1.) $C = 500$ pF
$Q = 0.326\ \mu$C
$d = 0.453$ mm

a.) $C = \dfrac{Q}{\Delta V}$ so $\Delta V = \dfrac{Q}{C} = \dfrac{0.346 \text{ x } 10^{-6}\text{C}}{500 \text{ x } 10^{-12}\text{F}} = \boxed{692 \text{ V}}$

b.) $C = \dfrac{\varepsilon_o A}{d}$ so $A = \dfrac{Cd}{\varepsilon_o} = \dfrac{(500 \text{ x } 10^{-12}\text{F})(0.453 \text{ x } 10^{-3}\text{m})}{\left(8.85 \text{ x } 10^{-12}\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}$

$A = \boxed{0.026 \text{ m}^2}$

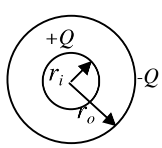
Check units:

$$A\,[=]\,\frac{(\text{F})(\text{m})}{\left(\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = [=]\,\frac{\left(\dfrac{\text{C}}{\text{V}}\right)(\text{m})}{\left(\dfrac{\text{C}^2}{\text{J}\cdot\text{m}}\right)}\,[=]\,\frac{\left(\dfrac{\text{C}}{\text{V}}\right)(\text{m})}{\left(\dfrac{\text{C}}{\text{V}\cdot\text{m}}\right)}\,[=]\ \text{m}^2$$

c.) $E = \dfrac{\Delta V}{d} = \dfrac{692 \text{ V}}{0.453 \text{ x } 10^{-3}\text{ m}} = \boxed{1.53 \text{ x } 10^6\ \dfrac{\text{V}}{\text{m}}}$

d.) $\sigma = \dfrac{Q}{A} = \dfrac{0.346 \text{ x } 10^{-6}\text{C}}{0.026 \text{ m}^2} = \boxed{13.3 \text{ x } 10^{-6}\ \dfrac{\text{C}}{\text{m}^2}}$

2.) a.)



$C = 150$ pF
$r_i = 0.20$ m

Between the spheres: $E = \dfrac{Q}{4\pi\varepsilon_o r^2}$ (using Gauss's Law)

$$V_o - V_i = -\int_{r_i}^{r_o} \vec{E}\cdot d\vec{\ell} = -\int_{r_i}^{r_o} \frac{Q}{4\pi\varepsilon_o r^2}dr = \frac{Q}{4\pi\varepsilon_o r}\Bigg]_{r_i}^{r_o}$$

$$V_o - V_i = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r_o} - \frac{1}{r_i}\right)$$

$$\frac{1}{C} = \frac{V_i - V_o}{Q} = \frac{1}{4\pi\varepsilon_o}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)$$

$$\frac{1}{r_o} = \frac{1}{r_i} - \frac{4\pi\varepsilon_o}{C} \quad \text{so} \quad \frac{1}{r_o} = \frac{C - 4\pi\varepsilon_o r_i}{C r_i}$$

$$r_o = \frac{C r_i}{C - 4\pi\varepsilon_o r_i} = \frac{(150 \text{ x } 10^{-12}\text{F})(0.20 \text{ m})}{(150 \text{ x } 10^{-12}\text{F}) - 4\pi\left(8.85 \text{ x } 10^{-12}\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.20 \text{ m})} = 0.235 \text{ m}$$

$r_o - r_i = 0.235 \text{ m} - 0.20 \text{ m} = \boxed{0.035 \text{ m}}$

2.)

b.)

$$Q = C\Delta V = \left(150 \text{ x } 10^{-12}\text{F}\right)\left(220 \text{ V}\right) = \boxed{3.3 \text{ x } 10^{-8}\text{C}}$$

c.)

inside: $A_i = 4\pi r_i^2$ and $\sigma_i = \frac{Q}{A_i} = \frac{\left(3.3 \text{ x } 10^{-8}\text{C}\right)}{4\pi\left(0.20 \text{ m}\right)^2} = \boxed{6.57 \text{ x } 10^{-8}\ \frac{\text{C}}{\text{m}^2}}$

outside: $A_o = 4\pi r_o^2$ and $\sigma_o = \frac{Q}{A_o} = \frac{\left(\text{-}3.3 \text{ x } 10^{-8}\text{C}\right)}{4\pi\left(0.235 \text{ m}\right)^2} = \boxed{-4.76 \text{ x } 10^{-8}\ \frac{\text{C}}{\text{m}^2}}$

3.)

$\Delta V = 140$ V, $r_i = 12$ cm, $r_o = 15$ cm

a.)

From problem 2: $\frac{1}{C} = \frac{V_i - V_o}{Q} = \frac{1}{4\pi\varepsilon_o}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)$ so $\frac{1}{C} = \frac{1}{4\pi\varepsilon_o}\left(\frac{r_o - r_i}{r_i r_o}\right)$

$$C = 4\pi\varepsilon_o\left(\frac{r_i r_o}{r_o - r_i}\right) = 4\pi\left(8.85 \text{ x } 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{\left(0.15 \text{ m}\right)\left(0.12 \text{ m}\right)}{\left(0.15 \text{ m}\right) - \left(0.12 \text{ m}\right)} = \boxed{66.7 \text{ pF}}$$

b.)

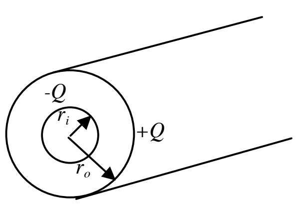
$Q = C\Delta V = \left(66.7 \text{ x } 10^{-12}\text{F}\right)\left(140 \text{ V}\right) = 9.34 \text{ x } 10^{-9}\text{C}$ and $E = \frac{Q}{4\pi\varepsilon_o r^2}$ (from Gauss's Law)

$$E = \frac{\left(9.34 \text{ x } 10^{-9}\text{C}\right)}{4\pi\left(8.85 \text{ x } 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(0.121 \text{ m}\right)^2} = \boxed{5734\ \frac{\text{N}}{\text{C}}}$$

c.)

$$E = \frac{\left(9.34 \text{ x } 10^{-9}\text{C}\right)}{4\pi\left(8.85 \text{ x } 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(0.149 \text{ m}\right)^2} = \boxed{3781\ \frac{\text{N}}{\text{C}}}$$

4.)



