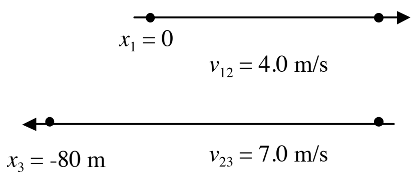
HO 1 Solutions

1.) a.)



$$s = \frac{d}{t} = \frac{d_{12} + d_{23}}{t_{12} + t_{23}}$$

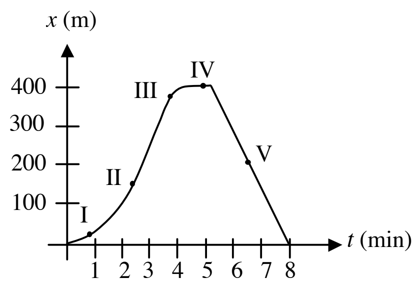
$$t_{12} = \frac{d_{12}}{s_{12}} = \frac{200\text{ m}}{\left(4\,\frac{\text{m}}{\text{s}}\right)} = 50\text{ s} \quad \text{and} \quad t_{23} = \frac{d_{23}}{s_{23}} = \frac{280\text{ m}}{\left(7\,\frac{\text{m}}{\text{s}}\right)} = 40\text{ s}$$

$$s = \frac{d}{t} = \frac{d_{12} + d_{23}}{t_{12} + t_{23}} = \frac{200\text{ m} + 280\text{ m}}{50\text{ s} + 40\text{ s}} = \boxed{5.3\ \frac{\text{m}}{\text{s}}}$$

b.)

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_3 - x_1}{t_3 - t_1} = \frac{-80\text{ m} - 0}{90\text{ s} - 0} = \boxed{-0.89\ \frac{\text{m}}{\text{s}}}$$

2.)



Velocity is the slope of the *x-t* graph.

a.) The slope is zero at point IV.

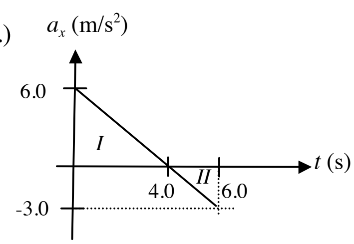
b.) The slope is constant and positive between points II and III.

c.) The slope is constant and negative at point V.

d.) The slope is increasing in magnitude between points I and II.

e.) The slope is decreasing in magnitude between points III and IV.

3.)



$$v_o = 12\ \frac{\text{m}}{\text{s}}$$

$$\Delta v = Area(a - t) = A_I + A_{II} = \frac{1}{2}(4\text{ s})\left(6\frac{\text{m}}{\text{s}^2}\right) + \frac{1}{2}(2\text{ s})\left(-3\frac{\text{m}}{\text{s}^2}\right) = 9\ \frac{\text{m}}{\text{s}}$$

$$\Delta v = 9\ \frac{\text{m}}{\text{s}} = v - v_o \text{ and } v = v_o + \Delta v = 12\ \frac{\text{m}}{\text{s}} + 9\ \frac{\text{m}}{\text{s}} = \boxed{21\ \frac{\text{m}}{\text{s}}}$$

4.) $\Delta x = 80\text{ m}$, $t = 7.00\text{ s}$, $v_2 = 15\ \frac{\text{m}}{\text{s}}$

a.)

$$\Delta x = \left(\frac{v_1 + v_2}{2}\right)t \quad \text{so} \quad v_1 = \frac{2\Delta x}{t} - v_2 = \frac{2(80\text{ m})}{7\text{ s}} - 15\ \frac{\text{m}}{\text{s}} = \boxed{7.86\ \frac{\text{m}}{\text{s}}}$$

b.)

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\left(15\ \frac{\text{m}}{\text{s}} - 7.86\ \frac{\text{m}}{\text{s}}\right)}{7\text{ s}} = \boxed{1.02\ \frac{\text{m}}{\text{s}^2}}$$

5.) $\Delta x = 420\text{ m}$, $v_1 = 0$, $t = 16.0\text{ s}$

$$\Delta x = \left(\frac{v_1 + v_2}{2}\right)t \quad \text{so} \quad v_2 = \frac{2\Delta x}{t} - v_1 = \frac{2(420\text{ m})}{16\text{ s}} - 0 = \boxed{52.5\ \frac{\text{m}}{\text{s}}}$$

6.) for free fall, $a = -g = -9.8\,\frac{\text{m}}{\text{s}^2}$ and $v_o = 5.00\ \frac{\text{m}}{\text{s}}$, $y_o = 40$ m

a.) $y = -\frac{1}{2}gt^2 + v_o t + y_o = \left(-4.9\frac{\text{m}}{\text{s}^2}\right)t^2 + \left(5.00\ \frac{\text{m}}{\text{s}}\right)t + 40\text{ m}$

at $t = 0.5$ s, $y(0.5\text{ s}) = \left(-4.9\frac{\text{m}}{\text{s}^2}\right)(0.5\text{ s})^2 + \left(5.00\ \frac{\text{m}}{\text{s}}\right)(0.5\text{ s}) + 40\text{ m} = \boxed{41.3\text{ m}}$

at $t = 2.0$ s, $y(2.0\text{ s}) = \left(-4.9\frac{\text{m}}{\text{s}^2}\right)(2.0\text{ s})^2 + \left(5.00\ \frac{\text{m}}{\text{s}}\right)(2.0\text{ s}) + 40\text{ m} = \boxed{30.4\text{ m}}$

