1.) a.) $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta = \left(5 \text{ x } 10^3 \text{ N/C}\right)\left(0.08 \text{ m}\right)^2\cos\left(53.1°\right) = \boxed{19.2 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}}}$

b.) The total flux through the cube must be zero, any flux entering the cube must also leave it.

2.) a.) $\Phi_{top} = \vec{E}\cdot\vec{A} = \left(\left(2.5 \text{ N/C}\right)\hat{i} - \left(4.2 \text{ N/C}\right)\hat{j}\right)\cdot\left(0.04 \text{ m}^2\right)\hat{k} = 0$

$\Phi_{bottom} = \vec{E}\cdot\vec{A} = \left(\left(2.5 \text{ N/C}\right)\hat{i} - \left(4.2 \text{ N/C}\right)\hat{j}\right)\cdot\left(0.04 \text{ m}^2\right)\left(-\hat{k}\right) = 0$

$\Phi_{left} = \vec{E}\cdot\vec{A} = \left(\left(2.5 \text{ N/C}\right)\hat{i} - \left(4.2 \text{ N/C}\right)\hat{j}\right)\cdot\left(0.04 \text{ m}^2\right)\left(-\hat{j}\right) = 0.168 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}}$

$\Phi_{right} = \vec{E}\cdot\vec{A} = \left(\left(2.5 \text{ N/C}\right)\hat{i} - \left(4.2 \text{ N/C}\right)\hat{j}\right)\cdot\left(0.04 \text{ m}^2\right)\hat{j} = -0.168 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}}$

$\Phi_{front} = \vec{E}\cdot\vec{A} = \left(\left(2.5 \text{ N/C}\right)\hat{i} - \left(4.2 \text{ N/C}\right)\hat{j}\right)\cdot\left(0.04 \text{ m}^2\right)\hat{i} = 0.1 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}}$

$\Phi_{back} = \vec{E}\cdot\vec{A} = \left(\left(2.5 \text{ N/C}\right)\hat{i} - \left(4.2 \text{ N/C}\right)\hat{j}\right)\cdot\left(0.04 \text{ m}^2\right)\left(-\hat{i}\right) = -0.1 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}}$

b.) $\Phi_{total} = 0$

3.) Using Gauss's Law $\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{4.8\text{x}10^{-6} \text{ C}}{8.85\text{x}10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}} = \boxed{5.42x10^5 \frac{\text{N}\cdot\text{m}^2}{\text{C}}}$

4.) Total flux is $\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{3.6\text{x}10^{-9} \text{ C}}{8.85\text{x}10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}} = 407.8 \frac{\text{N}\cdot\text{m}^2}{\text{C}}$

Since charge is at the center of the cube, equal amounts of flux will pass through each side

$\Phi_{side} = \frac{\Phi_{total}}{6} = \frac{407.8 \frac{\text{N}\cdot\text{m}^2}{\text{C}}}{6} = \boxed{67.8 \frac{\text{N}\cdot\text{m}^2}{\text{C}}}$

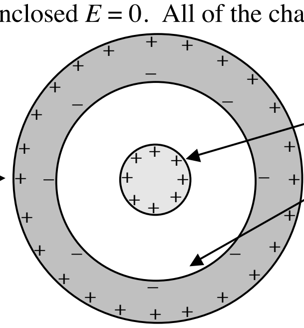
5.) a.) $\Phi = \frac{q_{enclosed}}{\varepsilon_o} = \frac{q_1}{\varepsilon_o} = \frac{5\text{x}10^{-9}\text{C}}{\varepsilon_o} = \boxed{565 \frac{\text{N}\cdot\text{m}^2}{\text{C}}}$

b and c.) $\Phi = \frac{q_{enclosed}}{\varepsilon_o} = \frac{q_1 + q_2}{\varepsilon_o} = \frac{5\text{x}10^{-9}\text{C} - 3\text{x}10^{-9}\text{C}}{\varepsilon_o} = \boxed{226 \frac{\text{N}\cdot\text{m}^2}{\text{C}}}$

6.) a.) $E = \frac{q}{4\pi\varepsilon_o r^2} = = \frac{0.15\text{x}10^{-9}\text{C}}{4\pi\varepsilon_o\left(0.7 \text{ m}\right)^2} = \boxed{2.76 \frac{\text{N}}{\text{C}}}$

b.) Inside the sphere no charge is enclosed $E = 0$. All of the charge is uniformly distributed on the surface of the sphere since it is a conductor.

7.) b.)



charge +5 nC on outer surface spherical

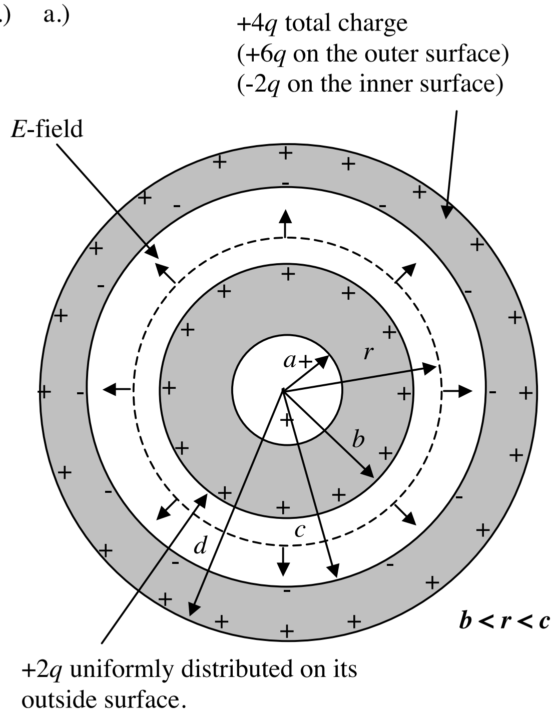
the total charge on the spherical shell is +7 nC. This is the sum of the charge on its inside and outside surfaces.

