1.) $I = 4.8$ A for 2 hours

$$I = \frac{\Delta Q}{\Delta t} \quad \text{so} \quad \Delta Q = I \cdot \Delta t = (4.8\ \text{A})(7200\ \text{s}) = \left(4.8\ \frac{\text{C}}{\text{s}}\right)(7200\ \text{s}) = \boxed{34{,}560\ \text{C}}$$

2.) $\Delta Q = 72$ C, $\Delta t = 1$ hr, $n = 5.8 \times 10^{28}$ e's/m^3, $D = 1.3$ mm

a.) $$I = \frac{\Delta Q}{\Delta t} = \frac{72\ \text{C}}{3600\ \text{s}} = \boxed{0.02\ \text{A}}$$

b.) $$I = nqv_d A \quad \text{so} \quad v_d = \frac{I}{nqA} = \frac{(0.02\ \text{A})}{(5.8 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\text{C})\left(\pi(0.65 \times 10^{-3}\text{m})^2\right)} = \boxed{1.62 \times 10^{-6}\ \frac{\text{m}}{\text{s}}}$$

3.) $\ell = 35.0$ m and $D = 2.06$ mm

for copper $\rho = 1.72 \times 10^{-8}\ \Omega\cdot\text{m}$ $$R_{Cu} = \frac{\rho\ell}{A} = \frac{(1.72 \times 10^{-8}\Omega\cdot\text{m})(35\ \text{m})}{\pi(1.025 \times 10^{-3}\text{m})^2} = \boxed{0.18\ \Omega}$$

for gold $\rho = 2.44 \times 10^{-8}\ \Omega\cdot\text{m}$ $$R_{Au} = \frac{\rho\ell}{A} = \frac{(2.44 \times 10^{-8}\Omega\cdot\text{m})(35\ \text{m})}{\pi(1.025 \times 10^{-3}\text{m})^2} = \boxed{0.26\ \Omega}$$

for silver $\rho = 1.47 \times 10^{-8}\ \Omega\cdot\text{m}$ $$R_{Cu} = \frac{\rho\ell}{A} = \frac{(1.47 \times 10^{-8}\Omega\cdot\text{m})(35\ \text{m})}{\pi(1.025 \times 10^{-3}\text{m})^2} = \boxed{0.16\ \Omega}$$

4.) $D_{Cu} = 2.2$ mm

$$\frac{R_{Cu}}{\ell} = \frac{\rho}{A} = \frac{(1.72 \times 10^{-8}\Omega\cdot\text{m})(35\ \text{m})}{\pi(1.1 \times 10^{-3}\text{m})^2} = 4.52 \times 10^{-3}\ \frac{\Omega}{\text{m}}$$

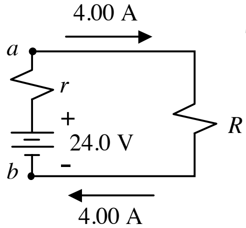
$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{\rho}{\pi \cdot r^2} \quad \text{so} \quad r = \sqrt{\frac{\rho\ell}{\pi \cdot R}} = \sqrt{\frac{(2.75 \times 10^{-8}\Omega\cdot\text{m})}{\pi\left(4.52 \times 10^{-3}\ \frac{\Omega}{\text{m}}\right)}} = 1.39 \times 10^{-3}\text{m}$$

$$D = 2r = 2(1.39 \times 10^{-3}\text{m}) = \boxed{2.78 \times 10^{-3}\text{m}}$$

5.) $R = 1.00\ \Omega$ for copper wire with diameter $D = 0.750$ mm and length ℓ

$$R = \frac{\rho\ell}{A} \quad \text{so} \quad \ell = \frac{RA}{\rho} = \frac{(1.00\ \Omega)\pi(0.375 \times 10^{-3}\text{m})^2}{(1.72 \times 10^{-8}\Omega\cdot\text{m})} = \boxed{26\ \text{m}}$$

6.)

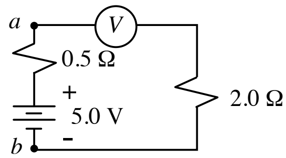


Terminal voltage is 20.2 V for the 24 V battery

a.) $$V_{ab} = \mathcal{E} - Ir \quad \text{so} \quad r = \frac{\mathcal{E} - V_{ab}}{I} = \frac{(24\ \text{V} - 20.2\ \text{V})}{4\ \text{A}} = \boxed{0.95\ \Omega}$$

b.) $$V_R = V_{ab} = 20.2\ \text{V} \quad \text{and} \quad V = IR \quad \text{or} \quad R = \frac{V}{I} = \frac{(20.2\ \text{V})}{(4\ \text{A})} = \boxed{5.05\ \Omega}$$

7.)



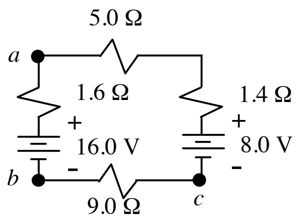
Ideal voltmeter has infinite resistance and there will be no current.

a.) $\boxed{I = 0}$

b.) $V_{ab} = \mathcal{E} - Ir = 5.0\text{ V} - 0 = \boxed{5.0\text{ V}}$

c.) Voltmeter will read the terminal voltage of the battery $V = V_{ab} = \boxed{5.0\text{ V}}$

8.)



a.) Kirchhoff's Loop rule:

Going clockwise starting at *b*

$$V_{16\text{V}} - V_{1.6\Omega} - V_{5.0\Omega} - V_{1.4\Omega} - V_{8\text{V}} - V_{9.0\Omega} = 0$$

$$16\text{ V} - I(1.6\ \Omega) - I(5.0\ \Omega) - I(1.4\ \Omega) - 8.0\text{ V} - I(9.0\ \Omega) = 0$$

$$8\text{ V} - I(17\ \Omega) = 0 \quad \text{and} \quad I = \frac{8\text{ V}}{17\ \Omega} = \boxed{0.471\text{ A} \quad \text{and is clockwise}}$$

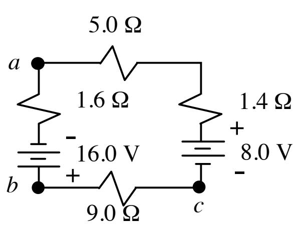
b.) $V_{ab} = \mathcal{E} - Ir = 16.0\text{ V} - (0.471\text{ A})(1.6\ \Omega) = \boxed{15.25\text{ V}}$

c.) $V_{ac} = V_a - V_c$ making point *c* the reference point (i.e. $V_c = 0$).

$$V_a = -I(9.0\ \Omega) + 16\text{ V} - I(1.6\ \Omega) = 16\text{ V} - (0.471\text{ A})(10.6\ \Omega) = 11.01\text{ V}$$

$$V_{ac} = V_a - V_c = 11.01\text{ V} - 0 = \boxed{11.01\text{ V}}$$

9.)