$v = -gt + v_o = -\left(9.8\frac{\text{m}}{\text{s}^2}\right)t + 5.00\ \frac{\text{m}}{\text{s}}$ so $v(0.5\text{ s}) = -\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.5\text{ s}) + 5.00\ \frac{\text{m}}{\text{s}} = \boxed{0.10\ \frac{\text{m}}{\text{s}}}$

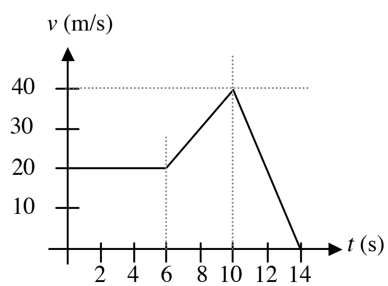
$v(2.0\text{ s}) = -\left(9.8\frac{\text{m}}{\text{s}^2}\right)(2.0\text{ s}) + 5.00\ \frac{\text{m}}{\text{s}} = \boxed{-14.6\ \frac{\text{m}}{\text{s}}}$

b.) $y = 0 \Rightarrow \Delta y = y - y_o = 0 - 40\text{ m} = -40\text{ m}$

$v^2 = v_o^2 + 2a\Delta y$ so $v = \pm\sqrt{v_o^2 + 2a\Delta y} = \pm\sqrt{\left(5.00\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(-40\text{ m})} = \pm 28.4\,\frac{\text{m}}{\text{s}} = \boxed{-28.4\ \frac{\text{m}}{\text{s}}}$

(Since the bag is going down (on its way to the ground) the velocity will be negative.)

7.)



a.) $a = slope = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$t = 3$ s, $a = slope = \boxed{0}$

$t = 7$ s, $a = slope = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\left(40\ \frac{\text{m}}{\text{s}} - 20\ \frac{\text{m}}{\text{s}}\right)}{(10\text{ s} - 6\text{ s})} = \boxed{5.0\ \frac{\text{m}}{\text{s}^2}}$

$t = 11$ s, $a = slope = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\left(0\ \frac{\text{m}}{\text{s}} - 40\ \frac{\text{m}}{\text{s}}\right)}{(14\text{ s} - 10\text{ s})} = \boxed{-10\ \frac{\text{m}}{\text{s}^2}}$

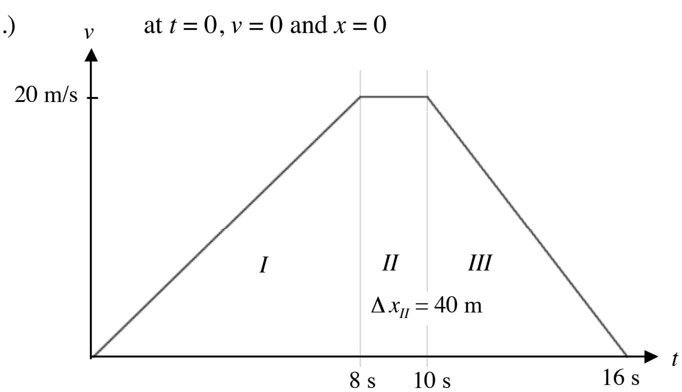
b.) $\Delta x = area$

$t = 5$ s, $\Delta x = \left(20\ \frac{\text{m}}{\text{s}}\right)(5\text{ s}) = \boxed{100\text{ m}}$

$t = 10$ s, $\Delta x = \left(20\ \frac{\text{m}}{\text{s}}\right)(10\text{ s}) + \frac{1}{2}\left(20\ \frac{\text{m}}{\text{s}}\right)(4\text{ s}) = 200\text{ m} + 40\text{ m} = \boxed{240\text{ m}}$

$t = 14$ s, $\Delta x = 240\text{ m} + \frac{1}{2}\left(40\ \frac{\text{m}}{\text{s}}\right)(4\text{ s}) = 240\text{ m} + 80\text{ m} = \boxed{320\text{ m}}$

8.) at $t = 0$, $v = 0$ and $x = 0$



$\Delta x = \Delta x_I + \Delta x_{II} + \Delta x_{III} = 180$ m

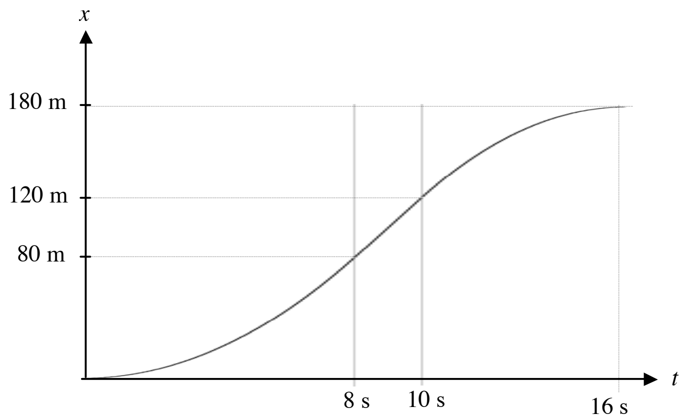
$\Delta x_I = area = \frac{1}{2}(8\text{ s})\left(20\frac{\text{m}}{\text{s}}\right) = 80$ m

$$\Delta x_{II} = 40\text{ m} = area = \left(20\frac{\text{m}}{\text{s}}\right)\Delta t_{II}$$