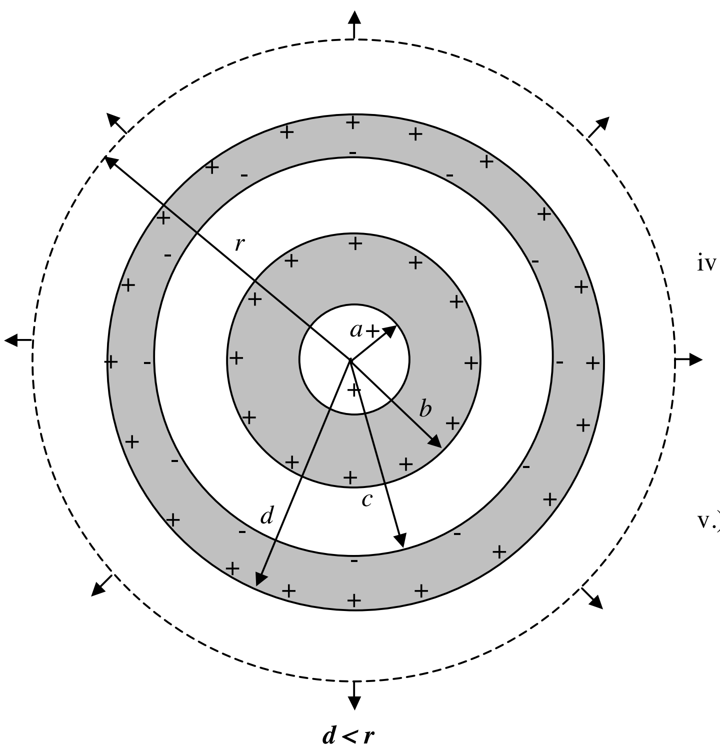
$q_{total} = q_{inside} + q_{outside}$
$+7 \text{ nC} = -5\text{nC} + q_{outside}$
Therefore $q_{outside} = +12$ nC

a.)
the charge attracts electrons to the inner surface of the conductive sphere. Its magnitude is the same as the inner charge. $q_{inside} = -5$nC

Ignoring the inside charge, there would be +7 nC on the outside surface of the spherical shell and none on its outside surface. The inner charge attracts –5 nC of electrons to the inner surface. On the outer surface is the combination of the original charge (+7 nC) and the charge "left behind" when –5nC moved to the inner surface: $q = 7 \text{ nC} + 5 \text{ nC} = 12$ nC.

8.) a.)





i.) when $\boldsymbol{r < a}$ there is no enclosed charge and $E = 0$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$

$$E 4\pi r^2 = 0$$

$$E = 0$$

ii.) when $\boldsymbol{a < r < b}$ there is no enclosed charge and $E = 0$. Also, the electric field is zero inside a conductor.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$

$$E 4\pi r^2 = 0$$

$$E = 0$$

iii.) when $\boldsymbol{b < r < c}$ the charge (+2q) of the smaller conductor is enclosed. Using Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\oint E \cdot dA = \frac{2q}{\varepsilon_o} \quad \text{(E and dA point in same direction.)}$$

$$E \oint dA = \frac{2q}{\varepsilon_o} \quad \text{(E is constant on this Gaussian surface)}$$

$$E 4\pi r^2 = \frac{2q}{\varepsilon_o} \quad \text{(Surface area of a sphere)}$$

$$E = \frac{2q}{4\pi r^2 \varepsilon_o} = \frac{q}{2\pi \varepsilon_o r^2} \qquad (b < r < a)$$

iv.) when $\boldsymbol{c < r < d}$, $E = 0$ because you are inside a conductor and the electric field is zero inside a conductor. (Also the enclosed charge is +2q + (-2q) = 0.)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$

$$E 4\pi r^2 = 0$$

$$E = 0$$

v.) when $\boldsymbol{d < r}$ the total charge enclosed is +6q (+2q on the inner sphere and +4q on the outer sphere).

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$E 4\pi r^2 = \frac{6q}{\varepsilon_o}$$

$$E = \frac{3q}{2\pi r^2 \varepsilon_o}$$

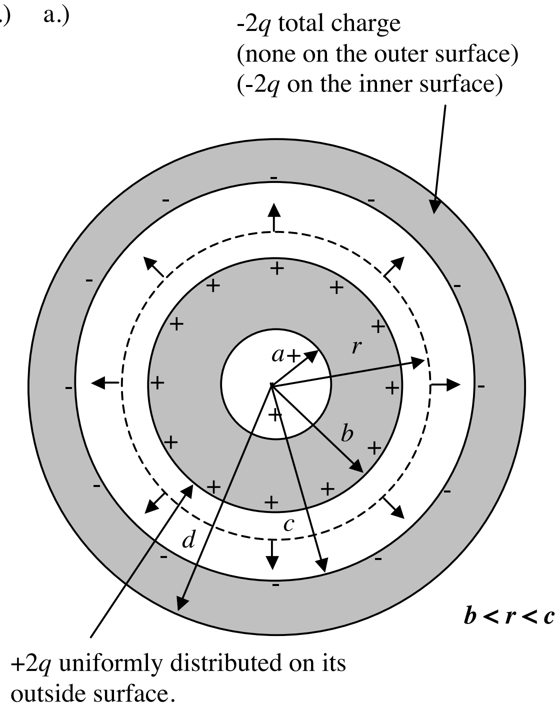
b.) i.) no charge (it is all forced to the outer surface.)

ii.) the +2q is all on the outer surface of the small shell

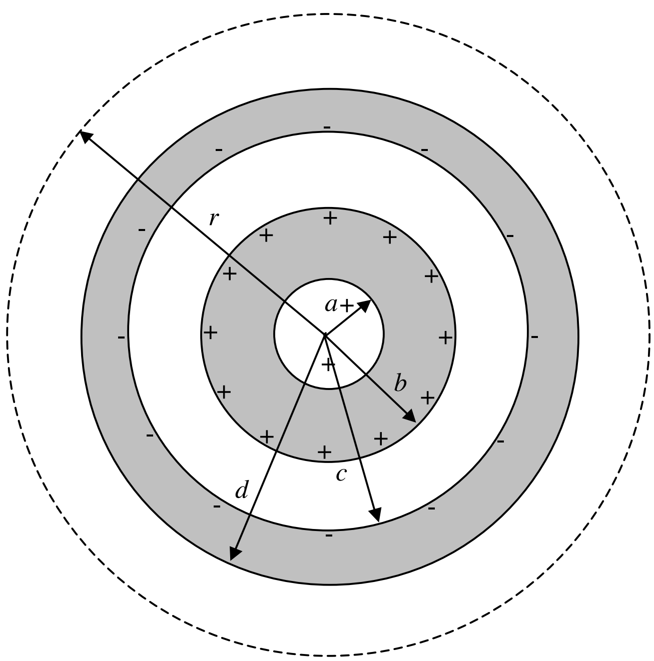
iii.) a charge of –2q is induced on the inner surface of the large shell. This results because electrons are attracted to the positive charge on the small shell.

iv.) the +2q "left behind" by the negative charge moving to the inner surface plus the original charge +4q placed there for a total charge of +6q.