a.) Kirchhoff's Loop rule:

Going counterclockwise starting at *a*

$$-V_{1.6\Omega} + V_{16\text{V}} - V_{9.0\Omega} + V_{8\text{V}} - V_{1.4\Omega} - V_{5.0\Omega} = 0$$

$$-I(1.6\ \Omega) + 16\text{ V} - I(9.0\ \Omega) + 8.0\text{ V} - I(1.4\ \Omega) - I(5.0\ \Omega) = 0$$

$$24\text{ V} - I(17\ \Omega) = 0 \quad \text{and} \quad I = \frac{24\text{ V}}{17\ \Omega} = \boxed{1.41\text{ A} \quad \text{and is counterclockwise}}$$

b.) $V_{ab} = \mathcal{E} - Ir = 16.0\text{ V} - (1.41\text{ A})(1.6\ \Omega) = \boxed{13.74\text{ V}}$

c.) $V_{ac} = V_a - V_c$ making point *c* the reference point (i.e. $V_c = 0$).

$$V_a = I(9.0\ \Omega) - 16\text{ V} + I(1.6\ \Omega) = (1.41\text{ A})(10.6\ \Omega) - 16\text{ V} = -1.05\text{ V}$$

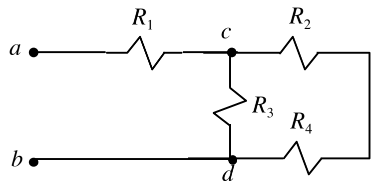
$$V_{ac} = V_a - V_c = -1.05\text{ V} - 0 = \boxed{-1.05\text{ V}}$$

10.) $P = 369$ W and $V = 18.0$ V $\qquad P = IV = \frac{V}{R}V = \frac{V^2}{R}$ so $R = \frac{V^2}{P} = \frac{(18\text{ V})^2}{369\text{ W}} = \boxed{0.878\ \Omega}$

11.) $V = 12.0$ V and $I = 0.29$ A $\qquad P = IV$ and $E = Pt = IVt = (0.29\text{ A})(12\text{ V})(16{,}200\text{ s}) = \boxed{5.64 \times 10^4\text{ J}}$

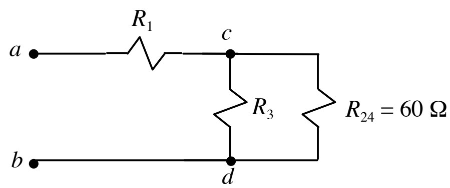
$t = 4.5\text{ hr} = 16{,}200\text{ s}$

1.)



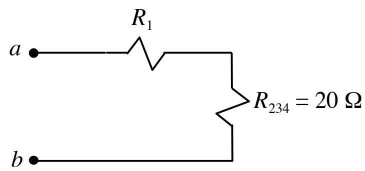
$R_1 = 12\ \Omega$, $R_2 = 20.0\ \Omega$, $R_3 = 30.0\ \Omega$, and $R_4 = 40.0\ \Omega$. The potential difference between a and b is 96 V.

a.)



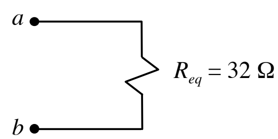
R_2 and R_4 are in series and can be replaced by their equivalent:

$$R_{24} = R_2 + R_4 = 20\ \Omega + 40\ \Omega = 60\ \Omega$$



R_{24} and R_3 are in parallel and can be replaced by their equivalent:

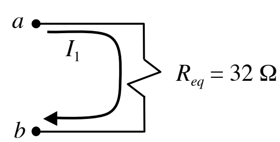
$$\frac{1}{R_{234}} = \frac{1}{R_{24}} + \frac{1}{R_3} = \frac{1}{60\ \Omega} + \frac{1}{30\ \Omega} \quad \text{and} \quad R_{234} = 20\ \Omega$$



R_{234} and R_1 are in series and can be replaced by their equivalent:

$$R_{eq} = R_{1234} = R_1 + R_{234} = 12\ \Omega + 20\ \Omega = 32\ \Omega$$

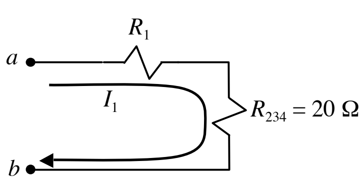
b.)



The current coming out of the battery is: $I_1 = \frac{V_{ab}}{R_{eq}} = \frac{96\text{ V}}{32\ \Omega} = 3\text{ A}$

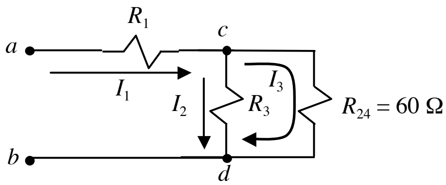
This is the current through R_1 and the equivalent R_{234}. Also,

$$V_1 = I_1R_1 = (3\text{ A})(12\ \Omega) = 36\text{ V} \quad \text{and} \quad V_{234} = I_1R_{234} = (3\text{ A})(20\ \Omega) = 60\text{ V}$$



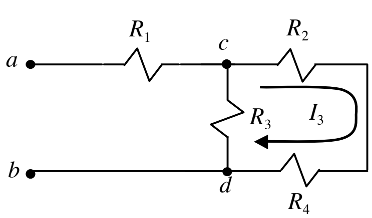
The voltage across R_3 and the equivalent R_{24} is the same as that across their equivalent R_{234} because they are in parallel.

So, $V_3 = V_{24} = V_{234} = 60\text{ V}$



Also, $I_2 = \frac{V_3}{R_3} = \frac{60\text{ V}}{30\ \Omega} = 2\text{ A}$ and $I_3 = \frac{V_{24}}{R_{24}} = \frac{60\text{ V}}{60\ \Omega} = 1\text{ A}$

(Note that $I_1 = I_2 + I_3$)



Finally, I_3 is the current in R_2 and R_4 because they are in series and have the same current as their equivalent.

and $V_2 = I_3R_2 = (1\text{ A})(20\ \Omega) = 20\text{ V}$

$V_4 = I_3R_4 = (1\text{ A})(40\ \Omega) = 40\text{ V}$

(Note that $V_{24} = V_2 + V_4$)

1.) b.) To summarize:

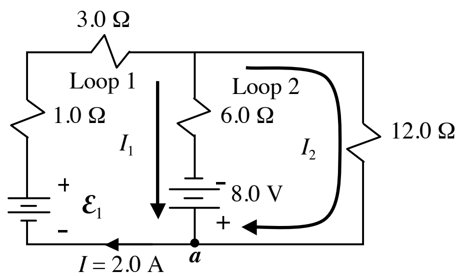
Resistor	Resistance	Current	Voltage	Power
R_1	12 Ω	3 A	36 V	108 W
R_2	20 Ω	1 A	20 V	20 W
R_3	30 Ω	2 A	60 V	120 W
R_4	40 Ω	1 A	40 V	40 W

c.)

$$\boxed{P = I_1 V_{ab} = (3\text{ A})(96\text{ V}) = 288\text{ W}}$$

This is same as the sum of the individual powers for each resistor.