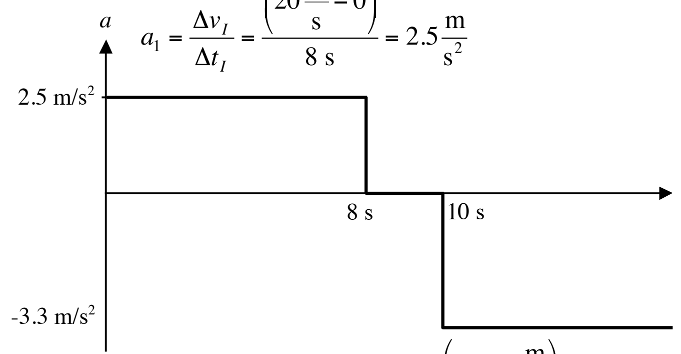
so $\quad$ so $\quad \Delta t_{II} = \dfrac{40\text{ m}}{\left(20\frac{\text{m}}{\text{s}}\right)} = 2\text{ s}$

$\Delta x_{III} = \Delta x - \Delta x_I - \Delta x_{II} = 180\text{ m} - 80\text{ m} - 40\text{ m} = 60\text{ m}$ $\quad$ and $\quad \Delta x_{III} = 60\text{ m} = area = \frac{1}{2}\left(20\frac{\text{m}}{\text{s}}\right)\Delta t_{III}$

so $\quad \Delta t_{III} = \dfrac{2(60\text{ m})}{\left(20\frac{\text{m}}{\text{s}}\right)} = 6\text{ s}$



$$a_1 = \frac{\Delta v_I}{\Delta t_I} = \frac{\left(20\frac{\text{m}}{\text{s}} - 0\right)}{8\text{ s}} = 2.5\frac{\text{m}}{\text{s}^2}$$



$$a_2 = \frac{\Delta v_{III}}{\Delta t_{III}} = \frac{\left(0 - 20\frac{\text{m}}{\text{s}}\right)}{6\text{ s}} = -3.3\frac{\text{m}}{\text{s}^2}$$

1.)

I. uniform acceleration $\Delta x_I = 3000\text{ m},\ v_o = 0,\ v = 24\ \frac{\text{m}}{\text{s}}$

II. constant velocity $t = 430\text{ s},\ v = 24\ \frac{\text{m}}{\text{s}}$

III. uniform acceleration $a = -0.065\ \frac{\text{m}}{\text{s}^2},\ v_o = 24\ \frac{\text{m}}{\text{s}},\ v = 0$

a.) for motion I,

$$v^2 = v_o^2 + 2a\Delta x \text{ so } a = \frac{\left(v^2 - v_o^2\right)}{2\Delta x} = \frac{\left(\left(24\ \frac{\text{m}}{\text{s}}\right)^2 - 0\right)}{2(3000\text{ m})} = \boxed{0.096\ \frac{\text{m}}{\text{s}^2}}$$

b.) for motion III,

$$v^2 = v_o^2 + 2a\Delta x \text{ so } \Delta x_{III} = \frac{\left(v^2 - v_o^2\right)}{2a} = \frac{\left(0 - \left(24\ \frac{\text{m}}{\text{s}}\right)^2\right)}{2\left(-0.065\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{4431\text{ m}}$$

c.) for motion III,

$$v = at + v_o = \left(-0.065\ \frac{\text{m}}{\text{s}^2}\right)(160\text{ s}) + 24\ \frac{\text{m}}{\text{s}} = \boxed{13.6\ \frac{\text{m}}{\text{s}}}$$

d.) $v_{av} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_{total}}{\Delta t_{total}}$

$$\Delta x_{total} = \Delta x_I + \Delta x_{II} + \Delta x_{III} \text{ and for II } \Delta x_{II} = vt = \left(24\ \frac{\text{m}}{\text{s}}\right)(430\text{ s}) = 10320\text{ m}$$

$$\Delta t_{total} = t_I + t_{II} + t_{III} \text{ and for I, } \Delta x = \frac{(v + v_o)}{2}t \text{ so } t_I = \frac{2\Delta x_I}{(v + v_o)} = \frac{2(3000\text{ m})}{\left(24\ \frac{\text{m}}{\text{s}} + 0\right)} = 250\text{ s}$$

$$\text{for III, } \Delta x = \frac{(v + v_o)}{2}t \text{ so } t_{III} = \frac{2\Delta x_{III}}{(v + v_o)} = \frac{2(4431\text{ m})}{\left(0 + 24\ \frac{\text{m}}{\text{s}}\right)} = 369.25\text{ s}$$

therefore: $\Delta x_{total} = \Delta x_I + \Delta x_{II} + \Delta x_{III} = 3000\text{ m} + 10320\text{ m} + 4431\text{ m} = 17751\text{ m}$

$$\Delta t_{total} = t_I + t_{II} + t_{III} = 250\text{ s} + 430\text{ s} + 369.25\text{ s} = 1049.25\text{ s}$$

and $v_{av} = \frac{\Delta x_{total}}{\Delta t_{total}} = \frac{17751\text{ m}}{1049.25\text{ s}} = \boxed{16.9\frac{\text{m}}{\text{s}}}$

2.)