$r_i = 2.5$ mm
$r_o = 4.0$ mm
$L = 3.5$ m
$V_{oi} = 0.350$ V

a.) $E = \frac{\lambda}{2\pi\varepsilon_o r}$ (using Gauss's Law)

$$V_o - V_i = -\int_{r_i}^{r_o} \vec{E}\cdot d\vec{\ell} = -\int_{r_i}^{r_o} \frac{\lambda}{2\pi\varepsilon_o r}dr = -\frac{\lambda}{2\pi\varepsilon_o}\ln(r)\Big]_{r_i}^{r_o}$$

$$V_o - V_i = \frac{\lambda_i}{2\pi\varepsilon_o}\left(\ln(r_i) - \ln(r_o)\right) = \frac{\lambda_i}{2\pi\varepsilon_o}\ln\left(\frac{r_i}{r_o}\right)$$

$$\lambda_i = \frac{2\pi\varepsilon_o\left(V_o - V_i\right)}{\ln\left(\frac{r_i}{r_o}\right)} = \frac{2\pi\left(8.85 \text{ x } 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(0.35 \text{ V - } 0\right)}{\ln\left(\frac{2.5 \text{ mm}}{4.0 \text{ mm}}\right)} = \text{-}4.14 \text{ x } 10^{-11}\ \frac{\text{C}}{\text{m}}$$

$$Q_i = \lambda_i L = \left(\text{-}4.14 \text{ x } 10^{-11}\ \frac{\text{C}}{\text{m}}\right)\left(3.5 \text{ m}\right) = \boxed{-1.45 \text{ x } 10^{-10}\text{C}}$$

$$\boxed{\begin{aligned} Q_i &= -1.45 \times 10^{-10}\,\text{C} \\ Q_o &= 1.45 \times 10^{-10}\,\text{C} \end{aligned}}$$

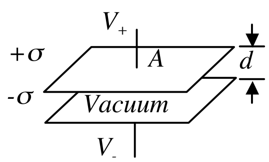
b.)

$$V_o - V_i = \frac{\lambda}{2\pi\varepsilon_o}\left(\ln(r_o) - \ln(r_i)\right) = \frac{\lambda}{2\pi\varepsilon_o}\ln\left(\frac{r_o}{r_i}\right)$$

$$V_o - V_i = \frac{Q_i}{2\pi\varepsilon_o L}\ln\left(\frac{r_o}{r_i}\right) \quad \text{so} \quad \frac{C}{L} = \frac{Q_i}{(V_o - V_i)L} = \frac{2\pi\varepsilon_o}{\ln\left(\frac{r_o}{r_i}\right)}$$

$$\frac{C}{L} = \frac{2\pi\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{\ln\left(\frac{4.0\text{ mm}}{2.5\text{ mm}}\right)} = \boxed{1.18 \times 10^{-10}\,\frac{\text{F}}{\text{m}}}$$

5.)



$d = 0.58$ cm

$A = (0.18\text{ m})^2 = 0.0324\text{ m}^2$

$V_+ - V_- = 50$ V

a.)

$$C = \frac{\varepsilon_o A}{d} = \frac{\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.0324\text{ m}^2)}{(0.58 \times 10^{-2}\text{m})} = \boxed{49.4\text{ pF}}$$

Check units:

$$C\,[=]\,\frac{\left(\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.0324\text{ m}^2)}{(0.58 \times 10^{-2}\text{m})}\,[=]\,\frac{\left(\frac{\text{C}^2}{\text{N}}\right)}{(\text{m})}\,[=]\,\frac{\left(\frac{\text{m}\cdot\text{C}}{\text{V}}\right)}{(\text{m})}\,[=]\,\frac{\text{C}}{\text{V}}\,[=]\,\text{F}$$

b.)

$$C = \frac{Q}{\Delta V} \quad \text{so} \quad Q = C\Delta V = (49.4 \times 10^{-12}\text{F})(50\text{ V}) = \boxed{2.47 \times 10^{-9}\text{C}}$$

c.)

for uniform E-fields $\quad V_{+-} = E\cdot d \quad$ so $\quad E = \frac{\Delta V}{d} = \frac{(50\text{ V})}{(0.58 \times 10^{-2}\text{m})} = \boxed{8.62 \times 10^3\,\frac{\text{V}}{\text{m}}}$

d.)

$$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}(2.47 \times 10^{-9}\text{C})(50\text{ V}) = \boxed{6.18 \times 10^{-8}\text{J}}$$

Check units:

$$U_C\,[=]\,\text{C}\cdot\Delta\text{V}\,[=]\,\text{C}\cdot\frac{\text{J}}{\text{C}}\,[=]\,\text{J}$$

5.)

e.) When battery is disconnected the charge cannot change so when d = 1.16 cm:

$$\boxed{Q = 2.47 \text{ x } 10^{-9}\text{C}}$$

$$C = \frac{\varepsilon_o A}{d} = \frac{\left(8.85 \text{ x } 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(0.0324 \text{ m}^2\right)}{\left(1.16 \text{ x } 10^{-2}\text{m}\right)} = \boxed{24.7 \text{ pF}}$$

$$C = \frac{Q}{\Delta V} \quad \text{so} \quad \Delta V = \frac{Q}{C} = \frac{\left(2.47 \text{ x } 10^{-9}\text{C}\right)}{\left(24.7 \text{ x } 10^{-12}\text{F}\right)} = \boxed{100 \text{ V}}$$

$$E = \frac{\Delta V}{d} = \frac{\left(100 \text{ V}\right)}{\left(1.16 \text{ x } 10^{-2}\text{m}\right)} = \boxed{8.62 \text{ x } 10^{3} \frac{\text{V}}{\text{m}}}$$

$$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}\left(2.47 \text{ x } 10^{-9}\text{C}\right)\left(100 \text{ V}\right) = \boxed{1.24 \text{ x } 10^{-7}\text{J}}$$

6.) When battery remains connected voltage is fixed and the charge can change so when d = 1.16 cm:

a.) as in (5e):

$$C = \boxed{24.7 \text{ pF}}$$

b.)

$$Q = C\Delta V = \left(24.7 \text{ x } 10^{-12}\text{F}\right)\left(50 \text{ V}\right) = \boxed{1.235 \text{ x } 10^{-9}\text{C}}$$

c .)