2.)



a.) Applying Kirchhoff's Loop Rule to Loop 1,

$$\mathcal{E}_1 + 8\text{ V} = (2\text{ A})(1\ \Omega) + (2\text{ A})(3\ \Omega) + I_1(6\ \Omega) \quad (1)$$

Applying Kirchhoff's Loop Rule to Loop 2,

$$8\text{ V} = I_1(6\ \Omega) - I_2(12\ \Omega) \quad (2)$$

Applying Kirchhoff's Junction Rule to node a,

$$I_1 + I_2 = I \quad \text{or} \quad I_2 = 2\text{ A} - I_1 \quad (3)$$

Combining equations (2) and (3): $8\text{ V} = I_1(6\ \Omega) - (2\text{ A} - I_1)(12\ \Omega) = -24\text{ V} + I_1(18\ \Omega)$

$$32\text{ V} = I_1(18\ \Omega) \quad \text{and} \quad I_1 = \frac{32\text{ V}}{18\ \Omega} = 1.78\text{ A}$$

Substituting this into (1): $\mathcal{E}_1 + 8\text{ V} = (2\text{ A})(1\ \Omega) + (2\text{ A})(3\ \Omega) + (1.78\text{ A})(6\ \Omega) = 18.67\text{ V}$

$$\mathcal{E}_1 = 18.67\text{ V} - 8\text{ V} = \boxed{10.67\text{ V}}$$

b.) $I_2 = 2\text{ A} - I_1 = 2\text{ A} - 1.78\text{ A} = \boxed{0.22\text{ A}}$

c.) $P_{1\Omega} = IV = I^2R = (2\text{ A})^2(1\ \Omega) = \boxed{4\text{ W}}$ $P_{3\Omega} = IV = I^2R = (2\text{ A})^2(3\ \Omega) = \boxed{12\text{ W}}$

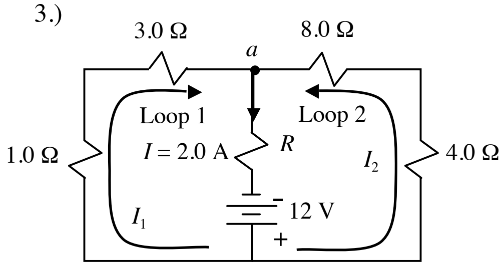
$P_{6\Omega} = IV = I^2R = (1.78\text{ A})^2(6\ \Omega) = \boxed{19\text{ W}}$ $P_{12\Omega} = IV = I^2R = (0.22\text{ A})^2(12\ \Omega) = \boxed{0.58\text{ W}}$

d.) $P_{10.67\text{V}} = IV = (2\text{ A})(10.67\text{ V}) = \boxed{21.34\text{ W}}$ $P_{8\text{V}} = IV = (1.78\text{ A})(8\text{ V}) = \boxed{14.24\text{ W}}$

Power from batteries is: $P_{batteries} = P_{10.67\text{V}} + P_{8\text{V}} = 21.34\text{ W} + 14.24\text{ W} = \boxed{35.6\text{ W}}$

Power dissipated in resistors: $P_{resistors} = P_{1\Omega} + P_{3\Omega} + P_{6\Omega} + P_{12\Omega} = 4\text{ W} + 12\text{ W} + 19\text{ W} + 0.58\text{ W} = \boxed{35.6\text{ W}}$

3.)



a.) Applying Kirchhoff's Loop Rule to Loop 1:

$$12\text{ V} = I_1(1\ \Omega) + I_1(3\ \Omega) + (2\text{ A})R = I_1(4\ \Omega) + (2\text{ A})R \quad (1)$$

Applying Kirchhoff's Loop Rule to Loop 2:

$$12\text{ V} = I_2(8\ \Omega) + I_2(4\ \Omega) + (2\text{ A})R = I_2(12\ \Omega) + (2\text{ A})R \quad (2)$$

Applying Kirchhoff's Junction Rule to node a,

$$I_1 + I_2 = I = 2\text{ A} \quad (3)$$

Subtracting equation (1) from equation (2): $0 = I_2(12\ \Omega) - I_1(4\ \Omega) = 3I_2 - I_1 \quad (4)$

Combining equation (3) and (4): $2\text{ A} = 4I_2$ and $I_2 = \dfrac{2\text{ A}}{4} = 0.5\text{ A}$ and

$$I_1 = I - I_2 = 2\text{ A} - 0.5\text{ A} = 1.5\text{ A}$$

Returning to equation (1): $12\text{ V} = I_1(4\ \Omega) + (2\text{ A})R = (1.5\text{ A})(4\ \Omega) + (2\text{ A})R = 6\text{ V} + (2\text{ A})R$

$$R = \frac{(12\text{ V} - 6\text{ V})}{(2\text{ A})} = \boxed{3\ \Omega}$$

b.) Found currents in part (a). Voltages are:

$V_{1\Omega} = I_1R = (1.5\text{ A})(1\ \Omega) = 1.5\text{ V}$ $\qquad V_{3\Omega} = I_1R = (1.5\text{ A})(3\ \Omega) = 4.5\text{ V}$

$V_{8\Omega} = I_2R = (0.5\text{ A})(8\ \Omega) = 4\text{ V}$ $\qquad V_{4\Omega} = I_2R = (0.5\text{ A})(4\ \Omega) = 2\text{ V}$ $\qquad V_R = IR = (2\text{ A})(3\ \Omega) = 6\text{ V}$

To summarize:

Resistance	Current	Voltage
3 Ω	2 A	6 V
1 Ω	1.5 A	1.5 V
3 Ω	1.5 A	4.5 V
8 Ω	0.5 A	4 V
4 Ω	0.5 A	2 V

a.) (Again)

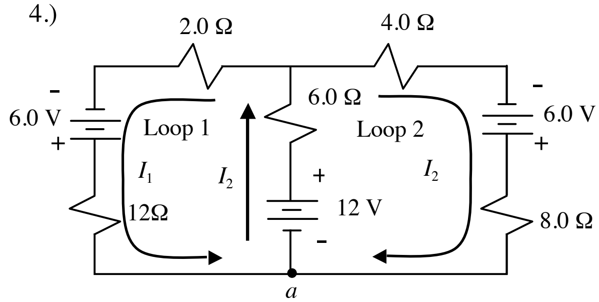
Currents can easily be obtained by recognizing that I_1 and I_2 are in parallel branches and divide the current I.