Car: $v_{Co} = 20\ \frac{\text{m}}{\text{s}}$, $x_{Co} = 0$, $a_C = 1.8\ \frac{\text{m}}{\text{s}^2}$, and $v_C = 25\ \frac{\text{m}}{\text{s}}$ then car remains at this speed

Truck $v_{To} = 18\ \frac{\text{m}}{\text{s}}$, $x_{To} = 50\ \text{m}$, and $a_T = 0$

a.) while the car is accelerating: $v_C^2 = v_{Co}^2 + 2a_C\Delta x_C$ so $\Delta x_C = x_C - x_{Co} = \frac{\left(v_C^2 - v_{Co}^2\right)}{2a_C}$

$$x_C - 0 = \frac{\left(\left(25\ \frac{\text{m}}{\text{s}}\right)^2 - \left(20\ \frac{\text{m}}{\text{s}}\right)^2\right)}{2\left(1.8\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{62.5\ \text{m}}$$

b.) Find the time in which the car is accelerating.

$$v_C = a_C t + v_{Co} \text{ so } t = \frac{\left(v_C - v_{Co}\right)}{a_C} = \frac{\left(\left(25\ \frac{\text{m}}{\text{s}}\right) - \left(20\ \frac{\text{m}}{\text{s}}\right)\right)}{\left(1.8\ \frac{\text{m}}{\text{s}^2}\right)} = 2.78\ \text{s}$$

Find the position of the truck after the car stops accelerating.

$$x_T = v_T t + x_{To} = \left(18\ \frac{\text{m}}{\text{s}}\right)\left(2.78\ \text{s}\right) + 50\ \text{m} = 100\ \text{m}$$

After the car stops accelerating:

The position for the truck is, $x_T = v_T t + x_{To} = \left(18\ \frac{\text{m}}{\text{s}}\right)t + 100\ \text{m}$

The position of the car is, $x_C = v_C t + x_{To} = \left(25\ \frac{\text{m}}{\text{s}}\right)t + 62.5\ \text{m}$

The car passes the truck is when $x_T = x_C$

$$\left(18\ \frac{\text{m}}{\text{s}}\right)t + 100\ \text{m} = \left(25\ \frac{\text{m}}{\text{s}}\right)t + 62.5\ \text{m}$$

or $$\left(7\ \frac{\text{m}}{\text{s}}\right)t = 37.5\ \text{m} \text{ and } t = \frac{\left(37.5\ \text{m}\right)}{\left(7\ \frac{\text{m}}{\text{s}}\right)} = \boxed{5.36\ \text{s}}$$

3.) Free fall so $a = -g = -9.8\frac{\text{m}}{\text{s}^2}$, $y_o = 30\ \text{m}$, and $v_o = 24.5\ \frac{\text{m}}{\text{s}}$

a.) at all times $a = -g = \boxed{-9.8\frac{\text{m}}{\text{s}^2}}$

b.) $t = 2.5\ \text{s}$ and $v = -gt + v_o = -\left(9.8\frac{\text{m}}{\text{s}^2}\right)\left(2.5\ \text{s}\right) + 24.5\ \frac{\text{m}}{\text{s}} = \boxed{0}$

c.) $v_{ave} = \frac{\Delta y}{\Delta t} = \frac{y - y_o}{t}$

The position at $t = 4.0\ \text{s}$ is, $y = -\frac{1}{2}gt^2 + v_o t + y_o = \left(-4.9\frac{\text{m}}{\text{s}^2}\right)\left(4.0\ \text{s}\right)^2 + \left(24.5\ \frac{\text{m}}{\text{s}}\right)\left(4.0\ \text{s}\right) + 30\ \text{m} = 49.6\ \text{m}$

so $v_{ave} = \frac{y - y_o}{t} = \frac{\left(49.6\ \text{m} - 30\ \text{m}\right)}{\left(4\ \text{s}\right)} = \boxed{4.9\ \frac{\text{m}}{\text{s}}}$

3.) (cont'd)

d.) $y = 24\text{ m} \Rightarrow \Delta y = y - y_o = 24\text{ m} - 30\text{ m} = -6\text{ m}$

$$v^2 = v_o^2 + 2a\Delta y \text{ so } v = \pm\sqrt{v_o^2 + 2a\Delta y} = \pm\sqrt{\left(24.5\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(-6\text{ m})} = \pm 26.8\frac{\text{m}}{\text{s}} = -26.8\ \frac{\text{m}}{\text{s}}$$

e.) on the ground $y = 0$ and $y = -\frac{1}{2}gt^2 + v_o t + y_o$ so $0 = \left(-4.9\frac{\text{m}}{\text{s}^2}\right)t^2 + \left(24.5\ \frac{\text{m}}{\text{s}}\right)t + 30\text{ m}$

