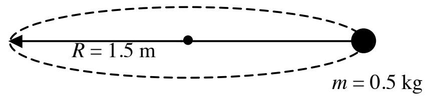
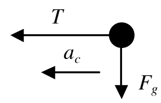
HO 18.1 Solutions

1.)

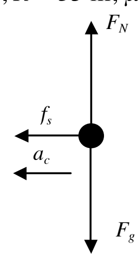




$$F_{net} = \sum F = ma$$
$$T = ma_c = m\frac{v^2}{R}$$
$$v = \sqrt{\frac{TR}{m}}$$

so $v_{max} = \sqrt{\frac{T_{max}R}{m}} = \sqrt{\frac{(50\text{ N})(1.5\text{ m})}{0.5\text{ kg}}} = \boxed{12.2\ \frac{\text{m}}{\text{s}}}$

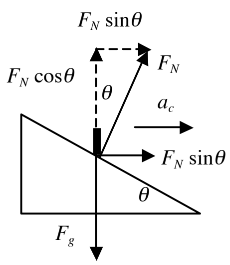
2.) $m = 1500\text{ kg}, R = 35\text{ m}, \mu_s = 0.50$



$$F_{net} = \sum F = ma$$
$$f_s = ma_c = m\frac{v^2}{R}$$
$$\mu_s mg = m\frac{v^2}{R}$$

and $v = \sqrt{\mu_s gR} = \sqrt{(0.50)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(35\text{ m})} = \boxed{13.1\ \frac{\text{m}}{\text{s}}}$

3.)



$R = 50\text{ m},\ v = 13.4\ \frac{\text{m}}{\text{s}}$

$$F_{net} = \sum F = ma$$

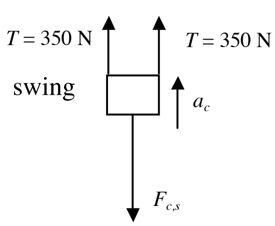
x-direction (1) $F_N \sin\theta = ma_c = m\frac{v^2}{R}$

y-direction (2) $F_N \cos\theta - F_g = 0$ so $F_N = \frac{F_g}{\cos\theta}$

into (1) $\frac{F_g}{\cos\theta}\sin\theta = F_g \tan\theta = mg\tan\theta = m\frac{v^2}{R}$

so $\tan\theta = \frac{v^2}{gR}$ or $\theta = \tan^{-1}\left(\frac{v^2}{gR}\right) = \tan^{-1}\left(\frac{\left(13.4\ \frac{\text{m}}{\text{s}}\right)^2}{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(50\text{ m})}\right) = \boxed{20.1°}$

4.)



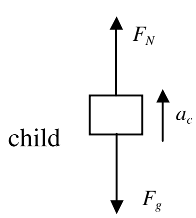
$R = 3\text{ m}, m = 40\text{ kg}$

a.) $F_{net} = \sum F = ma$

$$2T - F_{c,s} = ma_c = m\frac{v^2}{R}$$

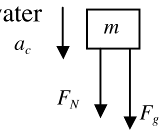
$$v = \sqrt{\frac{R(2T - F_{c,s})}{m}} = \sqrt{\frac{R(2T - mg)}{m}} = \sqrt{\frac{(3\text{ m})\left(2(350\text{ N}) - (40\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\right)}{40\text{ kg}}} = \boxed{4.81\ \frac{\text{m}}{\text{s}}}$$

b.)



$$F_{net} = \sum F = ma$$
$$F_N - F_g = ma_c = m\frac{v^2}{R}$$

and $F_N = m\frac{v^2}{R} + F_g = (40\text{ kg})\frac{\left(4.81\ \frac{\text{m}}{\text{s}}\right)^2}{3\text{ m}} + (40\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{700\text{ N}}$

5.) 