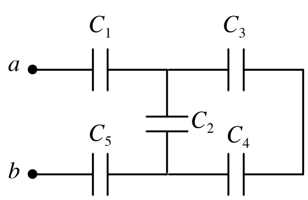
$$E = \frac{\Delta V}{d} = \frac{\left(50 \text{ V}\right)}{\left(1.16 \text{ x } 10^{-2}\text{m}\right)} = \boxed{4.31 \text{ x } 10^{3} \frac{\text{V}}{\text{m}}}$$

d.)

$$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}\left(1.235 \text{ x } 10^{-9}\text{C}\right)\left(50 \text{ V}\right) = \boxed{3.09 \text{ x } 10^{-8}\text{J}}$$

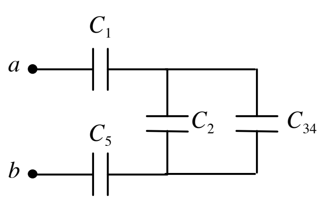
7.)

a.)



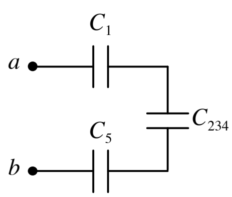
C_3 is in series with C_4 so they can be replaced by their equivalent.

$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{2.3 \ \mu\text{F}} + \frac{1}{2.3 \ \mu\text{F}} \quad \text{and} \quad C_{34} = 1.15 \ \mu\text{F}$$



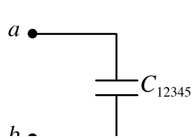
C_{34} is in parallel with C_2 so they can be replaced by their equivalent.

$$C_{234} = C_2 + C_{34} = 2.3 \ \mu\text{F} + 1.15 \ \mu\text{F} \quad \text{and} \quad C_{234} = 3.45 \ \mu\text{F}$$



C_1, C_{234}, and C_5 are all in series so they can be replaced by their equivalent.

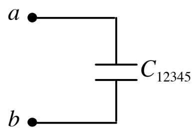
$$\frac{1}{C_{12345}} = \frac{1}{C_1} + \frac{1}{C_{234}} + \frac{1}{C_5} = \frac{1}{4.6 \ \mu\text{F}} + \frac{1}{3.45 \ \mu\text{F}} + \frac{1}{4.6 \ \mu\text{F}} \quad \text{and} \quad C_{12345} = 1.38 \ \mu\text{F}$$



So the equivalent capacitance is $\boxed{C_{eq} = 1.38 \ \mu\text{F}}$

7.)

b.)

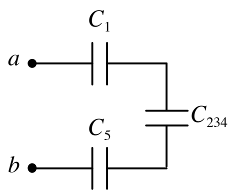


The charge on the equivalent capacitance is the charge on C_1, C_5, and C_{234} since capacitors in series have the same charge and the same charge as their equivalent.

$$Q_{12345} = C_{12345}\Delta V_{ab} = \left(1.38 \text{ x } 10^{-6}\text{F}\right)\left(540 \text{ V}\right) = 7.452 \text{ x } 10^{-4}\text{C}$$

so $$Q_1 = Q_5 = Q_{234} = 7.452 \text{ x } 10^{-4}\text{C}$$

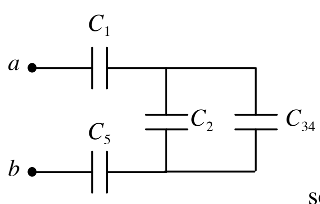
and $$\Delta V_1 = \frac{Q_1}{C_1} = \frac{\left(7.452 \text{ x } 10^{-4}\text{C}\right)}{\left(4.6 \text{ x } 10^{-6}\text{F}\right)} = 162 \text{ V and } \quad \Delta V_5 = \frac{Q_5}{C_5} = \frac{\left(7.452 \text{ x } 10^{-4}\text{C}\right)}{\left(4.6 \text{ x } 10^{-6}\text{F}\right)} = 162 \text{ V}$$



The voltage on the equivalent C_{234} is the same as the voltage on C_2 and the equivalent C_{34} since capacitors in parallel have the same voltage and the same voltage as their equivalent.

so $$\Delta V_2 = \Delta V_{34} = \Delta V_{234} = \frac{Q_{234}}{C_{234}} = \frac{\left(7.452 \text{ x } 10^{-4}\text{C}\right)}{\left(3.45 \text{ x } 10^{-6}\text{F}\right)} = 216 \text{ V}$$

and $$Q_2 = C_2\Delta V_2 = \left(2.3 \text{ x } 10^{-6}\text{F}\right)\left(216 \text{ V}\right) = 4.968 \text{ x } 10^{-4}\text{C}$$



The charge on the equivalent C_{34} is the same as the charge on C_3 and C_4 since capacitors in series have the same charge and the same charge as their equivalent.

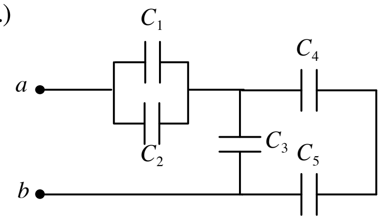
so $$Q_3 = Q_4 = Q_{34} = C_{34}\Delta V_{34} = \left(1.15 \text{ x } 10^{-6}\text{F}\right)\left(216 \text{ V}\right) = 2.484 \text{ x } 10^{-4}\text{C}$$

and $$\Delta V_3 = \frac{Q_3}{C_3} = \frac{\left(2.484 \text{ x } 10^{-4}\text{C}\right)}{\left(2.3 \text{ x } 10^{-6}\text{F}\right)} = 108 \text{ V and } \quad \Delta V_4 = \frac{Q_4}{C_4} = \frac{\left(2.484 \text{ x } 10^{-4}\text{C}\right)}{\left(2.3 \text{ x } 10^{-6}\text{F}\right)} = 108 \text{ V}$$

To summarize:

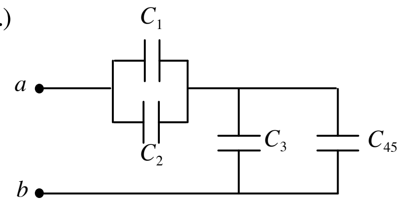
Capacitor	Capacitance	Charge	Voltage
C_1	4.6 x 10^{-6} F	7.452 x 10^{-4} C	162 V
C_2	2.3 x 10^{-6} F	4.968 x 10^{-4} C	216 V
C_3	2.3 x 10^{-6} F	2.484 x 10^{-4} C	108 V
C_4	2.3 x 10^{-6} F	2.484 x 10^{-4} C	108 V
C_5	4.6 x 10^{-6} F	7.452 x 10^{-4} C	162 V

1.)