Using quadratic formula: $\boxed{t = 6.0\text{ s}}$

4.) Burn phase: $\Delta y = 68\text{ m}$, $v_o = 0$, $v = 30\ \frac{\text{m}}{\text{s}}$

Free fall: $y_o = 68\text{ m}$, $a = -g = -9.8\frac{\text{m}}{\text{s}^2}$, $v_o = 30\ \frac{\text{m}}{\text{s}}$

a.) during burn phase: $v^2 = v_o^2 + 2a\Delta y$ so $a = \frac{(v^2 - v_o^2)}{2\Delta y} = \frac{\left(\left(30\ \frac{\text{m}}{\text{s}}\right)^2 - 0\right)}{2(68\text{ m})} = \boxed{6.62\ \frac{\text{m}}{\text{s}^2}}$

b.) at maximum height $v = 0$ and $v^2 = v_o^2 + 2a\Delta y$ and $\Delta y = \frac{(v^2 - v_o^2)}{2a}$

$$\text{Therefore: } y = \frac{(v^2 - v_o^2)}{2a} + y_o = \frac{\left(0 - \left(30\ \frac{\text{m}}{\text{s}}\right)^2\right)}{2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)} + 68\text{ m} = \boxed{114\text{ m}}$$

c.) on the ground $y = 0$ and $y = -\frac{1}{2}gt^2 + v_o t + y_o$ so $0 = \left(-4.9\frac{\text{m}}{\text{s}^2}\right)t^2 + \left(30\ \frac{\text{m}}{\text{s}}\right)t + 68\text{ m}$

Using quadratic formula: $\boxed{t = 7.88\text{ s}}$

alternatively the time equals the time to reach the maximum height plus the time to fall back to the ground

time to reach the max height is when $v = 0$ and $v = -gt + v_o$

so $t_1 = \frac{(v - v_o)}{-g} = \frac{\left(0 - 30\ \frac{\text{m}}{\text{s}}\right)}{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = 3.06\text{ s}$ and the time down is found from $y = -\frac{1}{2}gt^2 + v_o t + y_o$

where $y = 0$ and $v_o = 0$ so $0 = -\frac{1}{2}gt^2 + y_o$ and $t_2 = \sqrt{\frac{2y_o}{\text{g}}} = \sqrt{\frac{2(114\text{ m})}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = 4.82\text{ s}$

the total time is therefore $t = t_1 + t_2 = 3.06\text{ s} + 4.82\text{ s} = \boxed{7.88\text{ s}}$

d.) on the ground $y = 0 \Rightarrow \Delta y = y - y_o = 0\text{ m} - 68\text{ m} = -68\text{ m}$ and $v^2 = v_o^2 + 2a\Delta y$

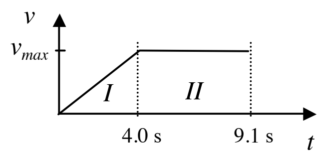
$$\text{so } v = \pm\sqrt{v_o^2 + 2a\Delta y} = \pm\sqrt{\left(30\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(-68\text{ m})} = \pm 47.2\frac{\text{m}}{\text{s}} = -47.2\ \frac{\text{m}}{\text{s}}$$

alternatively since we know the time for free fall

$$v = -gt + v_o = \left(-4.9\ \frac{\text{m}}{\text{s}^2}\right)(7.88\ \text{s}) + 30\ \frac{\text{m}}{\text{s}} = -47.2\ \frac{\text{m}}{\text{s}}$$

Either way the speed when the rocket returns to the ground $\boxed{v = 47.2\ \frac{\text{m}}{\text{s}}}$

5.) looking at the velocity-time graph



The area under the graph is the total displacement

$$\Delta x = Area_I + Area_{II} = 100\ \text{m}$$

so $100\ \text{m} = \frac{1}{2}(4.0\ \text{s})v_{max} + (5.1\ \text{s})v_{max}$ and $v_{max} = \frac{(100\ \text{m})}{(7.1\ \text{s})} = 14.09\ \frac{\text{m}}{\text{s}}$

The average acceleration for the first 4.0 s is therefore $a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\left(14.09\ \frac{\text{m}}{\text{s}} - 0\right)}{(4.0\ \text{s} - 0)} = \boxed{3.52\ \frac{\text{m}}{\text{s}^2}}$

alternatively

the velocity after the first 4.0 s is $v = at_1$ and the displacement is $\Delta x_I = \frac{1}{2}at^2 + v_o t = \frac{1}{2}a(4.0\ \text{s})^2 = (8.0\ \text{s}^2)a$

the displacement in the final 5.1 s is $\Delta x_{II} = vt = at_1 t_2 = a(4.0\ \text{s})(5.1\ \text{s}) = a(20.4\ \text{s}^2)$

the total displacement is 100 m so $\Delta x = 100\ \text{m} = \Delta x_I + \Delta x_{II} = (8.0\ \text{s}^2)a + (20.4\ \text{s}^2)a = (28.4\ \text{s}^2)a$

therefore the acceleration is $a = \frac{(100\ \text{m})}{(28.4\ \text{s}^2)} = \boxed{3.52\ \frac{\text{m}}{\text{s}^2}}$