9.) a.)



b < r < c



d < r

i.) when $\boldsymbol{r < a}$ there is no enclosed charge and $E = 0$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$
$$E4\pi r^2 = 0$$
$$E = 0$$

ii.) when $\boldsymbol{a < r < b}$ there is no enclosed charge and $E = 0$. Also, the electric field is zero inside a conductor.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$
$$E4\pi r^2 = 0$$
$$E = 0$$

iii.) when $\boldsymbol{b < r < c}$ the charge ($+2q$) of the smaller conductor is enclosed. Using Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$
$$\oint E \cdot dA = \frac{2q}{\varepsilon_o} \quad \text{(} E \text{ and } dA \text{ point in same direction.)}$$
$$E\oint dA = \frac{2q}{\varepsilon_o} \quad \text{(} E \text{ is constant on this Gaussian surface)}$$
$$E4\pi r^2 = \frac{2q}{\varepsilon_o} \quad \text{(Surface area of a sphere)}$$
$$E = \frac{2q}{4\pi r^2 \varepsilon_o} = \frac{q}{2\pi\varepsilon_o r^2} \qquad (b < r < a)$$

iv.) when $\boldsymbol{c < r < d}$, $E = 0$ because you are inside a conductor and the electric field is zero inside a conductor. (Also the enclosed charge is $+2q + (-2q) = 0$.)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$
$$E4\pi r^2 = 0$$
$$E = 0$$

v.) when $\boldsymbol{d < r}$ the total charge enclosed is still 0 (there is no charge on the outside surface of the large shell).

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$
$$E4\pi r^2 = 0$$
$$E = 0$$

b.) i.) no charge (it is all forced to the outer surface.)

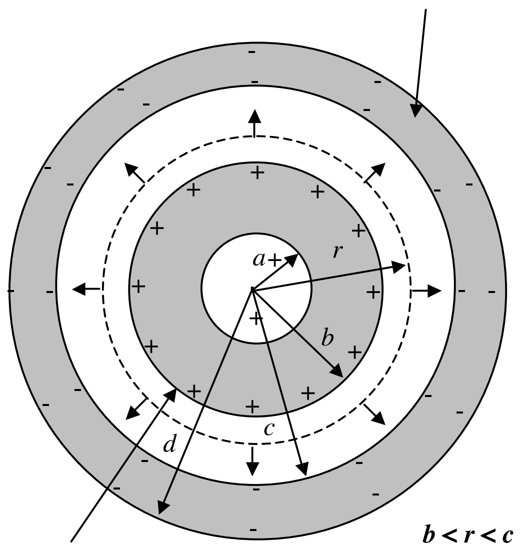
ii.) the $+2q$ is all on the outer surface of the small shell

iii.) a charge of $-2q$ is induced on the inner surface of the large shell. This results because electrons are attracted to the positive charge on the small shell.

iv.) there are no charges on the outer surface of the large shell. $q_{total} = q_{inside} + q_{outside}$ so $-2q = -2q + q_{outside}$ so $q_{outside} = 0$.

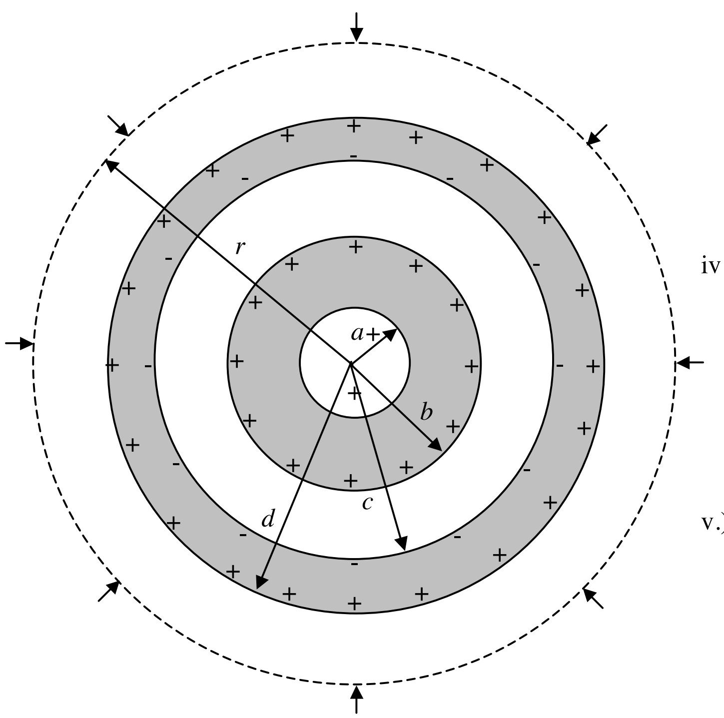
10.) a.)

-4q total charge
(-2q on the outer surface)
(-2q on the inner surface)



$\boldsymbol{b < r < c}$

+2q uniformly distributed on its outside surface.



$\boldsymbol{d < r}$

i.) when $\boldsymbol{r < a}$ there is no enclosed charge and $E = 0$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$
$$E 4\pi r^2 = 0$$
$$E = 0$$

ii.) when $\boldsymbol{a < r < b}$ there is no enclosed charge and $E = 0$. Also, the electric field is zero inside a conductor.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$
$$E 4\pi r^2 = 0$$
$$E = 0$$

iii.) when $\boldsymbol{b < r < c}$ the charge (+2q) of the smaller conductor is enclosed. Using Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$
$$\oint E \cdot dA = \frac{2q}{\varepsilon_o} \quad \text{(}E\text{ and }dA\text{ point in same direction.)}$$
$$E \oint dA = \frac{2q}{\varepsilon_o} \quad \text{(}E\text{ is constant on this Gaussian surface)}$$
$$E 4\pi r^2 = \frac{2q}{\varepsilon_o} \quad \text{(Surface area of a sphere)}$$
$$E = \frac{2q}{4\pi r^2 \varepsilon_o} = \frac{q}{2\pi \varepsilon_o r^2} \qquad (b < r < a)$$

iv.) when $\boldsymbol{c < r < d}$, $E = 0$ because you are inside a conductor and the electric field is zero inside a conductor. (Also the enclosed charge is +2q + (-2q) = 0.)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$
$$E 4\pi r^2 = 0$$
$$E = 0$$

v.) when $\boldsymbol{d < r}$ the total charge enclosed is -2q (+2q on the inner sphere and -4q on the outer sphere).

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$
$$E 4\pi r^2 = \frac{-2q}{\varepsilon_o}$$
$$E = \frac{-q}{2\pi r^2 \varepsilon_o}$$

b.) i.) no charge (it is all forced to the outer surface.)

ii.) the +2q is all on the outer surface of the small shell

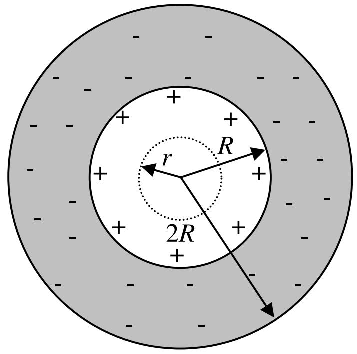
iii.) a charge of –2q is induced on the inner surface of the large shell. This results because electrons are attracted to the positive charge on the small shell.

iv.) since the total charge on the large shell is –4q and there is –2q on the inner surface there must be an additional –2q on the outer surface of the large shell.
$q_{total} = q_{inside} + q_{outside}$ so $-4q = -2q + q_{outside}$ so $q_{outside} = -2q$.