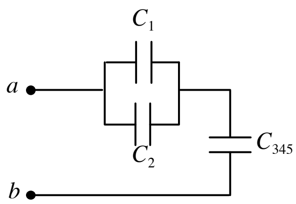
$C_1 = C_2 = 5.0\ \mu\text{F}$ and $C_3 = C_4 = C_5 = 10.0\ \mu\text{F}$, $V_{ab} = 120\ \text{V}$

a.)



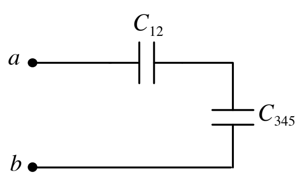
C_4 and C_5 are in series so they can be replaced by their equivalent.

$$\frac{1}{C_{45}} = \frac{1}{C_4} + \frac{1}{C_5} = \frac{1}{10\ \mu\text{F}} + \frac{1}{10\ \mu\text{F}} \quad \text{and} \quad C_{45} = 5\ \mu\text{F}$$



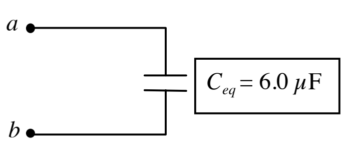
C_{45} and C_3 are in parallel so they can be replaced by their equivalent.

$$C_{345} = C_3 + C_{45} = 10\ \mu\text{F} + 5\ \mu\text{F} = 15\ \mu\text{F}$$



C_1 and C_2 are in parallel so they can be replaced by their equivalent.

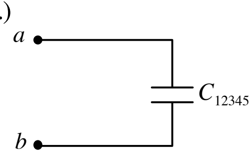
$$C_{12} = C_1 + C_2 = 5\ \mu\text{F} + 5\ \mu\text{F} = 10\ \mu\text{F}$$



C_{12} and C_{345} are in series so they can be replaced by their equivalent.

$$\frac{1}{C_{12345}} = \frac{1}{C_{12}} + \frac{1}{C_{345}} = \frac{1}{10\ \mu\text{F}} + \frac{1}{15\ \mu\text{F}} \quad \text{and} \quad C_{12345} = 6\ \mu\text{F}$$

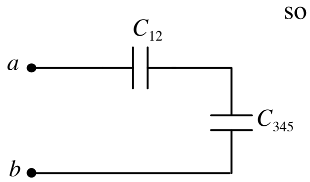
b.)



The charge on the equivalent capacitance is the charge on C_1, C_5, and C_{234} since capacitors in series have the same charge and the same charge as their equivalent.

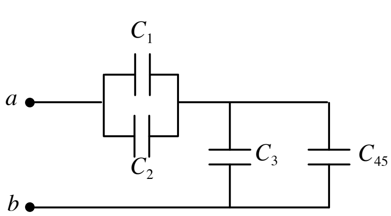
$$Q_{12345} = C_{12345}\Delta V_{ab} = \left(6.0 \text{ x } 10^{-6}\text{F}\right)\left(120\ \text{V}\right) = 7.20 \text{ x } 10^{-4}\text{C}$$

so $Q_{12} = Q_{345} = 7.20 \text{ x } 10^{-4}\text{C}$



The voltage on the equivalent C_1 is the same as the voltage on C_2 and the equivalent C_{12} since capacitors in parallel have the same voltage and the same voltage as their equivalent.

$$\Delta V_1 = \Delta V_2 = \Delta V_{12} = \frac{Q_{12}}{C_{12}} = \frac{7.20 \text{ x } 10^{-4}\text{C}}{10 \text{ x } 10^{-6}\text{F}} = 72\ \text{V}$$



It follows that:

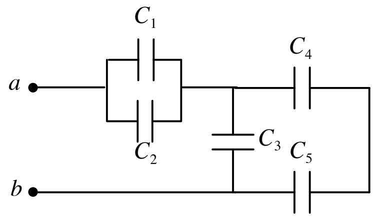
$$Q_1 = C_1\Delta V_1 = \left(5.0 \text{ x } 10^{-6}\text{F}\right)\left(72\ \text{V}\right) = 3.60 \text{ x } 10^{-4}\text{C}$$

and

$$Q_2 = C_2\Delta V_2 = \left(5.0 \text{ x } 10^{-6}\text{F}\right)\left(72\ \text{V}\right) = 3.60 \text{ x } 10^{-4}\text{C}$$

Also: $\Delta V_{345} = \dfrac{Q_{345}}{C_{345}} = \dfrac{\left(7.2 \text{ x } 10^{-4}\text{C}\right)}{\left(15 \text{ x } 10^{-6}\text{F}\right)} =$ 48 V which is the voltage on C_3 and C_{45} since they are in parallel and have the same voltage as their equivalent C_{345}. It follows that:

$$Q_3 = C_3\Delta V_3 = \left(10.0 \text{ x } 10^{-6}\text{F}\right)\left(48 \text{ V}\right) = 4.80 \text{ x } 10^{-4}\text{C} \text{ and } \quad Q_{45} = C_{45}\Delta V_{45} = \left(5.0 \text{ x } 10^{-6}\text{F}\right)\left(48 \text{ V}\right) = 2.40 \text{ x } 10^{-4}\text{C}$$