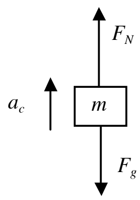
$F_{net} = \sum F = ma$

$F_N + F_g = ma_c = m\dfrac{v^2}{R}$ minimum speed is when the pail is no longer pushing on the water and $F_N = 0$.

so $F_g = m\dfrac{v^2}{R}$ or $mg = m\dfrac{v^2}{R}$ and $v = \sqrt{gR} = \sqrt{\left(9.8\ \dfrac{\text{m}}{\text{s}^2}\right)(1.0\ \text{m})} = \boxed{3.13\ \dfrac{\text{m}}{\text{s}}}$

6.) $m = 500$ kg

a.) at point A: $v = 20\ \dfrac{\text{m}}{\text{s}}$ and $R = 10$ m

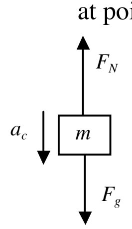


$F_{net} = \sum F = ma$

$F_N - F_g = ma_c = m\dfrac{v^2}{R}$

$$F_N = m\frac{v^2}{R} + F_g = (500\ \text{kg})\frac{\left(20\ \frac{\text{m}}{\text{s}}\right)^2}{10\ \text{m}} + (500\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{24{,}900\ \text{N}}$$

b.) at point B: $R = 15$ m and $F_N = 0$



$F_{net} = \sum F = ma$

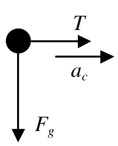
$F_g - F_N = ma_c = m\dfrac{v^2}{R}$

$F_g = mg = m\dfrac{v^2}{R}$

$$v = \sqrt{gR} = \sqrt{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(15\ \text{m})} = \boxed{12.1\ \frac{\text{m}}{\text{s}}}$$

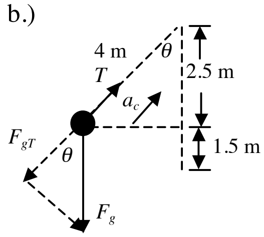
7.) $m = 70$ kg and $R = 10$ m

a.) at beginning man is at rest and rope is horizontal so $v = 0$



$F_{net} = \sum F = ma$ and $T = ma_c = m\dfrac{v^2}{R} = \boxed{0}$

b.)



$F_{net} = \sum F = ma$

$T - F_{gT} = ma_c = m\dfrac{v^2}{R}$

(F_{gT} is the component of the weight in the opposite direction of T)

$T - F_g\cos\theta = ma_c = m\dfrac{v^2}{R}$

Use Energy Conservation to get the speed at this point

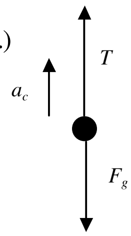
$K_1 + U_1 = K_2 + U_2$ starting from beginning $v_1 = 0$, $y_1 = 2.5$ m, and $y_2 = 0$

$U_1 = K_2$ and $mgy_1 = \frac{1}{2}mv_2^2$ so $v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.8\ \dfrac{\text{m}}{\text{s}^2}\right)(2.5\ \text{m})} = 7.0\ \dfrac{\text{m}}{\text{s}}$

$$T = m\frac{v^2}{R} + F_g\cos\theta = m\frac{v^2}{R} + mg\cos\theta = (70\ \text{kg})\frac{\left(7\ \frac{\text{m}}{\text{s}}\right)^2}{4\ \text{m}} + (70\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{2.5\ \text{m}}{4.0\ \text{m}}\right) = \boxed{1286\ \text{N}}$$

7.) (cont'd)

c.)



$$F_{net} = \sum F = ma$$
$$T - F_g = ma_c = m\frac{v^2}{R}$$
$$T = m\frac{v^2}{R} + F_g = m\frac{v^2}{R} + mg$$

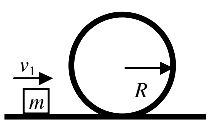
Use Energy Conservation to get the speed at this point

$K_1 + U_1 = K_2 + U_2$ starting from beginning $v_1 = 0$, $y_1 = 4.0\text{ m}$, and $y_2 = 0$

$U_1 = K_2$ and $mgy_1 = \frac{1}{2}mv_2^2$ so $v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(4.0\text{ m})} = 8.85\ \frac{\text{m}}{\text{s}}$

$$T = m\frac{v^2}{R} + mg = (70\text{ kg})\frac{\left(8.85\ \frac{\text{m}}{\text{s}}\right)^2}{4\text{ m}} + (70\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{2057\text{ N}}$$

8.)

