\hat{\mathbf{j}}\right) = -mg\Delta y$$

This is true for any segment Δd so the *total* work done by the gravitational force is

$$W_g = -mg\left(y_2 - y_1\right) = mg\left(y_1 - y_2\right) = U_{g_1} - U_{g_2}$$

Inclined Planes

On inclined planes, the change in gravitational potential energy depends up the vertical displacement *y* while moving a distance *d* along the incline.



$y = d\sin\theta$

Elastic Potential Energy

Recall that for springs, work is required to stretch or compress the spring from x_1 to x_2.

$$W = \tfrac{1}{2}kx_2^{\,2} - \tfrac{1}{2}kx_1^{\,2} \quad \text{(work done on spring)}$$

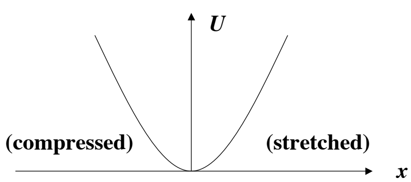
The work done by the spring is equal and opposite

$$W_e = \tfrac{1}{2}kx_1^{\,2} - \tfrac{1}{2}kx_2^{\,2} \quad \text{(work done by a spring)}$$

Elastic Potential Energy

As in gravitational work, the work done by the spring can be expressed in terms of potential energy

$$U_e = \tfrac{1}{2}kx^2 \quad \text{(elastic potential energy)}$$



Elastic Potential Energy

The work done by the spring can be expressed in terms of potential energy.

$$W_e = \tfrac{1}{2}kx_1^{\,2} - \tfrac{1}{2}kx_2^{\,2} = U_{e_1} - U_{e_2} = -\Delta U_e$$

$$\boxed{U_S = \tfrac{1}{2}k(\Delta x)^2}$$

Energy Conservation

Recall the work-energy theorem $W_{net} = \Delta KE$

$$W_{net} = W_g + W_e + W_{other} = -\Delta U_g + -\Delta U_e + W_{other}$$

$$-\left(U_{g_2} - U_{g_1}\right) + -\left(U_{e_2} - U_{e_1}\right) + W_{other} = KE_2 - KE_1$$

$$\boxed{KE_1 + U_{g_1} + U_{e_1} + W_{other} = KE_2 + U_{g_2} + U_{e_2}}$$

This is simply a statement of *conservation of energy.*

Conservative Forces

The work done by a *conservative force*

1.) **Can always be expressed as the difference between the initial and final values of a potential energy function.**
2.) **Is reversible.**
3.) **Is independent of the path of the body.**
4.) **When the starting and ending points are the same, the total work is zero.**

Nonconservative Forces

The work done by a *nonconservative force*

1.) Cannot be expressed as the difference between the initial and final values of a potential energy function.
2.) Is not reversible.
3.) Is dependent upon the path of the body.

Force and Potential Energy

If we are given a potential energy expression, we can find the corresponding force.

$$W = -\Delta U \qquad \text{or} \qquad F_x(x)\Delta x = -\Delta U$$

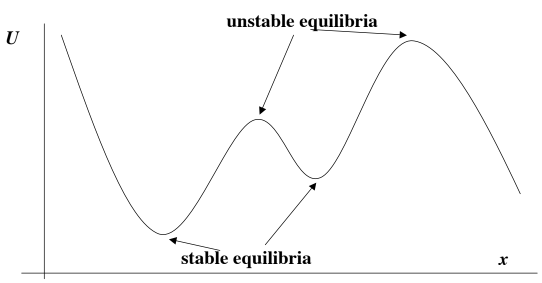
$$F_x(x) = -\frac{\Delta U}{\Delta x}$$

taking the limit as $\Delta x \to 0$

$$F_x(x) = -\frac{dU}{dx}$$

Energy Diagrams

An energy diagram is a graph of $U(x)$ versus x. The negative of the slope of the curve is equal to the force acting on the object.



Conservation of Mechanical Energy *E*

The mechanical energy E of a system is the sum of its potential energy U and the kinetic energy K of the objects within it:

$$E = K + U$$

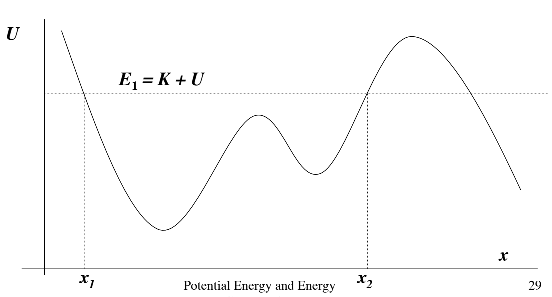
When only a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy E of the system, does not change.

$$E = K_1 + U_1 = K_2 + U_2$$

(Conservation of Mechanical Energy)

Energy Diagrams

The total mechanical energy E puts restrictions on the location of the particle.



Energy Diagrams

The total mechanical energy E puts restrictions on the location of the particle.