Finally the charge on C_4 and C_5 are the same as their equivalent C_{45} since they are in series and have the same charge and the same charge as their equivalent. Therefore:

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{\left(2.40 \text{ x } 10^{-4}\text{C}\right)}{\left(10.0 \text{ x } 10^{-6}\text{F}\right)} = 24 \text{ V} \quad \text{and} \quad \Delta V_5 = \frac{Q_5}{C_5} = \frac{\left(2.40 \text{ x } 10^{-4}\text{C}\right)}{\left(10.0 \text{ x } 10^{-6}\text{F}\right)} = 24 \text{ V}$$

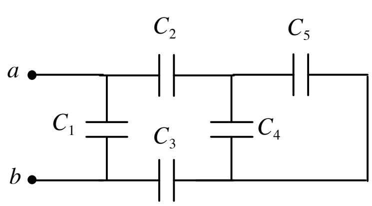
To summarize:

Capacitor	Capacitance	Charge	Voltage
C_1	5.0 x 10^{-6} F	3.60 x 10^{-4} C	72 V
C_2	5.0 x 10^{-6} F	3.60 x 10^{-4} C	72 V
C_3	10 x 10^{-6} F	4.80 x 10^{-4} C	48 V
C_4	10 x 10^{-6} F	2.40 x 10^{-4} C	24 V
C_5	10 x 10^{-6} F	2.40 x 10^{-4} C	24 V

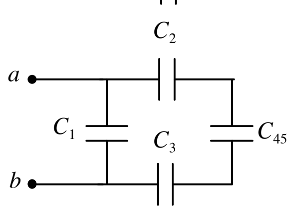
c.) $U_C = \dfrac{1}{2}Q_{eq}V_{ab} = \dfrac{1}{2}C_{eq}V_{ab}^2 = \dfrac{1}{2}\left(6.0 \text{ x } 10^{-6}\text{C}\right)\left(120 \text{ V}\right)^2 = \boxed{4.32 \text{ x } 10^{-2}\text{J}}$

2.)

a.)

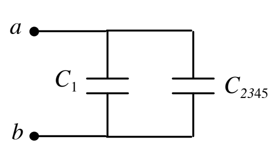


$C_1 = C_2 = C_3 = C_4 = C_5 = 5.0\ \mu\text{F}$. The potential difference between a and b is 100 V.



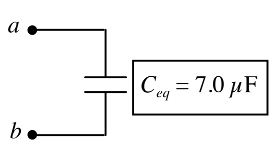
C_4 and C_5 are in parallel and can be replaced by their equivalent C_{45}.

$$C_{45} = C_4 + C_5 = 5.0\ \mu\text{F} + 5.0\ \mu\text{F} = 10\ \mu\text{F}$$



C_2, C_3, and C_{45} are in series and can be replaced by their equivalent C_{2345}.

$$\frac{1}{C_{2345}} = \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_{45}} = \frac{1}{5.0\ \mu\text{F}} + \frac{1}{5.0\ \mu\text{F}} + \frac{1}{10\ \mu\text{F}} \quad \text{and} \quad C_{2345} = 2.0\ \mu\text{F}$$

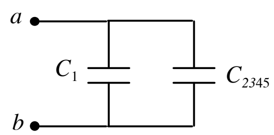


C_1 and C_{2345} are in parallel and can be replaced by their equivalent C_{12345}.

$$C_{eq} = C_{1234} = C_1 + C_{2345} = 5.0\ \mu\text{F} + 2.0\ \mu\text{F} = 7.0\ \mu\text{F}$$

2.)

b.)

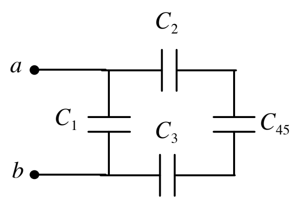


The voltage on the equivalent capacitance is the voltage on C_1 and C_{2345} since capacitors in parallel have the same voltage and the same voltage as their equivalent.

$$\Delta V_{12345} = \Delta V_1 = \Delta V_{2345} = V_{ab} = \text{ 100 V}$$

so $$Q_1 = C_1 V_1 = \left(5.0 \text{ x } 10^{-6}\text{F}\right)\left(100 \text{ V}\right) = \text{ 5.00 x } 10^{-4}\text{C}$$

and $$Q_{2345} = C_{2345} V_{2345} = \left(2.0 \text{ x } 10^{-6}\text{F}\right)\left(100 \text{ V}\right) = \text{ 2.00 x } 10^{-4}\text{C}$$

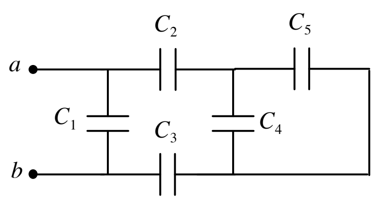


The charge on C_2, C_{45}, and C_3 is the same since they are in series and the same charge as their equivalent C_{2345}.

$$Q_2 = Q_{45} = Q_3 = Q_{2345} = \text{ 2.00 x } 10^{-4}\text{C}$$

so $$\Delta V_2 = \frac{Q_2}{C_2} = \frac{\left(2.00 \text{ x } 10^{-4}\text{C}\right)}{\left(5.0 \text{ x } 10^{-6}\text{F}\right)} = \text{ 40 V} \quad \text{and} \quad \Delta V_3 = \frac{Q_3}{C_3} = \frac{\left(2.00 \text{ x } 10^{-4}\text{C}\right)}{\left(5.0 \text{ x } 10^{-6}\text{F}\right)} = \text{ 40 V}$$

also $$\Delta V_{45} = \frac{Q_{45}}{C_{45}} = \frac{\left(2.00 \text{ x } 10^{-4}\text{C}\right)}{\left(10.0 \text{ x } 10^{-6}\text{F}\right)} = \text{ 20 V}$$



The voltage on C_4 and C_5 are the same and equal to the voltage on their equivalent C_{45} because they are in parallel.

$$\Delta V_4 = \Delta V_5 = \Delta V_{45} = \text{ 20 V}$$

so $$Q_4 = C_4 V_4 = \left(5.0 \text{ x } 10^{-6}\text{F}\right)\left(20 \text{ V}\right) = \text{ 1.00 x } 10^{-4}\text{C}$$