1.) a.) $\rho V = -Q$

$$V = \frac{4}{3}\pi\left((2R)^3 - R^3\right) = \frac{28}{3}\pi R^3$$

$$\rho = \frac{-Q}{V} = \boxed{\frac{-3Q}{28\pi R^3}}$$



$0 < r < R$

b.) for $\mathbf{0 < r < R}$, $q_{enclosed} = 0$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$

$$\oint E dA = E\oint dA = E4\pi r^2 = 0$$

$$\boxed{E = 0}$$

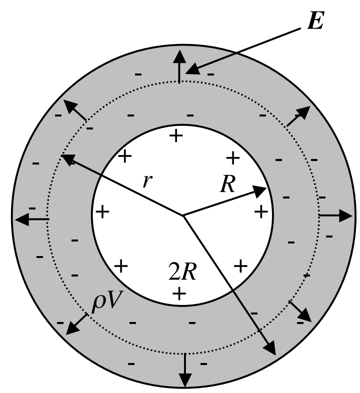
for $\mathbf{R < r < 2R}$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} \quad \text{and} \quad q_{enclosed} = Q + \rho V$$

$$q_{enclosed} = Q + \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R^3\right) = Q + \rho\frac{4}{3}\pi\left(r^3 - R^3\right)$$

$$q_{enclosed} = Q + \frac{-3Q}{28\pi R^3}\frac{4}{3}\pi\left(r^3 - R^3\right) = Q + \frac{-Q}{7R^3}R^3\left(\frac{r^3}{R^3} - 1\right)$$

$$q_{enclosed} = Q + \frac{-Q}{7}\left(\frac{r^3}{R^3} - 1\right) = Q\left(1 - \frac{1}{7}\left(\frac{r^3}{R^3} - 1\right)\right)$$



$R < r < 2R$

$$E4\pi r^2 = \frac{Q\left(1 - \frac{1}{7}\left(\frac{r^3}{R^3} - 1\right)\right)}{\varepsilon_o} \quad \text{and} \quad \boxed{E = \frac{Q\left(1 - \frac{1}{7}\left(\frac{r^3}{R^3} - 1\right)\right)}{4\pi r^2\varepsilon_o}}$$

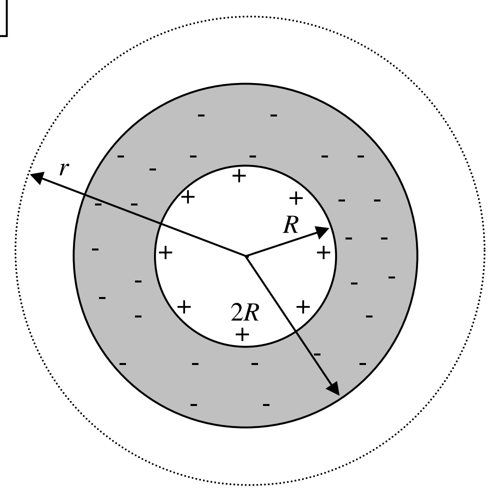
for $\mathbf{2R < r}$

$$q_{enclosed} = Q + -Q = 0$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$

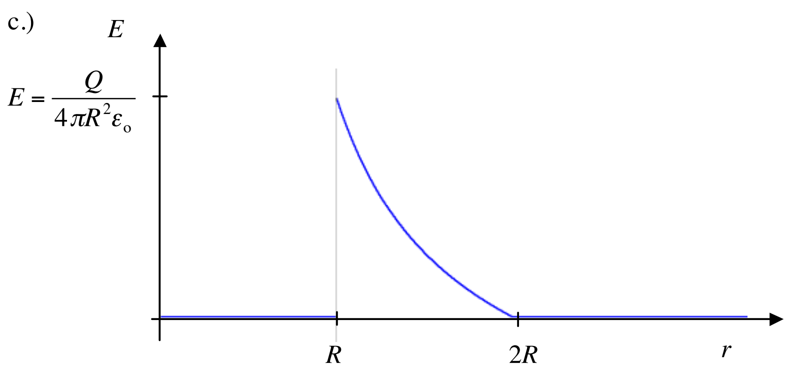
$$\oint E dA = E\oint dA = E4\pi r^2 = 0$$

$$\boxed{E = 0}$$



$2R < r$

c.)



The values for E at $r = R$ and $r = 2R$ make sense.

$$E(R) = \frac{Q\left(1-\frac{1}{7}\left(\frac{R^3}{R^3}-1\right)\right)}{4\pi R^2\varepsilon_o} = \frac{Q}{4\pi R^2\varepsilon_o}$$

and

$$E(2R) = \frac{Q\left(1-\frac{1}{7}\left(\frac{(2R)^3}{R^3}-1\right)\right)}{4\pi(2R)^2\varepsilon_o} = \frac{Q\left(1-\frac{1}{7}\left(\frac{8R^3}{R^3}-1\right)\right)}{4\pi(2R)^2\varepsilon_o} = \frac{Q\left(1-\frac{1}{7}(8-1)\right)}{4\pi(2R)^2\varepsilon_o}$$

$$E(2R) = \frac{Q\left(1-\frac{1}{7}(7)\right)}{4\pi(2R)^2\varepsilon_o} = \frac{Q(1-1)}{4\pi(2R)^2\varepsilon_o} = 0$$

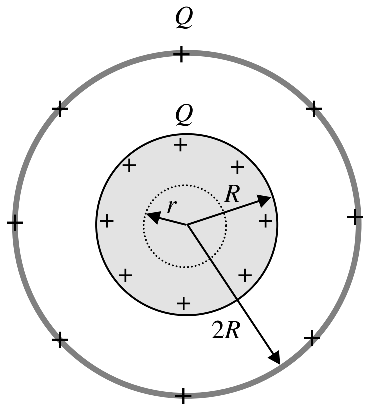
2.)