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I = \left(\frac{12\ \Omega}{4\ \Omega + 12\ \Omega}\right)(2\text{ A}) = 1.5\text{ A}$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I = \left(\frac{4\ \Omega}{4\ \Omega + 12\ \Omega}\right)(2\text{ A}) = 0.5\text{ A}$$

(R_1 is the total resistance of branch 1 and R_2 is the total resistance of branch 2.)

4.)



a.) Applying Kirchhoff's Loop Rule to Loop 1:

$$6\text{ V}+12\text{ V}=(12\ \Omega+2\ \Omega)I_1+(6\ \Omega)I_2$$
$$18\text{ V}=(14\ \Omega)I_1+(6\ \Omega)I_2 \quad (1)$$

Applying Kirchhoff's Loop Rule to Loop 2:

$$6\text{ V}+12\text{ V}=(6\ \Omega)I_2+(4\ \Omega+8\ \Omega)I_3$$
$$18\text{ V}=(6\ \Omega)I_2+(12\ \Omega)I_3 \quad (2)$$

Applying Kirchhoff's Junction Rule to node a,

$I_1+I_3=I_2$ (3) so $I_3=I_2-I_1$ (3*) and substituting this into equation (2) gives:

$$18\text{ V}=(6\ \Omega)I_2+(12\ \Omega)(I_2-I_1)$$
$$18\text{ V}=(-12\ \Omega)I_1+(18\ \Omega)I_2 \quad (2^*)$$

Multiplying equation (1) by 3 gives: $54\text{ V}=(42\ \Omega)I_1+(18\ \Omega)I_2$ (1*)

and subtracting equation (2*) from (1*) gives: $36\text{ V}=(54\ \Omega)I_1$ and $I_1=\dfrac{36\text{ V}}{54\ \Omega}=0.67\text{ A}$

From equation (1*): $I_2=\dfrac{18\text{ V}+(12\ \Omega)I_1}{18\ \Omega}=\dfrac{18\text{ V}+(12\ \Omega)(0.67\text{ A})}{18\ \Omega}=1.45\text{ A}$

Finally, from (3*): $I_3=I_2-I_1=1.45\text{ A}-0.67\text{ A}=0.78\text{ A}$

Voltages are:

$V_{2\Omega}=I_1R=(0.67\text{ A})(2\ \Omega)=1.34\text{ V}$ $\quad V_{12\Omega}=I_1R=(0.67\text{ A})(12\ \Omega)=8\text{ V}$

$V_{4\Omega}=I_2R=(0.78\text{ A})(4\ \Omega)=3.12\text{ V}$ $\quad V_{8\Omega}=I_2R=(0.78\text{ A})(8\ \Omega)=6.24\text{ V}$ $\quad V_{6\Omega}=I_2R=(1.45\text{ A})(6\ \Omega)=8.7\text{ V}$

To summarize:

Resistance	Current	Voltage
2 Ω	0.67 A	1.34 V
12 Ω	0.67 A	8 V
4 Ω	0.78 A	3.12 V
8 Ω	0.78 A	6.24 V
6 Ω	1.45 A	8.7 V

b.)

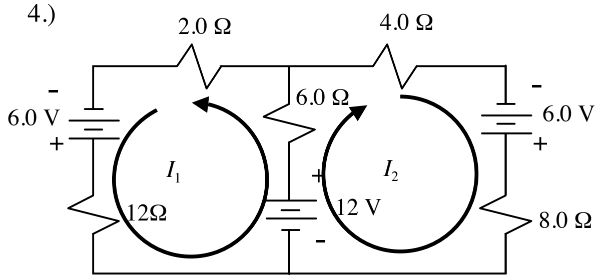
$$P_{6\text{V}}=I_1V=(0.67\text{ A})(6\text{ V})=\boxed{4\text{ W}}$$

$$P_{6\text{V}}=I_2V=(0.78\text{ A})(6\text{ V})=\boxed{4.7\text{ W}}$$

$$P_{12\text{V}}=IV=(1.45\text{ A})(12\text{ V})=\boxed{17.4\text{ W}}$$

Alternative solution using Mesh Currents:

4.)



a.) Using Loop Method (Mesh Currents):

$12\text{ V} + 6\text{ V} = (20\ \Omega)I_1 + (6\ \Omega)I_2$ (Loop 1)

$18\text{ V} = (20\ \Omega)I_1 + (6\ \Omega)I_2$ (1)

$18\text{ V} = (6\ \Omega)I_1 + (18\ \Omega)I_2$ (2) (Loop 2)

Multiplying equation (1) by (3) gives: $54\text{ V} = (60\ \Omega)I_1 + (18\ \Omega)I_2$

and subtracting equation (2) gives: $36\text{ V} = (54\ \Omega)I_1$ and $I_1 = \dfrac{36\text{ V}}{54\ \Omega} = 0.67\text{ A}$

From equation (1): $3\text{ A} = 3.33I_1 + I_2$ and $I_2 = 3\text{ A} - 3.33I_1 = 3\text{ A} - 3.33(0.67\text{ A}) = 0.78\text{ A}$

I_1 is the current in the 2 Ω and 12 Ω resistors.

I_2 is the current in the 4 Ω and 8 Ω resistors.

$I_1 + I_2 = 1.45$ A is the current in the 6 Ω resistor.

Voltages are:

$V_{2\Omega} = I_1R = (0.67\text{ A})(2\ \Omega) = 1.34\text{ V}$ $V_{12\Omega} = I_1R = (0.67\text{ A})(12\ \Omega) = 8\text{ V}$

$V_{4\Omega} = I_2R = (0.78\text{ A})(4\ \Omega) = 3.12\text{ V}$ $V_{8\Omega} = I_2R = (0.78\text{ A})(8\ \Omega) = 6.24\text{ V}$ $V_{6\Omega} = IR = (1.45\text{ A})(6\ \Omega) = 8.7\text{ V}$

To summarize:

Resistance	Current	Voltage
2 Ω	0.67 A	1.34 V
12 Ω	0.67 A	8 V
4 Ω	0.78 A	3.12 V
8 Ω	0.78 A	6.24 V
6 Ω	1.45 A	8.7 V

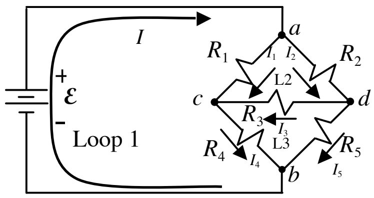
b.)