HO 3 Solutions

1.) $v(t) = a + bt^2$ where $a = 3.00\,\frac{\text{m}}{\text{s}}$ and $b = 0.200\,\frac{\text{m}}{\text{s}^3}$

a.) $a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$ and $v_1 = v(0) = \left(3.00\,\frac{\text{m}}{\text{s}}\right) + \left(0.200\,\frac{\text{m}}{\text{s}^3}\right)(0)^2 = 3.00\,\frac{\text{m}}{\text{s}}$

$$v_2 = v(5\text{s}) = \left(3.00\,\frac{\text{m}}{\text{s}}\right) + \left(0.200\,\frac{\text{m}}{\text{s}^3}\right)(5\text{s})^2 = 8.00\,\frac{\text{m}}{\text{s}}$$

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\left(8.00\,\frac{\text{m}}{\text{s}} - 3.00\,\frac{\text{m}}{\text{s}}\right)}{(5\text{s} - 0)} = \boxed{1.0\,\frac{\text{m}}{\text{s}^2}}$$

b.) $a = \frac{dv}{dt} = \frac{d}{dt}\left(a + bt^2\right) = 2bt = 2\left(0.200\,\frac{\text{m}}{\text{s}^3}\right)t$

$$a(8\text{ s}) = 2\left(0.200\,\frac{\text{m}}{\text{s}^3}\right)(8\text{ s}) = \boxed{3.2\,\frac{\text{m}}{\text{s}^2}}$$

$$a(13\text{ s}) = 2\left(0.200\,\frac{\text{m}}{\text{s}^3}\right)(13\text{ s}) = \boxed{5.2\,\frac{\text{m}}{\text{s}^2}}$$

$$a(15\text{ s}) = 2\left(0.200\,\frac{\text{m}}{\text{s}^3}\right)(15\text{ s}) = \boxed{6.0\,\frac{\text{m}}{\text{s}^2}}$$

2.) $x(t) = 3.42\text{ m} + \left(0.600\,\frac{\text{m}}{\text{s}^2}\right)t^2 - \left(0.100\,\frac{\text{m}}{\text{s}^6}\right)t^6$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(3.42\text{ m} + \left(0.600\,\frac{\text{m}}{\text{s}^2}\right)t^2 - \left(0.100\,\frac{\text{m}}{\text{s}^6}\right)t^6\right) = 2\left(0.600\,\frac{\text{m}}{\text{s}^2}\right)t - 6\left(0.100\,\frac{\text{m}}{\text{s}^6}\right)t^5$$

$$v(t) = \left(1.200\,\frac{\text{m}}{\text{s}^2} - \left(0.600\,\frac{\text{m}}{\text{s}^6}\right)t^4\right)t$$

so $v(t) = 0$ when $\left(1.200\,\frac{\text{m}}{\text{s}^2} - \left(0.600\,\frac{\text{m}}{\text{s}^6}\right)t^4\right) = 0$ and $t = 0$

$$\left(1.200\,\frac{\text{m}}{\text{s}^2} - \left(0.600\,\frac{\text{m}}{\text{s}^6}\right)t^4\right) = 0 \quad \text{when} \quad t = \left(\frac{1.200\,\frac{\text{m}}{\text{s}^2}}{0.600\,\frac{\text{m}}{\text{s}^6}}\right)^{1/4} = 1.19\text{ s}$$

$$x(0) = 3.42\text{ m} + \left(0.600\,\frac{\text{m}}{\text{s}^2}\right)0^2 - \left(0.100\,\frac{\text{m}}{\text{s}^6}\right)0^6 = \boxed{3.42\text{ m}}$$

$$x(1.19\text{ s}) = 3.42\text{ m} + \left(0.600\,\frac{\text{m}}{\text{s}^2}\right)(1.19\text{ s})^2 - \left(0.100\,\frac{\text{m}}{\text{s}^6}\right)(1.19\text{ s})^6 = \boxed{3.99\text{ m}}$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(1.200\,\frac{\text{m}}{\text{s}^2}\,t - \left(0.600\,\frac{\text{m}}{\text{s}^6}\right)t^5\right) = 1.200\,\frac{\text{m}}{\text{s}^2} - 5\left(0.600\,\frac{\text{m}}{\text{s}^6}\right)t^4 = 1.200\,\frac{\text{m}}{\text{s}^2} - \left(3.00\,\frac{\text{m}}{\text{s}^6}\right)t^4$$

$$a(0) = 1.200\frac{\text{m}}{\text{s}^2} - \left(3.00\frac{\text{m}}{\text{s}^6}\right)0^4 = \boxed{1.200\frac{\text{m}}{\text{s}^2}}$$

$$a(1.19\text{ s}) = 1.200\frac{\text{m}}{\text{s}^2} - \left(3.00\frac{\text{m}}{\text{s}^6}\right)(1.19\text{ s})^4 = \boxed{-4.82\frac{\text{m}}{\text{s}^2}}$$