a.) for $\mathbf{0 < r < R}$, $q_{enclosed} = 0$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = 0$$

$$\oint E dA = E\oint dA = E4\pi r^2 = 0$$

$$\boxed{E = 0}$$



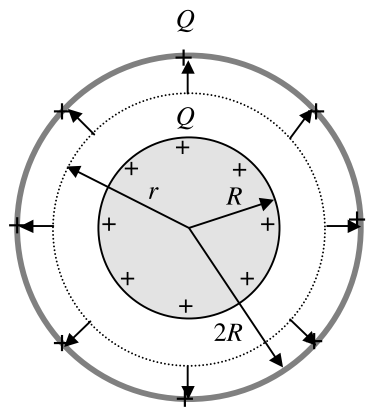
$0 < r < R$

for $\mathbf{R < r < 2R}$, $q_{enclosed} = Q$

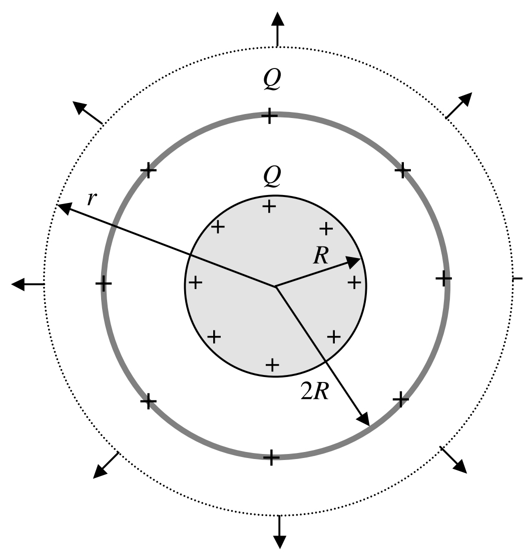
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

$$\oint E dA = E\oint dA = E4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$\boxed{E = \frac{Q}{4\pi r^2\varepsilon_o}}$$



$R < r < 2R$



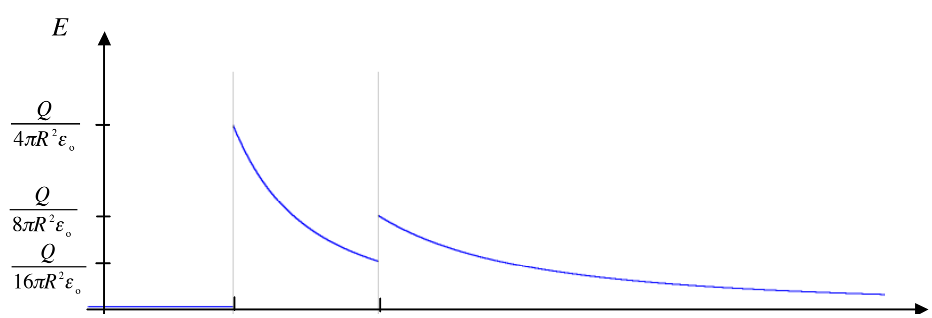
$2R < r$

for $\mathbf{r > 2R}$, $q_{enclosed} = Q + Q = 2Q$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{2Q}{\varepsilon_o}$$

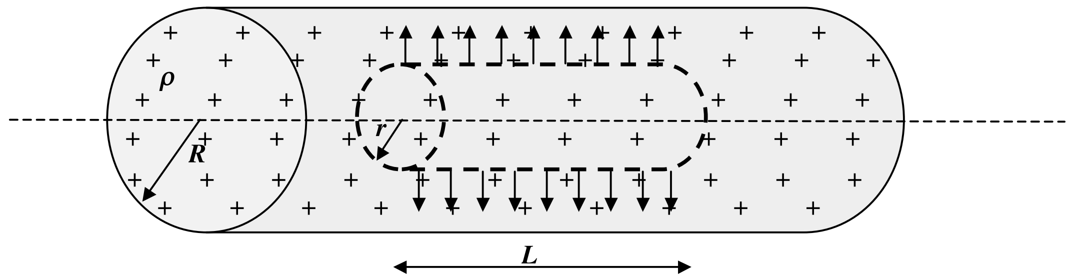
$$E4\pi r^2 = \frac{2Q}{\varepsilon_o} \quad \text{and} \quad \boxed{E = \frac{Q}{2\pi r^2\varepsilon_o}}$$

b.)

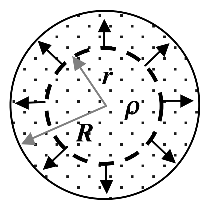


3.)

a.) ***r < R***



Gaussian surface is a cylinder radius r and length L. Its volume is $V = \pi r^2 L$ and it encloses a charge $\rho V = \rho\pi r^2 L$. The electric field on the surface is constant and perpendicular to the surface and has radial symmetry. No electric flux passes through the ends of the cylinder. Looking head-on along the axis of the cylinder:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

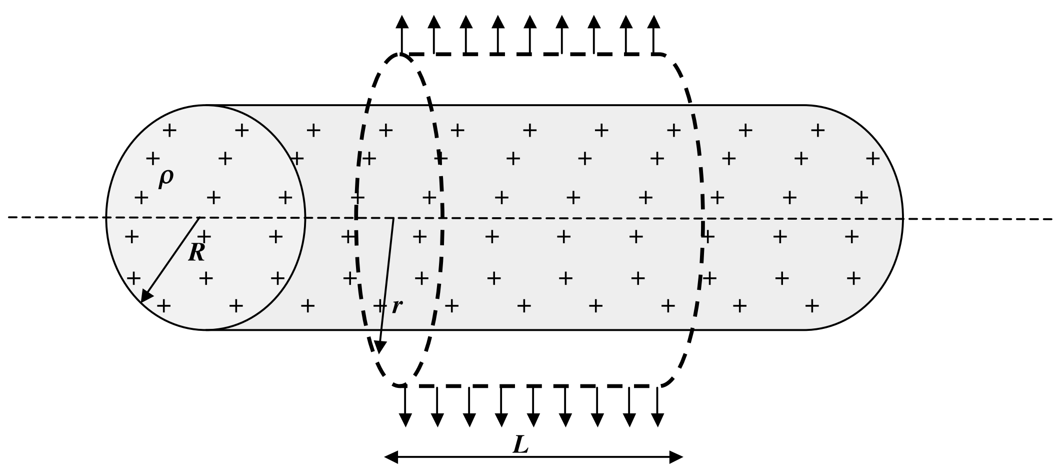
$$2\int_{\text{ends}} \vec{E}\cdot d\vec{A} + \int_{\text{cylinder}} \vec{E}\cdot d\vec{A} = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

$$\underset{E\perp\text{ to }A}{0} + \underset{E\text{ is constant and }\parallel\text{ to }A}{E\int dA} = \frac{\rho\pi r^2 L}{\varepsilon_o}$$

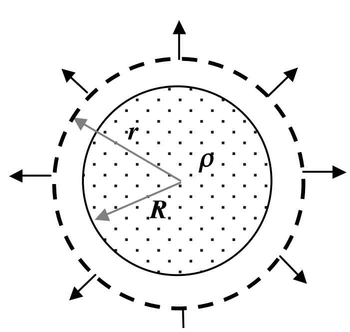
$$E(2\pi rL) = \frac{\rho\pi r^2 L}{\varepsilon_o}$$ (The surface area of the cylinder is $2\pi rL$.)