$P_{6\text{V}} = I_1V = (0.67\text{ A})(6\text{ V}) = \boxed{4\text{ W}}$

$P_{6\text{V}} = I_2V = (0.78\text{ A})(6\text{ V}) = \boxed{4.7\text{ W}}$

$P_{12\text{V}} = IV = (1.45\text{ A})(12\text{ V}) = \boxed{17.4\text{ W}}$

5.)



$R_1 = R_2 = R_3 = R_4 = 1.0\ \Omega$, and $R_5 = 2.0\ \Omega$.
The voltage across the battery is 13 V and it has no internal resistance.

a.)

Applying Kirchhoff's Loop rule on Loop 1:

$\mathcal{E} = I_1R_1 + I_4R_4$ or $13\text{ V} = I_1(1\ \Omega) + I_4(1\ \Omega)$ (1)

Applying Kirchhoff's Loop rule on Loop 2:

$0 = I_1R_1 - I_3R_3 - I_2R_2$ or $0 = I_1(1\ \Omega) - I_2(1\ \Omega) - I_3(1\ \Omega)$ (2)

Applying Kirchhoff's Loop rule on Loop 3:

$0 = I_3R_3 + I_4R_4 - I_5R_5$ or $0 = I_3(1\ \Omega) + I_4(1\ \Omega) + I_5(2\ \Omega)$ (3)

Applying Kirchhoff's Junction rule on junction *a*:

$I = I_1 + I_2$ and $0 = I - I_1 - I_2$ (4)

Applying Kirchhoff's Junction rule on junction *c*:

$I_1 + I_3 = I_4$ and $0 = I_1 + I_3 - I_4$ (5)

Applying Kirchhoff's Junction rule on junction *d*:

$I_2 = I_3 + I_5$ and $0 = I_2 - I_3 - I_5$ (6)

We have six equations with six unknowns. Setting up a matrix gives:

$$\begin{bmatrix} 0 & 1\ \Omega & 0 & 0 & 1\ \Omega & 0 \\ 0 & 1\ \Omega & -1\ \Omega & -1\ \Omega & 0 & 0 \\ 0 & 0 & 0 & 1\ \Omega & 1\ \Omega & -2\ \Omega \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} I \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 13\text{ V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

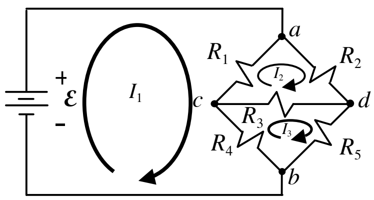
using matrix operations on a calculator: $I = 11$ A, $I_1 = 6$ A, $I_2 = 5$ A, $I_3 = 1$ A, $I_4 = 7$ A, and $I_5 = 4$ A

Resistor	Resistance	Current	Voltage
R_1	1 Ω	6 A	6 V
R_2	1 Ω	5 A	5 V
R_3	1 Ω	1 A	1 V
R_4	1 Ω	7 A	7 V
R_5	2 Ω	4 A	8 V

b.) The equivalent resistance is found using the current (I) from the battery. (Nothing is in parallel or series so ordinary methods for finding equivalent resistance will not work.)

$$V = IR \quad \text{and} \quad \mathcal{E} = IR_{eq} \quad \text{so} \quad R_{eq} = \frac{\mathcal{E}}{I} = \frac{13\text{ V}}{11\text{ A}} = \boxed{1.18\ \Omega}$$

(Solution using Mesh Currents)



$R_1 = R_2 = R_3 = R_4 = 1.0\ \Omega$, and $R_5 = 2.0\ \Omega$.
The voltage across the battery is 13 V and it has no internal resistance.

a.)

Applying Kirchhoff's Loop rule on Loop 1 (the loop containing I_1):

$$\mathcal{E} = I_1(R_1 + R_4) - I_2R_1 - I_3R_4 \quad \text{or} \quad 13\text{ V} = I_1(2\ \Omega) - I_2(1\ \Omega) - I_3(1\ \Omega) \quad (1)$$

Applying Kirchhoff's Loop rule on Loop 2 (the loop containing I_2):

$$0 = -I_1R_1 + I_2(R_1 + R_2 + R_3) - I_3R_3 \quad \text{or} \quad 0 = -I_1(1\ \Omega) + I_2(3\ \Omega) - I_3(1\ \Omega) \quad (2)$$

Applying Kirchhoff's Loop rule on Loop 3 (the loop containing I_3):

$$0 = -I_1R_4 - I_2R_3 + I_3(R_3 + R_4 + R_5) \quad \text{or} \quad 0 = -I_1(1\ \Omega) - I_2(1\ \Omega) + I_3(4\ \Omega) \quad (3)$$

We have three equations with three unknowns. Setting up a matrix gives:

$$\begin{bmatrix} 2\ \Omega & -1\ \Omega & -1\ \Omega \\ -1\ \Omega & 3\ \Omega & -1\ \Omega \\ -1\ \Omega & -1\ \Omega & 4\ \Omega \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 13\text{ V} \\ 0 \\ 0 \end{bmatrix}$$

using matrix operations on a calculator: $I_1 = 11$ A, $I_2 = 5$ A, and $I_3 = 4$ A

for R_1 the current through it is $I_1 - I_2 = 11\text{ A} - 5\text{ A} = 6\text{ A}$

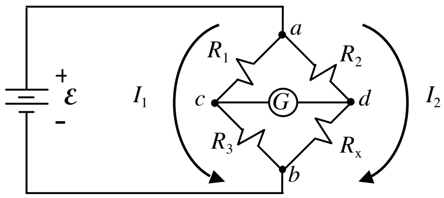
for R_2 the current through it is $I_2 = 5\text{ A}$

for R_3 the current through it is $I_2 - I_3 = 5\text{ A} - 4\text{ A} = 1\text{ A}$

for R_4 the current through it is $I_1 - I_3 = 11\text{ A} - 4\text{ A} = 7\text{ A}$

for R_5 the current through it is $I_3 = 4\text{ A}$

6.)



With no current through the galvanometer, the current through R_1 and R_3 is the same and equal to I_1.

Also, the current through R_2 and R_x is the same and equal to I_2.

The potential at point c is the same as the potential at point d (There is no current through the meter, therefore there is no voltage drop since $V_G = I_G R_G = 0$). Therefore, $V_{ac} = V_{ad}$ and $V_{cb} = V_{db}$.