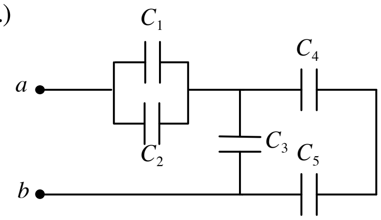
$$Q_5 = C_5 V_5 = \left(5.0 \text{ x } 10^{-6}\text{F}\right)\left(20 \text{ V}\right) = \text{ 1.00 x } 10^{-4}\text{C}$$

To summarize:

Capacitor	Capacitance	Charge	Voltage
C_1	5.0 x 10^{-6} F	5.0 x 10^{-4} C	100 V
C_2	5.0 x 10^{-6} F	2.0 x 10^{-4} C	40 V
C_3	5.0 x 10^{-6} F	2.0 x 10^{-4} C	40 V
C_4	5.0 x 10^{-6} F	1.0 x 10^{-4} C	20 V
C_5	5.0 x 10^{-6} F	1.0 x 10^{-4} C	20 V

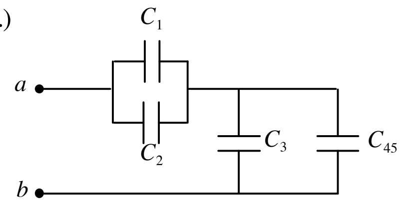
c.) $$U_C = \frac{1}{2} Q_{eq} V_{ab} = \frac{1}{2} C_{eq} V_{ab}^2 = \frac{1}{2}\left(7.0 \text{ x } 10^{-6}\text{C}\right)\left(100 \text{ V}\right)^2 = \boxed{3.50 \text{ x } 10^{-2}\text{J}}$$

3.)



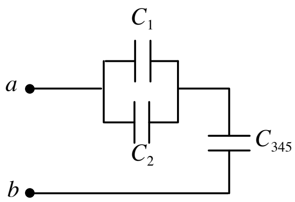
$C_1 = C_5 = 6.0\ \mu\text{F}$, $C_3 = 3.6\ \mu\text{F}$ and $C_2 = C_4 = 4.0\ \mu\text{F}$, $V_{ab} = 120$ V

a.)



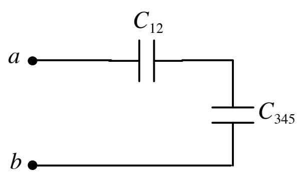
C_4 and C_5 are in series so they can be replaced by their equivalent.

$$\frac{1}{C_{45}} = \frac{1}{C_4} + \frac{1}{C_5} = \frac{1}{4.0\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}} \quad \text{and} \quad C_{45} = 2.4\ \mu\text{F}$$



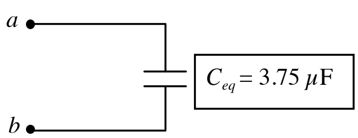
C_{45} and C_3 are in parallel so they can be replaced by their equivalent.

$$C_{345} = C_3 + C_{45} = 3.6\ \mu\text{F} + 2.4\ \mu\text{F} = 6.0\ \mu\text{F}$$



C_1 and C_2 are in parallel so they can be replaced by their equivalent.

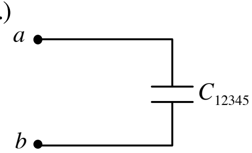
$$C_{12} = C_1 + C_2 = 6.0\ \mu\text{F} + 4.0\ \mu\text{F} = 10\ \mu\text{F}$$



C_{12} and C_{345} are in series so they can be replaced by their equivalent.

$$\frac{1}{C_{12345}} = \frac{1}{C_{12}} + \frac{1}{C_{345}} = \frac{1}{10\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}} \quad \text{and} \quad C_{12345} = 3.75\ \mu\text{F}$$

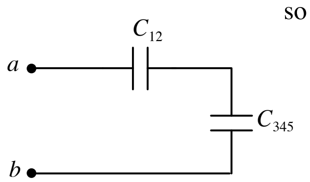
b.)



The charge on the equivalent capacitance is the charge on C_{12}, C_5, and C_{234} since capacitors in series have the same charge and the same charge as their equivalent.

$$Q_{12345} = C_{12345}V_{ab} = \left(3.75 \text{ x } 10^{-6}\text{F}\right)\left(120 \text{ V}\right) = 4.5 \text{ x } 10^{-4}\text{C}$$

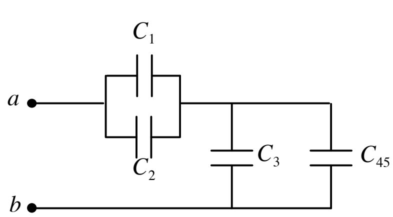
so $\quad Q_{12} = Q_{345} = 4.5 \text{ x } 10^{-4}\text{C}$



The voltage on the equivalent C_1 is the same as the voltage on C_2 and the equivalent C_{12} since capacitors in parallel have the same voltage and the same voltage as their equivalent.

$$\Delta V_1 = \Delta V_2 = \Delta V_{12} = \frac{Q_{12}}{C_{12}} = \frac{4.5 \text{ x } 10^{-4}\text{C}}{10 \text{ x } 10^{-6}\text{F}} = 45 \text{ V}$$

It follows that:

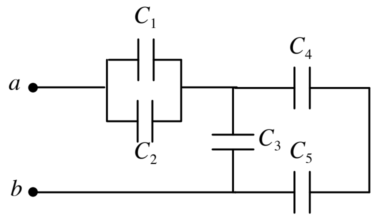


$$Q_1 = C_1\Delta V_1 = \left(6.0 \text{ x } 10^{-6}\text{F}\right)\left(45 \text{ V}\right) = 2.70 \text{ x } 10^{-4}\text{C}$$

and $\quad Q_2 = C_2\Delta V_2\left(4.0 \text{ x } 10^{-6}\text{F}\right)\left(45 \text{ V}\right) = 1.80 \text{ x } 10^{-4}\text{C}$