$$\boxed{E = \frac{\rho r}{2\varepsilon_o}}$$

R < r



Gaussian surface is a cylinder radius r and length L. It encloses a charge $\rho V = \rho\pi R^2 L$ (only the solid cylinder with radius R contains charges). The electric field on the surface is again constant and perpendicular to the surface with radial symmetry. No electric flux passes through the ends of the cylinder. Looking head-on along the axis of the cylinder:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho\pi R^2 L}{\varepsilon_o}$$

$$2\int_{\text{ends}} \vec{E}\cdot d\vec{A} + \int_{\text{cylinder}} \vec{E}\cdot d\vec{A} = \frac{\rho\pi R^2 L}{\varepsilon_o}$$

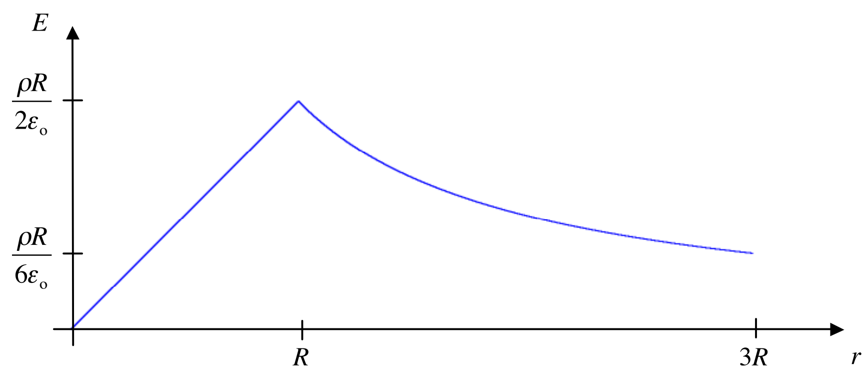
$$0 + E\int dA = \frac{\rho\pi R^2 L}{\varepsilon_o}$$

$$E(2\pi rL) = \frac{\rho\pi R^2 L}{\varepsilon_o} \quad \text{so} \quad \boxed{E = \frac{\rho R^2}{2\varepsilon_o r}}$$

b.)

for $r < R$, $E = \dfrac{\rho r}{2\varepsilon_o}$ and is a linear increasing function of r.

for $r > R$, $E = \dfrac{\rho R^2}{2\varepsilon_o r}$ and decreases with a $\dfrac{1}{r}$ dependence.



Note that the electric field E has piece-wise continuity at $r = R$ and $E(r < R) = E(r > R)$.

$$E(r < R) = \frac{\rho R}{2\varepsilon_o} \quad \text{and} \quad E(r > R) = \frac{\rho R^2}{2\varepsilon_o R} = \frac{\rho R}{2\varepsilon_o}$$

c.) to express in terms of linear density λ one must relate λ to volume charge density ρ.

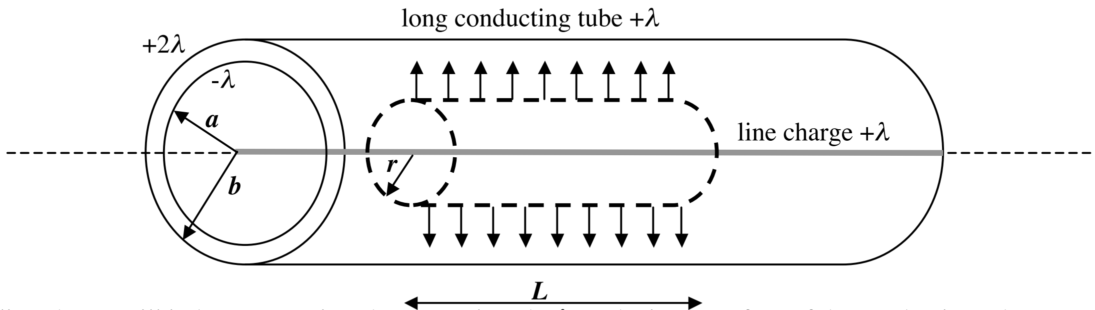
volume charge density x cross-sectional area = linear charge density

$\rho \pi R^2 = \lambda$ or $\rho = \dfrac{\lambda}{\pi R^2}$

for $r < R$, $E = \dfrac{\lambda r}{2\pi\varepsilon_o R^2}$

for $r > R$, $E = \dfrac{\lambda}{2\pi\varepsilon_o r}$ (same as for an infinite line of charge)

4.)

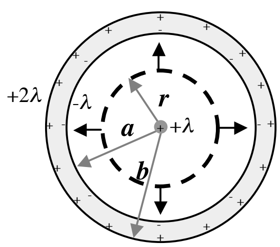


The positive line charge will induce a negative charge per length -λ on the inner surface of the conducting tube. In order to have a total charge per unit length of +λ, a charge of +2λ must exist on its outer surface

a.)

i.) ***r* < *a***

The Gaussian surface is cylindrical with length L and radius r as in the previous example. The electric field on the surface is again constant and perpendicular to the surface with radial symmetry. No electric flux passes through the ends of the cylinder. Looking head-on along the axis of the cylinder:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\lambda L}{\varepsilon_o} \qquad \text{(only line charge } +\lambda \text{ is enclosed)}$$

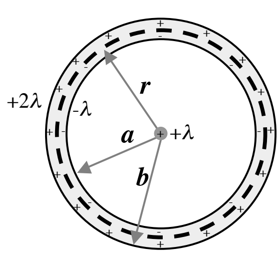
$$2\int_{\text{ends}} \vec{E}\cdot d\vec{A} + \int_{\text{cylinder}} \vec{E}\cdot d\vec{A} = \frac{\lambda L}{\varepsilon_o}$$

$$0 + E\int dA = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o} \quad \text{so} \quad \boxed{E = \frac{\lambda}{2\pi\varepsilon_o r}}$$

ii.) ***a* < *r* < *b***

Inside the conductive cylinder no electric field can exist. The enclosed charge per unit length is the sum of the positive charge on the line-charge +λ and the negative induced charge on the inner surface of the cylinder -λ.



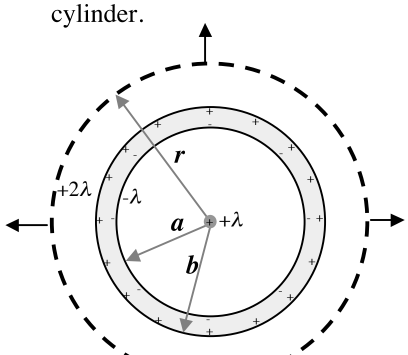
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{(\lambda-\lambda)L}{\varepsilon_o} = \frac{0L}{\varepsilon_o}$$

$$\oint EdA = E\oint dA = E2\pi rL = 0$$

$$\boxed{E = 0}$$

iii.) ***b* < *r***

The enclosed charge per unit length is the sum of the positive charge on the line-charge +λ and the negative induced charge on the inner surface of the cylinder -λ and the positive charge +2λ on the outer surface of the cylinder.