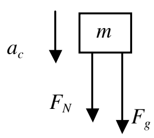


$R = 10\text{ m}$, $v_1 = 25\text{ m/s}$, and $m = 3\text{ kg}$

Use Energy Conservation to get the speed at the top

$K_1 + U_1 = K_2 + U_2$ with $y_1 = 0$ and $y_2 = 2R = 20\text{ m}$
$K_1 = K_2 + U_2$
$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgy_2$

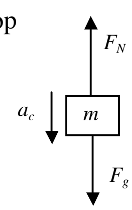
so $v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{\left(25\ \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(20\text{ m})} = \boxed{15.3\ \frac{\text{m}}{\text{s}}}$



$$F_{net} = \sum F = ma$$
$$F_N + F_g = ma_c = m\frac{v^2}{R} \quad \text{so} \quad F_N = m\frac{v^2}{R} - F_g = m\frac{v^2}{R} - mg$$
$$F_N = (3\text{ kg})\frac{\left(15.3\ \frac{\text{m}}{\text{s}}\right)^2}{10\text{ m}} - (3\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{40.8\text{ N}}$$

9.) $R = 7.5\text{ m}$

a.) at the top



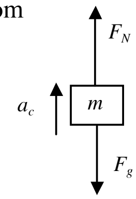
$$F_{net} = \sum F = ma$$
$$F_g - F_N = ma_c = m\frac{v^2}{R}$$

"weightless when $F_N = 0$" and $F_g = mg = m\frac{v^2}{R}$ so $g = \frac{v^2}{R}$

$$v = \sqrt{gR} = \sqrt{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(7.5\text{ m})} = \boxed{8.57\ \frac{\text{m}}{\text{s}}}$$

9.) $R = 7.5\text{ m}$

b.) at the bottom



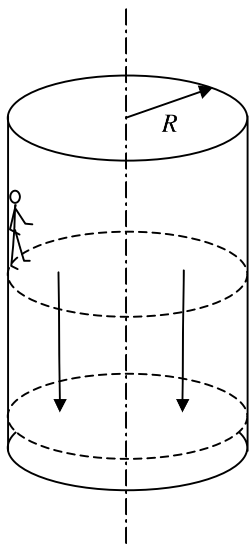
$$F_{net} = \sum F = ma$$

$$F_N - F_g = ma_c = m\frac{v^2}{R}$$

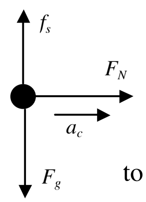
$$F_N = m\frac{v^2}{R} + F_g = m\frac{v^2}{R} + mg$$

$$F_N = (60\text{ kg})\frac{\left(8.57\ \frac{\text{m}}{\text{s}}\right)^2}{10\text{ m}} + (60\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{1176\text{ N}}$$

10.)



$$R = 5.0\text{ m},\ \omega = 0.50\ \frac{\text{rev}}{\text{s}}\left(\frac{2\pi\ \text{rad}}{1\text{ rev}}\right) = \pi\ \frac{\text{rad}}{\text{s}}$$



$$F_{net} = \sum F = ma$$

$$F_N = ma_c = m\omega^2 R$$

to not slip $\quad F_g = f_s = \mu_s F_N$

$$mg = \mu_s F_N = \mu_s m\omega^2 R$$

so $\quad \mu_s = \dfrac{g}{\omega^2 R} = \dfrac{9.8\ \frac{\text{m}}{\text{s}^2}}{\left(\pi\ \frac{\text{rad}}{\text{s}}\right)^2 (5.0\text{ m})} = \boxed{0.20}$