Also: $\Delta V_{345} = \dfrac{Q_{345}}{C_{345}} = \dfrac{\left(4.5 \text{ x } 10^{-4}\text{C}\right)}{\left(6.0 \text{ x } 10^{-6}\text{F}\right)} = 75 \text{ V}$ which is the voltage on C_3 and C_{45} since they are in parallel and have the same voltage as their equivalent C_{345}. It follows that:

$$Q_3 = C_3\Delta V_3 = \left(3.6 \text{ x } 10^{-6}\text{F}\right)\left(75 \text{ V}\right) = 2.70 \text{ x } 10^{-4}\text{C} \quad \text{and} \quad Q_{45} = C_{45}\Delta V_{45} = \left(2.4 \text{ x } 10^{-6}\text{F}\right)\left(75 \text{ V}\right) = 1.80 \text{ x } 10^{-4}\text{C}$$



Finally the charge on C_4 and C_5 are the same as their equivalent C_{45} since they are in series and have the same charge and the same charge as their equivalent. Therefore:

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{\left(1.80 \text{ x } 10^{-4}\text{C}\right)}{\left(4.0 \text{ x } 10^{-6}\text{F}\right)} = 45 \text{ V} \quad \text{and} \quad \Delta V_5 = \frac{Q_5}{C_5} = \frac{\left(1.80 \text{ x } 10^{-4}\text{C}\right)}{\left(6.0 \text{ x } 10^{-6}\text{F}\right)} = 30 \text{ V}$$

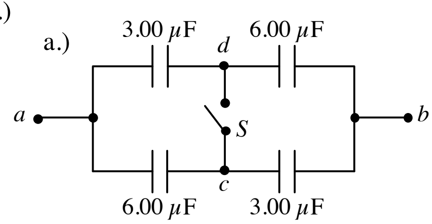
To summarize:

Capacitor	Capacitance	Charge	Voltage
C_1	6.0 x 10^{-6} F	2.7 x 10^{-4} C	45 V
C_2	4.0 x 10^{-6} F	1.8 x 10^{-4} C	45 V
C_3	3.6 x 10^{-6} F	2.7 x 10^{-4} C	75 V
C_4	4.0 x 10^{-6} F	1.8 x 10^{-4} C	45 V
C_5	6.0 x 10^{-6} F	1.8 x 10^{-4} C	30 V

c.) $U_C = \dfrac{1}{2}Q_{eq}V_{ab} = \dfrac{1}{2}C_{eq}V_{ab}^2 = \dfrac{1}{2}\left(3.75 \text{ x } 10^{-6}\text{C}\right)\left(120 \text{ V}\right)^2 = \boxed{2.70 \text{ x } 10^{-2}\text{J}}$

4.)

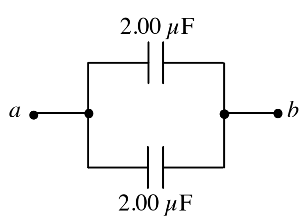
a.)



$V_{ab} = 360$ V

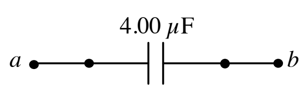
With switch S open the two sets of 3.00 μF and 6.00 μF capacitors are in series and can be replaced by their equivalent.

$$\frac{1}{C_{36}} = \frac{1}{3.00 \ \mu\text{F}} + \frac{1}{6.00 \ \mu\text{F}} \quad \text{and} \quad C_{36} = 2.00 \ \mu\text{F}$$



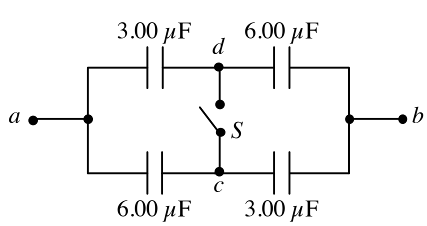
These are in parallel and the equivalent capacitance is:

$$C_{eq} = 2.00 \ \mu\text{F} + 2.00 \ \mu\text{F} = 4.00 \ \mu\text{F}$$



The voltage on the 4.00 μF equivalent is the same as the voltage on the two 2.00 μF equivalents and is equal to $V_{ab} = 360$ V. The charges are:

$$Q = C\Delta V = \left(2.00 \text{ x } 10^{-6}\text{F}\right)\left(360 \text{ V}\right) = 7.20 \text{ x } 10^{-4}\text{C}$$



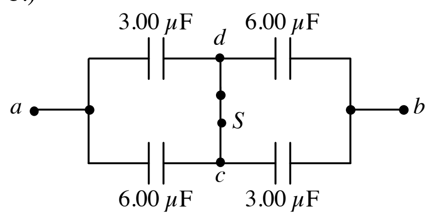
This is the charge on the 3.00 μF and 6.00 μF capacitors since they are in series. There voltages are:

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{\left(7.20 \text{ x } 10^{-4}\text{C}\right)}{\left(3.00 \text{ x } 10^{-6}\text{F}\right)} = 240 \text{ V} \quad \text{and} \quad \Delta V_6 = \frac{Q_6}{C_6} = \frac{\left(7.20 \text{ x } 10^{-4}\text{C}\right)}{\left(6.00 \text{ x } 10^{-6}\text{F}\right)} = 120 \text{ V}$$

If $V_b = 0$ then $V_a = 360$ V and $V_d = V_a - V_3 = 360 \text{ V} - 240 \text{ V} = 120 \text{ V}$ and $V_c = V_a - V_6 = 360 \text{ V} - 120 \text{ V} = 240 \text{ V}$.

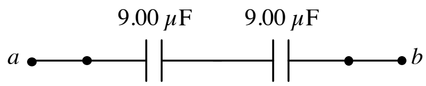
Therefore: $V_{cd} = V_c - V_d = 240 \text{ V} - 120 \text{ V} = 120 \text{ V}$

b.)