Using Ohm's Law:

for R_1 $\quad V_{R1} = V_{ac} = I_1 R_1$ $\quad$ and for R_2 $\quad V_{R2} = V_{ad} = I_2 R_2$

since $V_{ac} = V_{ad}$ it follows that $\quad I_1 R_1 = I_2 R_2$ or $\quad I_1 = I_2 \dfrac{R_2}{R_1}$ (1)

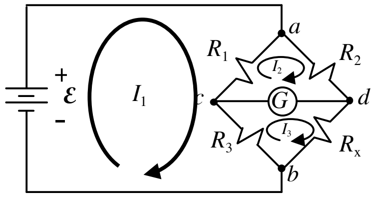
for R_3 $\quad V_{R3} = V_{cb} = I_1 R_3$ $\quad$ and for R_x $\quad V_{Rx} = V_{db} = I_2 R_x$

since $V_{cb} = V_{db}$ it follows that $\quad I_1 R_3 = I_2 R_x$ or $\quad I_1 = I_2 \dfrac{R_x}{R_3}$ (2)

Comparing equations 1 and 2 it follows that: $\quad I_2 \dfrac{R_2}{R_1} = I_2 \dfrac{R_x}{R_3}$ $\quad$ and $\quad \dfrac{R_2}{R_1} = \dfrac{R_x}{R_3}$

Therefore: $\quad R_x = \dfrac{R_2 R_3}{R_1}$

Method 2:



When bridge is balanced there is no current through the galvanometer.

Therefore: $I_2 = I_3$

Applying Kirchhoff's Loop rule on Loop 2 (the loop containing I_2):

$$0 = -I_1 R_1 + I_2 (R_1 + R_2) \quad (1)$$

Applying Kirchhoff's Loop rule on Loop 3 (the loop containing I_3):

$$0 = -I_1 R_3 + I_3 (R_3 + R_x) = -I_1 R_3 + I_2 (R_3 + R_x) \quad (2)$$

From equation 1 it follows that: $\quad I_2 = \dfrac{I_1 R_1}{(R_1 + R_2)}$ and from equation 2 it follows that: $\quad I_2 = \dfrac{I_1 R_3}{(R_3 + R_x)}$

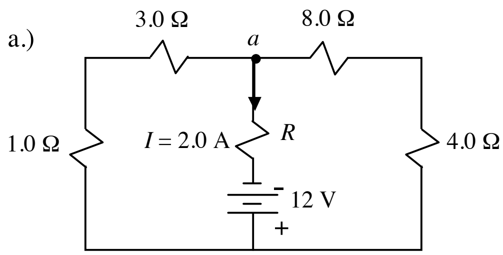
Therefore: $\quad \dfrac{I_1 R_1}{(R_1 + R_2)} = \dfrac{I_1 R_3}{(R_3 + R_x)}$ $\quad$ and $\quad R_1 (R_3 + R_x) = R_3 (R_1 + R_2)$ $\quad$ and $\quad R_1 R_3 + R_1 R_x = R_3 R_1 + R_3 R_2$

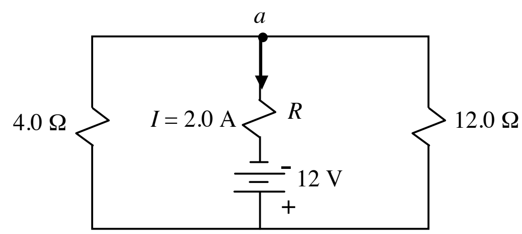
so $\quad R_1 R_x = R_3 R_2$ $\quad$ and $\quad R_x = \dfrac{R_3 R_2}{R_1}$

Another look at Problem 3:

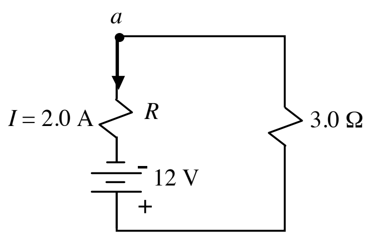
3.)

a.)





The equivalent resistance for the entire circuit is: $R_{eq} = \frac{V_{12V}}{I} = \frac{12\text{ V}}{2\text{ A}} = 6\ \Omega$

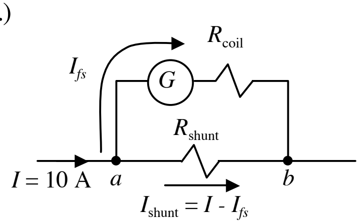


$R_{eq} = R + 3\ \Omega = 6\ \Omega$ so $R = 3\ \Omega$

HO 38 Solutions

1.) $R_{\text{coil}} = 50.0\ \Omega$, $I_{fs} = 300\ \mu\text{A}$

a.)



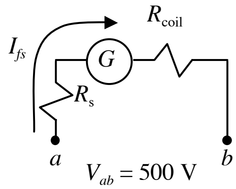
Ammeter reading 10 A full scale.

Voltage on R_{coil} and R_{shunt} are the same (they are parallel).

$$V_{\text{coil}} = V_{\text{shunt}} \quad \text{or} \quad I_{fs}R_{\text{coil}} = I_{\text{shunt}}R_{\text{shunt}} = \left(I - I_{fs}\right)R_{\text{shunt}}$$

$$R_{\text{shunt}} = \frac{I_{fs}R_{\text{coil}}}{\left(I - I_{fs}\right)} = \frac{\left(300 \text{ x } 10^{-6}\text{ A}\right)\left(50\ \Omega\right)}{\left(10\text{ A} - 300 \text{ x } 10^{-6}\text{ A}\right)} = \boxed{1.5 \text{ x } 10^{-3}\Omega}$$

b.)

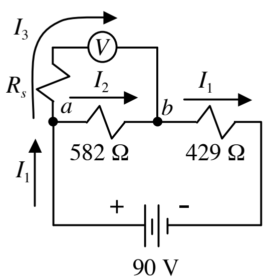


Voltmeter reading 500 V full scale.

$$V_{ab} = V_s + V_{coil} = I_{fs}R_s + I_{fs}R_{\text{coil}}$$

$$R_s = \frac{V_{ab} - I_{fs}R_{\text{coil}}}{I_{fs}} = \frac{\left(500\text{ V} - \left(300 \text{ x } 10^{-6}\text{ A}\right)\left(50\ \Omega\right)\right)}{\left(300 \text{ x } 10^{-6}\text{ A}\right)} = \boxed{1.67 \text{ x } 10^{6}\Omega}$$

2.)



a.) Voltmeter reads 44.6 V so $V_{ab} = 44.6$ V.

$$V_{ab} = I_2R \quad \text{so} \quad I_2 = \frac{V_{ab}}{R} = \frac{44.6\text{ V}}{582\ \Omega} = 0.0766\text{ A}$$

Applying Kirchhoff's Loop rule to loop with battery:

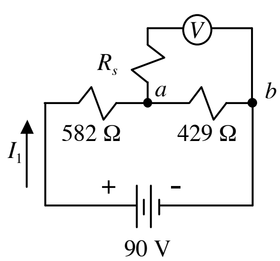
$$90\text{ V} = 44.6\text{ V} + I_1\left(429\ \Omega\right) \quad \text{and} \quad I_1 = \frac{\left(90\text{ V} - 44.6\text{ V}\right)}{\left(429\ \Omega\right)} = 0.1058\text{ A}$$

At junction *a*: $I_1 = I_2 + I_3$ or $I_3 = I_1 - I_2 = 0.1058\text{ A} - 0.0766\text{ A} = 0.0292\text{ A}$

Finally the meter is parallel to 582 Ω resistor and has same voltage.

$$V_{ab} = I_3R_s \quad \text{and} \quad R_s = \frac{V_{ab}}{I_3} = \frac{44.6\text{ V}}{0.0292\text{ A}} = \boxed{1527\ \Omega}$$

b.)