3.) $x(t) = \left(21 + 22t - 6.0t^2\right)\text{ m}$

$$v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\left(21 + 22(3) - 6(3)^2\right)\text{ m} - \left(21 + 22(1) - 6(1)^2\right)\text{ m}}{3\text{ s} - 1\text{ s}} = \boxed{-2.0\ \frac{\text{m}}{\text{s}}}$$

4.) $x(t) = \left(2.0t^3 - 6.0t^2 + 4.0\right)\text{ m}$

$$a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \quad \text{and} \quad v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(2.0t^3 - 6.0t^2 + 4.0\right) = \left(6.0t^2 - 12.0t\right)\frac{\text{m}}{\text{s}}$$

$$a_{ave} = \frac{\left(6.0(3)^2 - 12.0(3)\right)\frac{\text{m}}{\text{s}} - \left(6.0(1)^2 - 12.0(1)\right)\frac{\text{m}}{\text{s}}}{3\text{ s} - 1\text{ s}} = \boxed{12\ \frac{\text{m}}{\text{s}^2}}$$

5.) $x(t) = \left(24t - 2.0t^3\right)\text{ m}$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(24t - 2.0t^3\right) = \left(24 - 6.0t^2\right)\frac{\text{m}}{\text{s}}$$

not moving when $v(t) = \left(24 - 6.0t^2\right)\frac{\text{m}}{\text{s}} = 0$ or $t = \sqrt{\frac{24}{6}}\text{ s} = 2\text{ s}$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(24 - 6.0t^2\right) = (-12.0t)\frac{\text{m}}{\text{s}^2} \quad \text{and} \quad a(2\text{ s}) = (-12.0(2))\frac{\text{m}}{\text{s}^2} = -24\ \frac{\text{m}}{\text{s}^2} \quad \text{so} \quad |a(2\text{ s})| = \boxed{24\ \frac{\text{m}}{\text{s}^2}}$$

6.) $x_1 = 2.0\text{ m}$ and $x_2 = 8.0\text{ m}$ when $t = 2.5\text{ s}$

when $x_2 = 8.0\text{ m}$, $v_2 = 2.8\frac{\text{m}}{\text{s}}$

$$x_2 = \left(\frac{v_1 + v_2}{2}\right)t + x_1 \quad \text{so} \quad v_1 = \frac{2(x_2 - x_1)}{t} - v_2 = \frac{2(8-2)\text{m}}{2.5\text{ s}} - 2.8\frac{\text{m}}{\text{s}} = 2.0\frac{\text{m}}{\text{s}}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{2.8\frac{\text{m}}{\text{s}} - 2.0\frac{\text{m}}{\text{s}}}{2.5\text{ s}} = \boxed{0.32\frac{\text{m}}{\text{s}^2}}$$

7.) $a = \alpha t$ and $\alpha = 1.5\ \text{m/s}^3$

a.) since $a = \frac{dv}{dt}$, the velocity can be obtained from acceleration using the anti-derivative of acceleration

$v = \alpha\frac{t^2}{2} + C_1$ and the constant C_1 can be found by applying the boundary condition $v(1\ \text{s}) = 5.0\ \text{m/s}$.

$C_1 = v - \alpha\frac{t^2}{2}$ so $C_1 = \left(5.0\frac{\text{m}}{\text{s}}\right) - \left(1.5\frac{\text{m}}{\text{s}^3}\right)\frac{(1\ \text{s})^2}{2} = 4.25\frac{\text{m}}{\text{s}}$ and $v = \left(1.5\frac{\text{m}}{\text{s}^3}\right)\frac{t^2}{2} + 4.25\frac{\text{m}}{\text{s}}$

therefore, $v(2.0\ \text{s}) = \left(1.5\frac{\text{m}}{\text{s}^3}\right)\frac{(2.0\ \text{s})^2}{2} + 4.25\frac{\text{m}}{\text{s}} = \boxed{7.25\frac{\text{m}}{\text{s}}}$

b.) since $v = \frac{dx}{dt}$, the position can be obtained from velocity using the anti-derivative of velocity

$x = \alpha\frac{t^3}{6} + C_1 t + C_2$ and the constant C_2 can be found by applying the boundary condition $x(1\ \text{s}) = 6.0\ \text{m}$

$C_2 = x - \alpha\frac{t^3}{6} - C_1 t$ so $C_2 = (6.0\ \text{m}) - \left(1.5\frac{\text{m}}{\text{s}^3}\right)\frac{(1\ \text{s})^3}{6} - \left(4.25\frac{\text{m}}{\text{s}}\right)(1\ \text{s}) = 1.5\ \text{m}$

and $x(t) = \left(1.5\frac{\text{m}}{\text{s}^3}\right)\frac{t^3}{6} + \left(4.25\frac{\text{m}}{\text{s}}\right)t + 1.5\ \text{m}$ therefore, $x(2.0\ \text{s}) = \left(1.5\frac{\text{m}}{\text{s}^3}\right)\frac{(2.0\ \text{s})^3}{6} + \left(4.25\frac{\text{m}}{\text{s}}\right)(2.0\ \text{s}) + 1.5\ \text{m} = \boxed{12\ \text{m}}$

8.) $a = At - Bt^2$, where $A = 1.20\frac{\text{m}}{\text{s}^3}$ and $B = 0.120\frac{\text{m}}{\text{s}^4}$

a.) since $a = \frac{dv}{dt}$, the velocity can be obtained from acceleration using the anti-derivative of acceleration

$v(t) = A\frac{t^2}{2} - B\frac{t^3}{3} + C_1$ and the constant C_1 can be found by applying the boundary condition $v(0) = 0$.