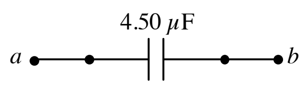
With switch S closed the two sets of 3.00 μF and 6.00 μF capacitors are in parallel and can be replaced by their equivalent.

$$C_{36} = 3.00 \ \mu\text{F} + 6.00 \ \mu\text{F} = 9.00 \ \mu\text{F}$$



The two 9.00 μF equivalents are in series so the equivalent capacitance is:

$$\frac{1}{C_{eq}} = \frac{1}{9.00 \ \mu\text{F}} + \frac{1}{9.00 \ \mu\text{F}} \quad \text{and} \quad C_{eq} = 4.50 \ \mu\text{F}$$



The charge on the two 9.00 μF equivalents is the same as the charge on the 4.50 μF equivalent capacitance.

$$Q = C\Delta V = \left(4.50 \text{ x } 10^{-6}\text{F}\right)\left(360 \text{ V}\right) = 1.62 \text{ x } 10^{-4}\text{C}$$

The voltages on the 9.00 μF equivalents are the same: $\Delta V_9 = \dfrac{Q_9}{C_9} = \dfrac{\left(1.62 \text{ x } 10^{-4}\text{C}\right)}{\left(9.00 \text{ x } 10^{-6}\text{F}\right)} = 180 \text{ V}$

This is the voltage on the 3.00 μF and 6.00 μF capacitors because they are in parallel and all capacitors have voltages $\boxed{\Delta V = 180 \text{ V.}}$

c.) Based upon equivalent capacitances:

$$Q_a = C_{eq}V_{ab} = \left(4.00 \text{ x } 10^{-6}\text{F}\right)\left(360 \text{ V}\right) = 1.44 \text{ x } 10^{-3}\text{C}$$

$$Q_b = C_{eq}V_{ab} = \left(4.50 \text{ x } 10^{-6}\text{F}\right)\left(360 \text{ V}\right) = 1.62 \text{ x } 10^{-3}\text{C}$$

and the charge that flows through switch S is: $Q_b - Q_a = 1.62 \text{ x } 10^{-3}\text{C} - 1.44 \text{ x } 10^{-3}\text{C} = \boxed{1.80 \text{ x } 10^{-4}\text{C}}$

5.) Cylindrical air capacitor $L = 25.0$ m and $U_c = 5.40 \text{ x } 10^{-9}$ J when $V = 3.00$ V

a.) $U_C = \dfrac{1}{2}Q\Delta V$ so $Q = \dfrac{2U_C}{\Delta V} = \dfrac{2\left(5.40 \text{ x } 10^{-9}\text{J}\right)}{\left(3.00 \text{ V}\right)} = \boxed{3.6 \text{ x } 10^{-9}\text{C}}$

b.) for cylindrical capacitors

$$\Delta V = \frac{Q}{2\pi\varepsilon_o L}\ln\left(\frac{r_o}{r_i}\right) \text{ so } \ln\left(\frac{r_o}{r_i}\right) = \frac{2\pi\varepsilon_o L\Delta V}{Q} \quad \text{and} \quad \frac{r_o}{r_i} = \exp\left(\frac{2\pi\varepsilon_o L\Delta V}{Q}\right)$$

$$\frac{r_o}{r_i} = \exp\left(\frac{2\pi\left(8.85 \text{ x } 10^{-12}\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(25 \text{ m}\right)\left(3.00 \text{ V}\right)}{\left(3.6 \text{ x } 10^{-9}\text{C}\right)}\right) = 3.19 \quad \text{and} \quad \boxed{\frac{r_i}{r_o} = 0.314}$$

6.)

parallel-plate capacitor with dielectric constant $\kappa = 3.40$ and dielectric strength $E = 2.00 \text{ x } 10^7$ V/m

capacitance $C = 1.37$ nF and voltage must be at least $\Delta V = 6000$ V.

$$\Delta V = E \cdot d \quad \text{so} \quad d = \frac{\Delta V}{E} = \frac{(6000 \text{ V})}{\left(2.00 \text{ x } 10^7 \frac{\text{V}}{\text{m}}\right)} = 3.00 \text{ x } 10^{-4} \text{ m}$$

$$C = \frac{\kappa \varepsilon_o A}{d} \quad \text{so} \quad A = \frac{Cd}{\kappa \varepsilon_o} = \frac{\left(1.37 \text{ x } 10^{-9} \text{F}\right)\left(3.00 \text{ x } 10^{-4} \text{ m}\right)}{3.4\left(8.85 \text{ x } 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)} = \boxed{1.37 \text{ x } 10^{-2} \text{m}^2}$$

7.) parallel-plate capacitor with $d = 1.60$ mm and dielectric constant $\kappa = 4.50$ and dielectric strength $E = 1.40 \text{ x } 10^6$ V/m

a.)

$$E = \frac{\sigma}{\kappa \varepsilon_o} \quad \text{so} \quad \sigma = \kappa \varepsilon_o E = 4.50\left(8.85 \text{ x } 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)\left(1.40 \text{ x } 10^6 \frac{\text{V}}{\text{m}}\right) = \boxed{5.58 \text{ x } 10^{-5} \frac{\text{C}}{\text{m}^2}}$$

b.)

$$\sigma_i = \sigma\left(1 - \frac{1}{\kappa}\right) = 5.58 \text{ x } 10^{-5} \frac{\text{C}}{\text{m}^2}\left(1 - \frac{1}{4.50}\right) = \boxed{4.34 \text{ x } 10^{-5} \frac{\text{C}}{\text{m}^2}}$$

8.) The two slabs are in series (they have the same charge density) with same area A and thickness $\frac{d}{2}$

$$C_1 = \frac{\kappa_1 \varepsilon_o A}{\frac{d}{2}} = \frac{2\kappa_1 \varepsilon_o A}{d} \quad \text{and} \quad C_2 = \frac{2\kappa_2 \varepsilon_o A}{d}$$

Since they are in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{2\kappa_1 \varepsilon_o A}{d}} + \frac{1}{\frac{2\kappa_2 \varepsilon_o A}{d}} = \frac{d}{2\varepsilon_o A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right) = \frac{d}{2\varepsilon_o A}\left(\frac{\kappa_2 + \kappa_1}{\kappa_1 \kappa_2}\right)$$

$$\boxed{C_{eq} = \frac{2\varepsilon_o A}{d}\left(\frac{\kappa_1 \kappa_2}{\kappa_2 + \kappa_1}\right)}$$