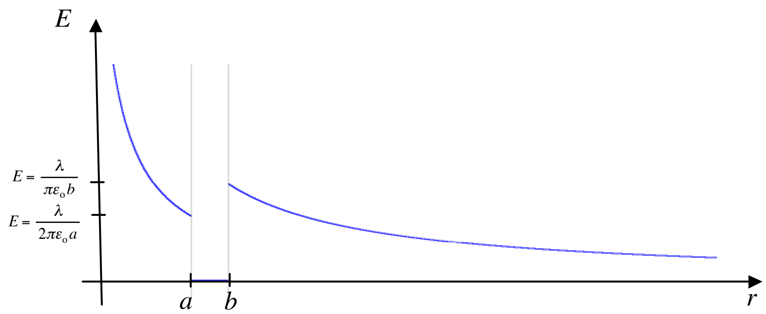
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{(\lambda-\lambda+2\lambda)L}{\varepsilon_o} = \frac{2\lambda L}{\varepsilon_o}$$

$$2\int_{\text{ends}} \vec{E}\cdot d\vec{A} + \int_{\text{cylinder}} \vec{E}\cdot d\vec{A} = \frac{2\lambda L}{\varepsilon_o}$$

$$0 + E\int dA = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{2\lambda L}{\varepsilon_o} \quad \text{so} \quad \boxed{E = \frac{\lambda}{\pi\varepsilon_o r}}$$

4.)



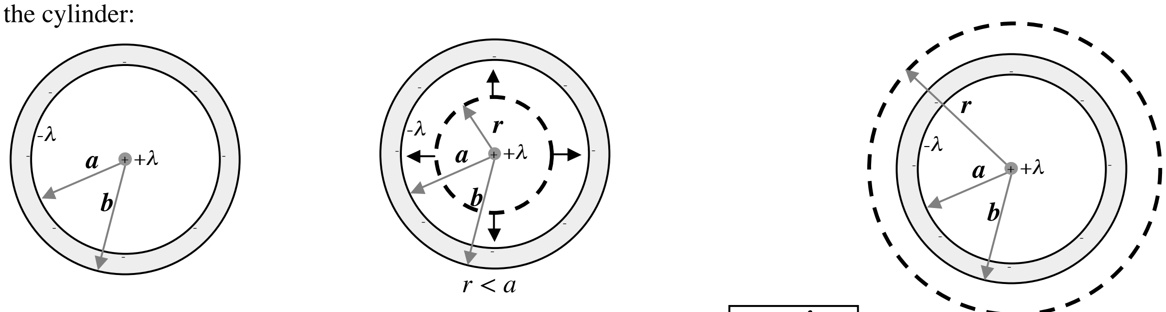
b.)

i.) The charge per unit length is -λ on the inner surface of the tube.

ii.) The charge per unit length is +2λ on the outer surface of the tube.

5.)

The positive line charge will induce a negative charge per length -λ on the inner surface of the conducting tube. In order to have a total charge per unit length of -λ, no charge is present on its outer surface. Looking head-on along the axis of the cylinder:

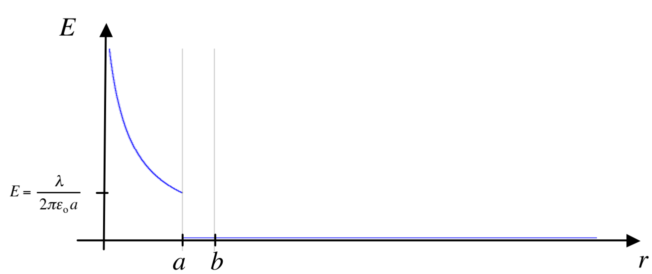


a.)

i.) **($r < a$)** Same as problem 4 only the line +λ charge is enclosed and $\boxed{E = \frac{\lambda}{2\pi\varepsilon_o r}}$.

ii.) **($a < r < b$)** Same as problem 4 and the cylinder encloses the line charge +λ and the induced charge -λ on the inner surface of the cylinder and there is no net charge enclosed and $\boxed{E = 0}$.

iii.) **($b < r$)** Since there are no charges on the outer surface of the cylinder the net charge remains zero and $\boxed{E = 0}$.

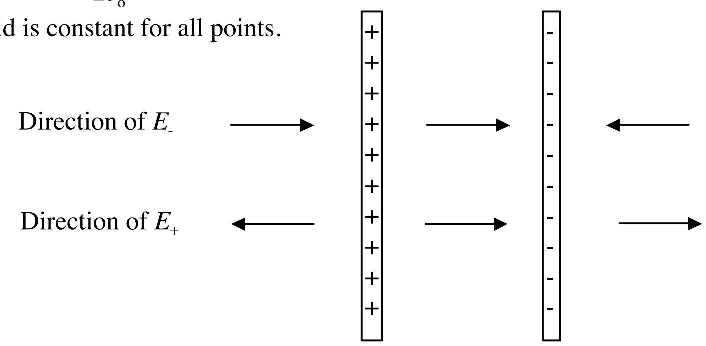


b.)

i.) The charge per unit length is -λ on the inner surface of the tube.

ii.) There is no charge on the outer surface of the tube.

6.) For an infinite sheet charge $E = \dfrac{\sigma}{2\varepsilon_o}$ and is directed away from positively charged sheets and towards negatively charged sheets. The field is constant for all points.

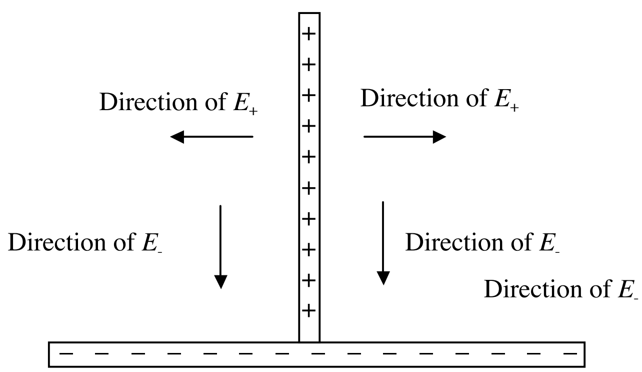


So between the sheets both fields point in the same direction (in this case the +x-direction).

$$E = E_+ + E_- = \frac{\sigma_+}{2\varepsilon_o} + \frac{\sigma_-}{2\varepsilon_o} = \frac{1}{2\varepsilon_o}(\sigma_+ + \sigma_-) = \frac{1}{2\varepsilon_o}\left(0.20\text{x}10^{-9}\ \frac{\text{C}}{\text{m}^2} + 0.60\text{x}10^{-9}\ \frac{\text{C}}{\text{m}^2}\right) = 45\ \frac{\text{N}}{\text{C}}$$

$$\boxed{\vec{E} = \left(45\ \frac{\text{N}}{\text{C}}\right)\hat{i}}$$

7.)



