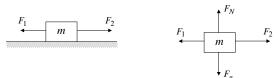
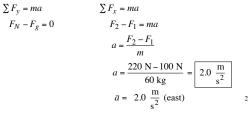
Example 1:

 $m = 60 \text{ kg}, \vec{F}_1 = 100 \text{ N} \text{ (west)}, \vec{F}_2 = 220 \text{ N} \text{ (east)}, a = ?$



Net force equations using Newton's 2nd Law:



Example 2:

Example 1:

A scale is fixed to the bottom of an elevator. A 50 kg box is on the scale. What is the scale reading when

Two forces parallel to the ground act upon a box with a

strength of 100 N. The other force is directed east and has a strength of 220 N. Find the acceleration of the

Forces

1

3

5

mass of 60 kg. One force is directed west and has a

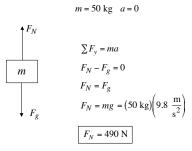
box assuming the surface is frictionless.

- a.) the elevator is at rest?
- b.) the elevator accelerates upward at 2.0 m/s^2 ?
- c.) the elevator accelerates downward at 1.5 m/s²?
- d.) the elevator moves at constant velocity?



Example 2:

a.) the elevator is at rest



Example 2:

b.) the elevator accelerates upward at 2.0 m/s²

$$m = 50 \text{ kg } a = 2.0 \frac{\text{m}}{\text{s}^2} \text{ (upward)}$$

$$\uparrow F_N$$

$$\downarrow F_N = ma$$

$$F_N - F_g = ma \text{ (direction is consistent with a)}$$

$$F_N = ma + F_g$$

$$F_N = ma + mg$$

$$F_N = (50 \text{ kg}) \left(2 \frac{\text{m}}{\text{s}^2}\right) + (50 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right)$$

$$F_N = 590 \text{ N}$$

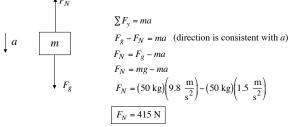
Example 2:

c.) the elevator accelerates downward at 1.5 m/s²

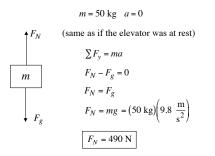
$$n = 50 \text{ kg}$$
 $a = 1.5 \frac{\text{m}}{\text{s}^2}$ (downward)

4

6



d.) the elevator moves at constant velocity



Example 3:

Two masses are suspended using cords with negligible mass.

- a.) Draw force diagrams for each mass.
- b.) Find the tensions in the cords if $m_1 = 25$ kg and $m_2 = 55$ kg.



$m_1 = 25 \text{ kg}, m_2 = 55 \text{ kg}$	$T_1 = ?, T_2 = ?$
$ \begin{array}{c} \uparrow T_1 \\ \hline m_1 \\ F_{g_1} \downarrow T_2 \end{array} $	$\begin{array}{c} & T_2 \\ \hline m_2 \\ \hline & F_{g_2} \end{array}$
$\sum F_y = ma$	
$T_1 - F_{g_1} - T_2 = 0$	$T_2 - F_{g_2} = 0$
$T_1 = F_{g_1} + T_2$	$T_2 = F_{g_2}$
$T_1 = m_1 g + m_2 g$	$T_2 = m_2 g$
$\left(\frac{m}{s^2}\right) + (55 \text{ kg})\left(9.8 \frac{m}{s^2}\right)$	$T_2 = (55 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right)$
$T_1 = 784 \text{ N}$	$T_2 = 539 \text{ N}$ 9
	T_{1} T_{1} T_{2} $F_{g_{1}} \downarrow T_{2}$ $\Sigma F_{y} = ma$ $T_{1} - F_{g_{1}} - T_{2} = 0$ $T_{1} = F_{g_{1}} + T_{2}$ $T_{1} = m_{1}g + m_{2}g$ $\frac{m}{s^{2}} + (55 \text{ kg}) \left(9.8 \frac{m}{s^{2}}\right)$

Example 4:

7

11

A 25 kg box is at rest on a rough horizontal surface. The coefficients of static and kinetic friction are 0.30 and 0.15 respectively.

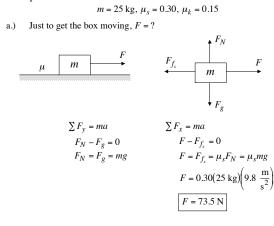
Forces

- a.) How much force is needed to just set the box in motion?
- b.) What is the acceleration of the box if a horizontal force of 100 N is applied to it?
- c.) How much force is needed to move the box at a constant velocity of 15 m/s?
- d.) How much force is needed to move the box with an acceleration of 1.5 m /s²?