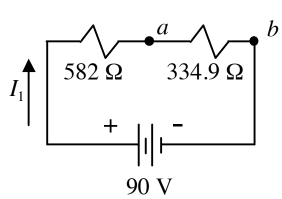


The meter resistance is parallel to 429 Ω resistor and their equivalent resistance is:

$$\frac{1}{R_{ab}} = \frac{1}{1527\ \Omega} + \frac{1}{429\ \Omega} \quad \text{and} \quad R_{ab} = 334.9\ \Omega$$

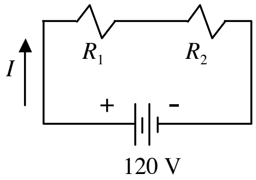
This is in series with the 582 Ω resistor so their equivalent resistance is:

$$R_{eq} = 334.9\ \Omega + 582\ \Omega = 916.9\ \Omega$$



Also: $I_1 = \dfrac{V_{90\text{V}}}{R_{eq}} = \dfrac{90\text{ V}}{916.9\ \Omega} = 0.0982\text{ A}$

The voltmeter reads V_{ab} and $V_{ab} = I_1R_{ab} = \left(0.0982\text{ A}\right)\left(334.9\ \Omega\right) = \boxed{32.9\text{ V}}$

3.) 

The meters will deflect full-scale when the voltage across them is 150 V:

For meter 1: $I_{fs1} = \dfrac{V}{R_1} = \dfrac{150\text{ V}}{15 \text{ x } 10^3\,\Omega} = 0.01\text{ A}$

For meter 2: $I_{fs2} = \dfrac{V}{R_2} = \dfrac{150\text{ V}}{150 \text{ x } 10^3\,\Omega} = 0.001\text{ A}$

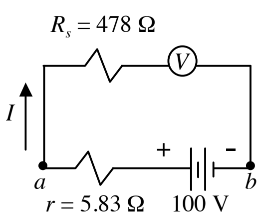
The actual current through each is: $I = \dfrac{V_{120\text{V}}}{R_1 + R_2} = \dfrac{120\text{ V}}{\left(15 \text{ x } 10^3\,\Omega + 150 \text{ x } 10^3\,\Omega\right)} = 7.27 \text{ x } 10^{-4}\text{ A}$

Reading on each meter will be: $V_{meter} = V_{fs}\left(\dfrac{I}{I_{fs}}\right)$

so $V_{meter1} = V_{fs1}\left(\dfrac{I}{I_{fs1}}\right) = (150\text{ V})\left(\dfrac{7.27 \text{ x } 10^{-4}\text{ A}}{0.01\text{ A}}\right) = \boxed{10.9\text{ V}}$

(Note that the sum is 120 V)

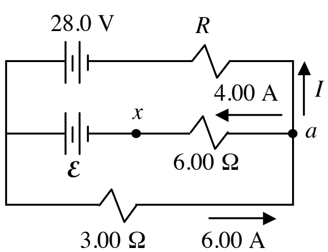
and $V_{meter2} = V_{fs2}\left(\dfrac{I}{I_{fs2}}\right) = (150\text{ V})\left(\dfrac{7.27 \text{ x } 10^{-4}\text{ A}}{0.001\text{ A}}\right) = \boxed{109.1\text{ V}}$

4.) 

Applying Kirchhoff's Loop rule: $V_{100\text{V}} = Ir + IR_s$

$$I = \frac{V_{100\text{V}}}{(r + R_s)} = \frac{100\text{ V}}{(5.83\,\Omega + 478\,\Omega)} = 0.207\text{ A}$$

The terminal voltage V_{ab} is: $V_{ab} = V_{100\text{V}} - Ir = 100\text{ V} - (0.207\text{ A})(5.83\,\Omega) = \boxed{98.8\text{ V}}$

5.) 

a.) Applying Kirchhoff's Junction rule at junction *a*:

$6\text{ A} = I + 4\text{ A}$ so $I = 6\text{ A} - 4\text{ A} = \boxed{2\text{ A}}$

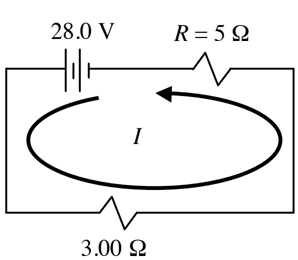
b.) The 3 Ω resistor is parallel to the 28 V battery and *R* so they have the same voltage.

$V_{3\Omega} = IR = (6\text{ A})(3\,\Omega) = 18\text{ V}$ and $V_{3\Omega} = V_{28\text{V}} - IR$

Therefore: $R = \dfrac{(V_{28\text{V}} - V_{3\Omega})}{I} = \dfrac{(28\text{ V} - 18\text{ V})}{2\text{ A}} = \boxed{5\,\Omega}$

c.) The 3 Ω resistor is also parallel to *emf* $\mathcal{E}$ and the 6 Ω resistor so they have the same voltage.

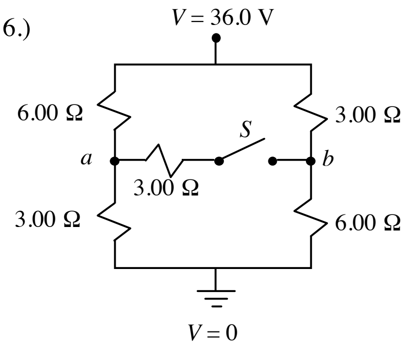
$V_{3\Omega} = \mathcal{E} - IR$ and $\mathcal{E} = V_{3\Omega} + IR = 18\text{ V} + (4\text{ A})(6\,\Omega) = \boxed{42\text{ V}}$

d.) 