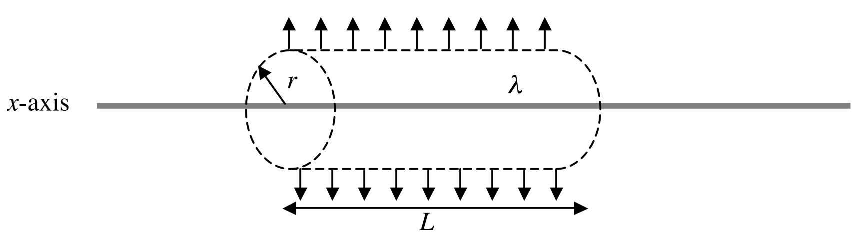
Assuming we are on the right side of the positive sheet:

$$\vec{E} = E_+\hat{i} - E_-\hat{j} = \frac{\sigma_+}{2\varepsilon_o}\hat{i} - \frac{\sigma_-}{2\varepsilon_o}\hat{j}$$

$$\vec{E} = \frac{60\text{ x }10^{-12}}{2\varepsilon_o}\hat{i} - \frac{80\text{ x }10^{-12}}{2\varepsilon_o}\hat{j} = \left(3.39\hat{i} - 4.52\hat{j}\right)\frac{\text{N}}{\text{C}}$$

$$\boxed{\vec{E} = 5.65\ \frac{\text{N}}{\text{C}}\angle -53.1^\circ}$$

8.)



The electric field has radial symmetry and the appropriate Gaussian surface is a cylinder of radius r and length L. The electric field on cylindrical surface is constant and perpendicular to the surface. No electric flux passes through the ends of the cylinder.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\lambda L}{\varepsilon_o}$$

$$2\underset{\text{ends}}{\int} \vec{E}\cdot d\vec{A} + \underset{\text{cylinder}}{\int} \vec{E}\cdot d\vec{A} = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

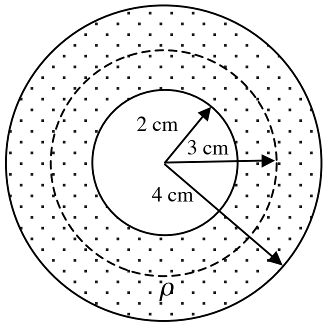
$$\lambda = \frac{2.0\text{ nC}}{0.20\text{ m}} = 10\ \frac{\text{nC}}{\text{m}}$$ and $r = 2.0$ m on the y-axis

$$\vec{E} = \frac{\left(10\text{x}10^{-9}\ \frac{\text{C}}{\text{m}}\right)}{2\pi\left(8.854\text{x}10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)2.0\text{ m}}\hat{j} = \boxed{90\ \frac{\text{N}}{\text{C}}\hat{j}}$$

9.) Looking at a cross-sectional view of the charged cylinder:

$Q = 6\ \mu\text{C}$

$L = 4\ \text{m}$



The charge density is: $\rho = \dfrac{Q}{V} = \dfrac{Q}{\pi\left(r_o^{\,2} - r_i^{\,2}\right)L}$

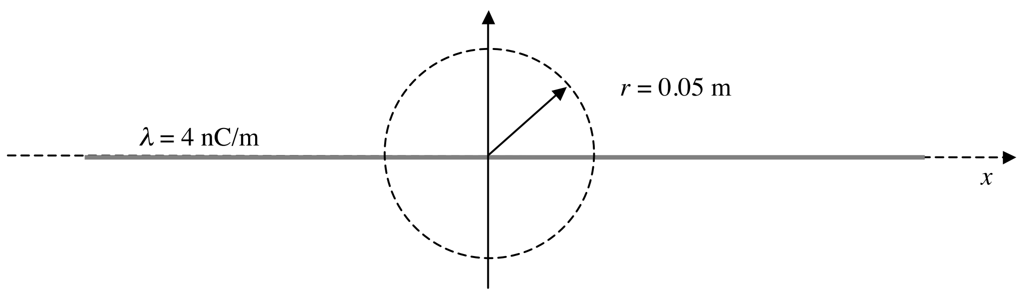
Since the point is located on the y-axis the field points in the positive y-direction.

$$\rho = \frac{\left(6\text{x}10^{-6}\ \text{C}\right)}{\pi\left((0.04\ \text{m})^2 - (0.02\ \text{m})^2\right)4\ \text{m}} = 4.0\text{x}10^{-4}\ \frac{\text{C}}{\text{m}^3}$$

Using Gauss's Law the flux through the 3 cm cylinder (length 2 m) is related to the enclosed charge .

$$\Phi_E = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho V}{\varepsilon_o} = \frac{\left(4.0\text{x}10^{-4}\ \frac{\text{C}}{\text{m}^3}\right)\pi(2\ \text{m})\left((0.03\ \text{m})^2 - (0.02\ \text{m})^2\right)}{\left(8.854\text{x}10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = \boxed{1.4\ \text{x}\ 10^5\ \frac{\text{N}}{\text{C}}\cdot\text{m}^2}$$

10.)



The sphere encloses 0.1 m of the line charge the electric flux using Gauss's Law is:

$$\Phi_E = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\lambda L}{\varepsilon_o} = \frac{\left(4.0\text{x}10^{-9}\ \frac{\text{C}}{\text{m}}\right)(0.1\ \text{m})}{\left(8.854\text{x}10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = \boxed{45.2\ \frac{\text{N}}{\text{C}}\cdot\text{m}^2}$$

11.) Nonconducting sphere radius $R = 0.12$ m and uniform charge density $\rho = 5\ \text{nC/m}^3$.

a.) $r = 0.05\ \text{m} < R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho V}{\varepsilon_o} = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_o}$$

$$E\int dA = \frac{\rho 4\pi r^3}{3\varepsilon_o}$$

$$E4\pi r^2 = \frac{\rho 4\pi r^3}{3\varepsilon_o}$$

$$E = \frac{\rho r}{3\varepsilon_o}$$

$$E = \frac{\left(5\text{x}10^{-9}\ \frac{\text{C}}{\text{m}^3}\right)(0.05\ \text{m})}{3\left(8.854\text{x}10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = 9.4\ \frac{\text{N}}{\text{C}}$$

a.) $r = 0.24\ \text{m} > R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho V}{\varepsilon_o} = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_o}$$

$$E\int dA = \frac{\rho 4\pi R^3}{3\varepsilon_o}$$

$$E4\pi r^2 = \frac{\rho 4\pi R^3}{3\varepsilon_o}$$

$$E = \frac{\rho R^3}{3\varepsilon_o r^2}$$

$$E = \frac{\left(5\text{x}10^{-9}\ \frac{\text{C}}{\text{m}^3}\right)(0.12\ \text{m})^3}{3\left(8.854\text{x}10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.24\ \text{m})^2} = 5.6\ \frac{\text{N}}{\text{C}}$$