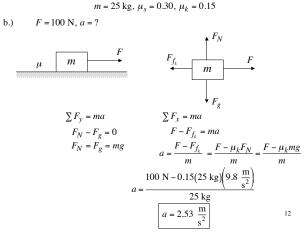
10

Example 4:

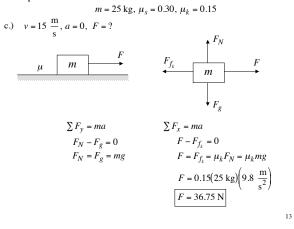
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Example 4:

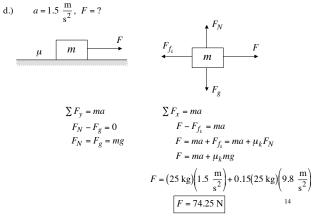


Example 4:



Example 4:

 $m = 25 \text{ kg}, \mu_s = 0.30, \mu_k = 0.15$



Example 5:

$$\mu_k \qquad F = 50 \text{ N}$$

 $a = 2.0 \text{ m/s}^2$

A 50 N force is applied to a 15 kg box causing it to accelerate at a rate of 2.0 m/s^2 .

a.) Draw a force diagram for the box.

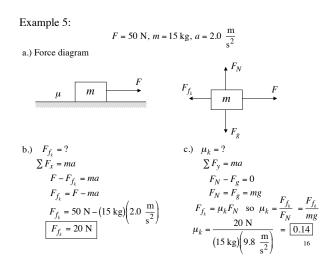
b.) Find the frictional force.

c.) Find the coefficient of kinetic (sliding) friction.

Forces

15

17



Example 6:

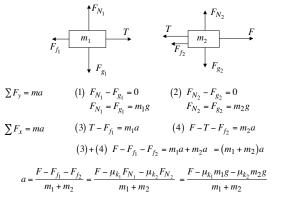
Two boxes are connected by a cord with negligible mass, as shown in the figure below. A force of 60 N is applied horizontally to the 5.0 kg box causing the boxes to accelerate to the right. The coefficient of kinetic friction between the boxes and the surface is 0.25. Find the magnitude of the acceleration of the boxes and the tension in the cord that connects them.

$$T = 60 \text{ N}$$

Forces

Example 6:

 $m_1 = 10.0 \text{ kg}, m_2 = 5.0 \text{ kg}, F = 60 \text{ N}, \mu_{k_1} = \mu_{k_2} = 0.25, a = ? \text{ and } T = ?$



Example 6:

$$m_{1} = 10.0 \text{ kg}, m_{2} = 5.0 \text{ kg}, F = 60 \text{ N}, \mu_{k_{1}} = \mu_{k_{2}} = 0.25, a = ? \text{ and } T = ?$$

$$a = \frac{F - \mu_{k_{1}}m_{1}g - \mu_{k_{2}}m_{2}g}{m_{1} + m_{2}} = \frac{60 \text{ N} - 0.25(10 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^{2}}\right) - 0.25(5 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^{2}}\right)}{10 \text{ kg} + 5 \text{ kg}}$$

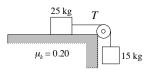
$$\boxed{a = 1.55 \frac{\text{m}}{\text{s}^{2}}}$$

$$(3) T - F_{f_{1}} = m_{1}a \qquad (4) F - T - F_{f_{2}} = m_{2}a$$

$$\text{using (3) } T = m_{1}a + F_{f_{1}} = m_{1}a + \mu_{k_{1}}F_{N_{1}} = m_{1}a + \mu_{k_{1}}m_{1}g$$

$$T = (10 \text{ kg})\left(1.55 \frac{\text{m}}{\text{s}^{2}}\right) + 0.25(10 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^{2}}\right)$$

$$\boxed{T = 40 \text{ N}}$$



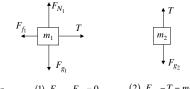
Example 7:

Assuming that the system above is initially at rest and that the pulley is frictionless and of negligible mass

- Find the acceleration of both boxes and the a.) tension in the cord that connects them.
- How much horizontal force must be applied to b.) the 25 kg box to move both boxes at a constant velocity of 15 m/s to the left?