$C_1 = v(t) - A\frac{t^2}{2} + B\frac{t^3}{3}$ so $C_1 = 0$ and $v(t) = \left(1.20\frac{\text{m}}{\text{s}^3}\right)\frac{t^2}{2} - \left(0.120\frac{\text{m}}{\text{s}^4}\right)\frac{t^3}{3} = \boxed{\left(0.60\frac{\text{m}}{\text{s}^3}\right)t^2 - \left(0.040\frac{\text{m}}{\text{s}^4}\right)t^3}$

since $v = \frac{dx}{dt}$, the position can be obtained from velocity using the anti-derivative of velocity

$x(t) = A\frac{t^3}{6} - B\frac{t^4}{12} + C_2$ and the constant C_2 can be found by applying the boundary condition $x(0) = 0$

$C_2 = x(t) - A\frac{t^3}{6} + B\frac{t^4}{12}$ so $C_2 = 0$ and $x(t) = \left(1.20\frac{\text{m}}{\text{s}^3}\right)\frac{t^3}{6} - \left(0.120\frac{\text{m}}{\text{s}^4}\right)\frac{t^4}{12} = \boxed{\left(0.20\frac{\text{m}}{\text{s}^3}\right)t^3 - \left(0.01\frac{\text{m}}{\text{s}^4}\right)t^4}$

b.) maximum velocity occurs when $a = \frac{dv}{dt} = 0$ and $a = At - Bt^2 = 0$ when $t = \frac{A}{B} = \frac{\left(1.20\frac{\text{m}}{\text{s}^3}\right)}{\left(0.120\frac{\text{m}}{\text{s}^4}\right)} = 10\ \text{s}$

therefore maximum velocity is $v(10\ \text{s}) = \left(0.60\frac{\text{m}}{\text{s}^3}\right)(10\ \text{s})^2 - \left(0.040\frac{\text{m}}{\text{s}^4}\right)(10\ \text{s})^3 = \boxed{20\frac{\text{m}}{\text{s}}}$

9.) $v(t) = \alpha - \beta t^2$ where $\alpha = 5.00$ m/s and $\beta = 2.00$ m/s^3

a.) since $v = \frac{dx}{dt}$, the position can be obtained from velocity using the anti-derivative of velocity

$$x(t) = \alpha t - \beta\frac{t^3}{3} + C_1$$ and the constant C_1 can be found by applying the boundary condition $x(0) = 0$

$$C_1 = x(t) - \alpha t + \beta\frac{t^3}{3}$$ so $C_1 = 0$ and $x(t) = \alpha t - \beta\frac{t^3}{3} = \left(5.00\ \frac{\text{m}}{\text{s}}\right)t - \left(2.00\ \frac{\text{m}}{\text{s}^3}\right)\frac{t^3}{3} = \boxed{\left(5.00\ \frac{\text{m}}{\text{s}}\right)t - \left(0.667\ \frac{\text{m}}{\text{s}^3}\right)t^3}$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\alpha - \beta t^2\right) = -2\beta t = -2\left(2.00\ \frac{\text{m}}{\text{s}^3}\right)t = \boxed{\left(-4.00\ \frac{\text{m}}{\text{s}^3}\right)t}$$

b.) maximum displacement occurs when $v = \frac{dx}{dt} = 0$ or $v(t) = \alpha - \beta t^2 = 0$ and $t = \sqrt{\frac{\alpha}{\beta}} = \sqrt{\frac{\left(5.00\ \frac{\text{m}}{\text{s}}\right)}{\left(2.00\ \frac{\text{m}}{\text{s}^3}\right)}} = \sqrt{2.5}$ s

$$x\left(\sqrt{2.5}\ \text{s}\right) = \left(5.00\ \frac{\text{m}}{\text{s}}\right)\left(\sqrt{2.5}\ \text{s}\right) - \left(0.667\ \frac{\text{m}}{\text{s}^3}\right)\left(\sqrt{2.5}\ \text{s}\right)^3 = \boxed{5.27\ \text{m}}$$

10.) a.) for free fall, $a = -g = -9.8\frac{\text{m}}{\text{s}^2}$ and at maximum height $v = 0$ and $\Delta y = 0.52$ m

$v^2 = v_o^2 + 2a\Delta y$ so at maximum height $0 = v_o^2 + 2a\Delta y$ or $v_o = \sqrt{-2a\Delta y} = \sqrt{-2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(0.52\ \text{m})} = \boxed{3.19\ \frac{\text{m}}{\text{s}}}$

b.) when the flea returns to the ground $\Delta y = 0$

$\Delta y = -\frac{1}{2}gt^2 + v_o t$ and when it returns to the ground $0 = -\frac{1}{2}gt^2 + v_o t$ or $t = \frac{2v_o}{g} = \frac{2\left(3.19\ \frac{\text{m}}{\text{s}}\right)}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.65\ \text{s}}$