Forces

Example 7a: $m_1 = 25 \text{ kg}, m_2 = 15 \text{ kg}, \mu_{k_1} = 0.20, a = ? \text{ and } T = ?$



$$\sum F_y = ma \qquad (1) \ F_{N_1} - F_{g_1} = 0 \qquad (2) \ F_{g_2} - T = m_2 a$$
$$F_{N_1} = F_{g_1} = m_1 g$$

 $\sum F_x = ma \qquad (3) T - F_{f_1} = m_1 a \qquad \text{no } x \text{ - forces}$

 $(2) + (3) F_{g_2} - F_{f_1} = m_1 a + m_2 a = (m_1 + m_2)a$

$$a = \frac{F_{g_2} - F_{f_1}}{m_1 + m_2} = \frac{m_2 g - \mu_{k_1} F_{N_1}}{m_1 + m_2} = \frac{m_2 g - \mu_{k_1} m_1 g}{m_1 + m_2}$$

21

23

Example 7a: $m_1 = 25 \text{ kg}, m_2 = 15 \text{ kg}, \mu_{k_1} = 0.20, a = ? \text{ and } T = ?$

$$a = \frac{m_2 g - \mu_{k_1} m_1 g}{m_1 + m_2} = \frac{(15 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right) - 0.20(25 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right)}{15 \text{ kg} + 25 \text{ kg}}$$

$$\boxed{a = 2.45 \frac{\text{m}}{\text{s}^2}}$$
(3) $T - F_{f_1} = m_1 a$
(2) $F_{g_2} - T = m_2 a$
using (3) $T = m_1 a + F_{f_1} = m_1 a + \mu_{k_1} F_{N_1} = m_1 a + \mu_{k_1} m_1 g$
 $T = (25 \text{ kg}) \left(2.45 \frac{\text{m}}{\text{s}^2}\right) + 0.20(25 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right)$

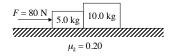
$$\boxed{T = 110.25 \text{ N}}$$

22

20



Example 7b: $m_1 = 25 \text{ kg}, m_2 = 15 \text{ kg}, \mu_{k_1} = 0.20, a = 0, v = 15 \frac{\text{m}}{\text{s}} \text{ to the left, } F = ?$ $f_{F_{N_1}}$ $f_{F_{N_2}}$ $f_{F_{N_2}}$ $f_{F_{N_2}}$ $f_{F_{N_2}} = 0$ $f_{F_{N_1}} = f_{F_{N_1}} = 0$ $f_{F_{N_2}} = m_1g$ $\sum F_x = ma$ $f_{N_1} = F_{F_{N_1}} = 0$ no x - forces $f_{N_1} = f_{F_{N_2}} = 0$ $(2) + (3) F - F_{g_2} - F_{f_1} = 0$ $F = F_{g_2} + F_{f_1} = m_2 g + \mu_{k_1} F_{N_1} = m_2 g + \mu_{k_1} m_1 g$ $F = (15 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right) + 0.20(25 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right)$ F = 196 N



Example 8:

A force of 80 N is applied to a 5.0 kg block that is in contact with a 10 kg block. Both blocks are initially at rest on a horizontal surface and the coefficient of kinetic friction for both blocks is 0.20 on this surface. Find the force that the 10 kg block exerts on the 5.0 kg block.

Forces

Example 8:

$$m_1 = 5.0 \text{ kg}, m_2 = 10.0 \text{ kg}, \mu_{k_1} = \mu_{k_2} = 0.20, F = 80 \text{ N}, a = ? \text{ and } F_{2,1} = ?$$

$$F \xrightarrow{\qquad F_{N_1}} F_{2,1} \xrightarrow{\qquad F_{1,2}} m_2 \xrightarrow{\qquad F_{f_2}} F_{f_1} \xrightarrow{\qquad F_{f_2}} F_{f_2}$$

$$\sum F_y = ma$$
 (1) $F_{N_1} - F_{g_1} = 0$ (2) $F_{N_2} - F_{g_2} = 0$
 $F_{N_1} = F_{g_1} = m_1 g$ $F_{N_2} = F_{g_2} = m_2 g$

$$\sum F_x = ma \qquad (3) \ F - F_{f_1} - F_{2,1} = m_1 a \quad (4) \ F_{1,2} - F_{f_2} = m_2 a$$

$$(3) + (4) \quad F - F_{f_1} - F_{f_2} = m_1 a + m_2 a \quad (F_{1,2} = F_{2,1})$$

$$F - F_{f_1} - F_{f_2} = (m_1 + m_2)a \text{ so } a = \frac{F - F_{f_1} - F_{f_2}}{m_1 + m_2} = \frac{F - \mu_{k_1}F_{N_1} - \mu_{k_2}F_{N_2}}{m_1 + m_2}$$
25

Example 8:

 $m_1 = 5.0 \text{ kg}, m_2 = 10.0 \text{ kg}, \mu_{k_1} = \mu_{k_2} = 0.20, F = 80 \text{ N}, a = ? \text{ and } F_{2,1} = ?$

$$a = \frac{F - \mu_{k_1} F_{N_1} - \mu_{k_2} F_{N_2}}{m_1 + m_2} = \frac{F - \mu_{k_1} m_1 g - \mu_{k_2} m_2 g}{m_1 + m_2}$$

$$a = \frac{80 \text{ N} - 0.20(5.0 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right) - 0.20(10.0 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right)}{5.0 \text{ kg} + 10.0 \text{ kg}}$$

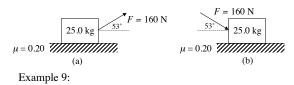
$$\boxed{a = 3.37 \frac{\text{m}}{\text{s}^2}}$$

$$(3) F - F_{f_1} - F_{2,1} = m_1 a \quad (4) F_{1,2} - F_{f_2} = m_2 a$$

$$\text{using (4)} F_{1,2} = m_2 a + F_{f_2} = m_2 a + \mu_{k_2} F_{N_2} = m_2 a + \mu_{k_2} m_2 g$$

$$F_{1,2} = (10 \text{ kg}) \left(3.37 \frac{\text{m}}{\text{s}^2}\right) + 0.20(10 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right)$$

$$\boxed{F_{2,1} = F_{1,2} = 53.3 \text{ N}}$$
26



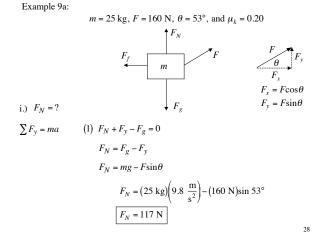
A 25 kg box is being pulled by a force that makes a 53° angle above the horizontal as shown in figure (a).

i.) Find the normal force acting on the box.

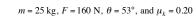
ii.) Find the acceleration of the box.

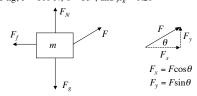
Repeat part (i) and (ii) if the force is applied below the horizontal as shown in figure (b).

Forces 27



Example 9a:

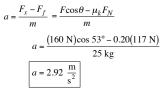




29

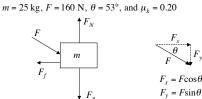
ii.) a = ?

 $\sum F_x = ma \qquad (2) F_x - F_f = ma$



m =

Example 9b:



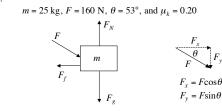
i.) $F_N = ?$

$$\sum F_y = ma$$
 (1) $F_N - F_y - F_g = 0$
 $F_N = F_g + F_y$
 $F_N = mg + F\sin\theta$
 $F_N = (25 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2}\right) + (160 \text{ N})\sin 53^\circ$
 $\overline{F_N = 373 \text{ N}}$

30

Example 9b:

ii.) a = ?



$$\sum F_x = ma$$
(2) $F_x - F_f = ma$

$$a = \frac{F_x - F_f}{m} = \frac{F\cos\theta - \mu_k F_N}{m}$$

$$a = \frac{(160 \text{ N})\cos 53^\circ - 0.20(373 \text{ N})}{25 \text{ kg}}$$

$$a = 0.87 \frac{\text{m}}{\text{s}^2}$$
31

 $\begin{array}{c} 245 \text{ } N \\ \mu = 0.15 \quad 37^{\circ} \end{array}$

Example 10:

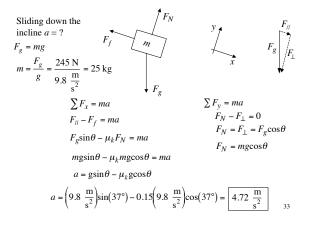
A box with a weight of 245 N held at rest on a 37° incline. The coefficient of kinetic friction between the box and the inclined surface is 0.15.

- a.) Find the acceleration of the box if it is allowed to slide down the incline.
- b.) What force (parallel to the incline) should be applied to the box if the block is to slide down the incline at a constant velocity?
- c.) What force (parallel to the incline) should be applied to the box if the block is to move up the incline with an acceleration of 3.0 m/s²?

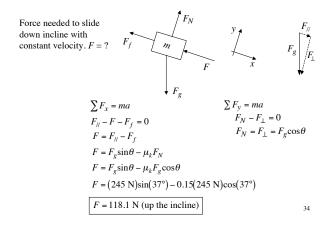
Forces

32

Example 10a $\theta = 37^{\circ}, F_{g} = 245 \text{ N}, \text{ and } \mu_{k} = 0.15$







Example 10c $\theta = 37^{\circ}$, $F_g = 245$ N, m = 25 kg, and $\mu_k = 0.15$