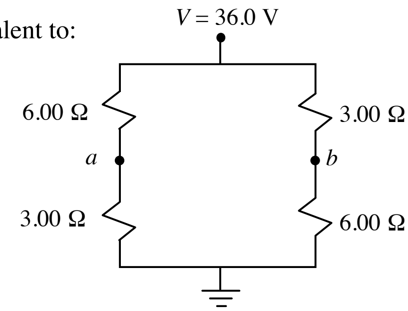
Applying Kirchhoff's Loop rule: $V_{28\text{V}} = V_R + V_{3\Omega} = IR + IR_{3\Omega}$

$$I = \frac{V_{28\text{V}}}{(R + R_{3\Omega})} = \frac{28\text{ V}}{(5\,\Omega + 3\,\Omega)} = 3.5\text{ A}$$

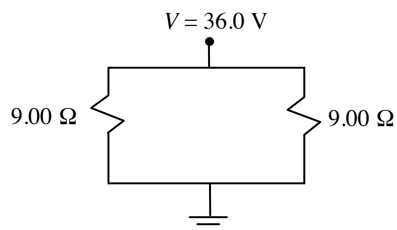
6.)



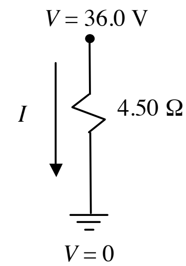
a.) With switch open the circuit is equivalent to:



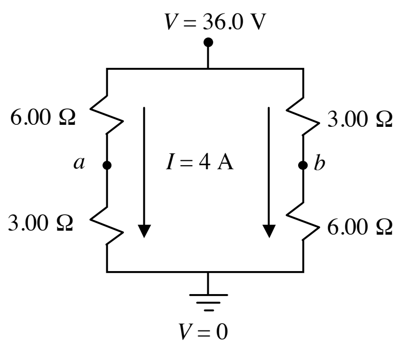
This reduces to :



and finally:



The current from the battery is: $I = \dfrac{V_{36\text{V}}}{R_{eq}} = \dfrac{36\text{ V}}{4.5\ \Omega} = 8\text{ A}$ and this is split equally between the 9 Ω equivalents.

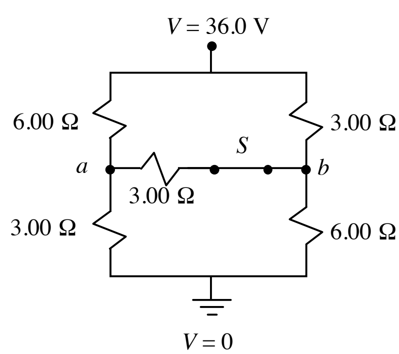


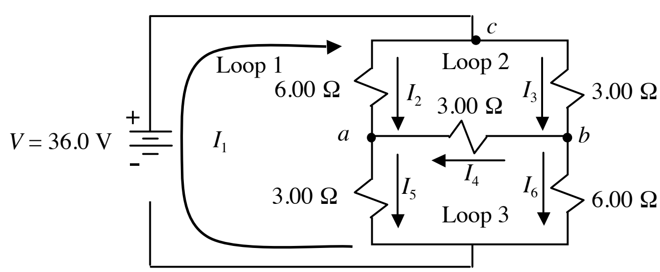
So: $V_a = V_{36\text{V}} - V_{6\Omega} = V_{36\text{V}} - I_{6\Omega}R_{6\Omega} = 36\text{ V} - (4\text{ A})(6\ \Omega) = 12\text{ V}$

$V_b = V_{36\text{V}} - V_{3\Omega} = V_{36\text{V}} - I_{3\Omega}R_{3\Omega} = 36\text{ V} - (4\text{ A})(3\ \Omega) = 24\text{ V}$

It follows that: $V_{ab} = V_a - V_b = 12\text{ V} - 24\text{ V} = \boxed{-12\text{ V}}$

b.) Closing the switch the circuit becomes:





The circuit is like the *bridge* circuit on HO 36 with nothing in parallel or series.

Applying Kirchhoff's Loop rule on Loop 1:

$$36\text{ V} = I_2(6\ \Omega) + I_5(3\ \Omega) \quad (1)$$

Applying Kirchhoff's Loop rule on Loop 2:

$$0 = I_3(3\ \Omega) + I_4(3\ \Omega) - I_2(6\ \Omega) \quad (2)$$

6.) b.)

Applying Kirchhoff's Loop rule on Loop 3:

$$0 = I_4(3\ \Omega) + I_5(3\ \Omega) - I_6(6\ \Omega) \quad (3)$$

Applying Kirchhoff's Junction rule on junction *c*:

$$I_1 = I_2 + I_3 \text{ and } 0 = I_1 - I_2 - I_3$$

Applying Kirchhoff's Junction rule on junction *a*:

$$I_2 + I_4 = I_5 \text{ and } 0 = I_2 + I_4 - I_5$$

Applying Kirchhoff's Junction rule on junction *b*:

$$I_3 = I_4 + I_6 \text{ and } 0 = I_3 - I_4 - I_6$$

We have six equations with six unknowns. Setting up a matrix gives:

$$\begin{bmatrix} 0 & 6\ \Omega & 0 & 0 & 3\ \Omega & 0 \\ 0 & 0 & 0 & 3\ \Omega & 3\ \Omega & -6\ \Omega \\ 0 & -6\ \Omega & 3\ \Omega & 3\ \Omega & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 36\text{ V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using matrix operations on a calculator: $I_1 = 8.57$ A, $I_2 = 3.43$ A, $I_3 = 5.14$ A, $I_4 = 1.71$ A, $I_5 = 5.14$ A, and $I_6 = 3.43$ A

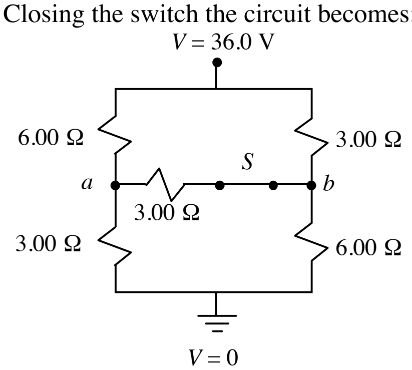
The current through the switch is : $I_{switch} = I_4 = \boxed{1.71\text{ A}}$

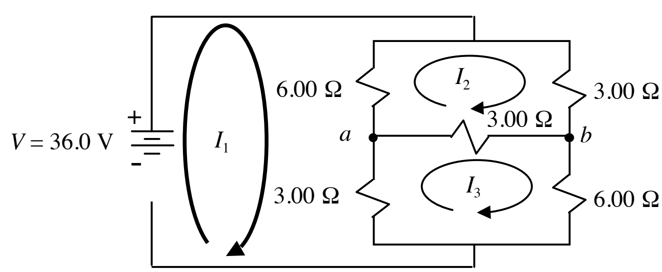
c.) The equivalent resistance is found using the current (I_1) from the battery. (Nothing is in parallel or series so ordinary methods for finding equivalent resistance will not work.)

$$V = IR \quad \text{and} \quad V_{36\text{V}} = I_1 R_{eq} \text{ so } \quad R_{eq} = \frac{V_{36\text{V}}}{I_1} = \frac{36\text{ V}}{8.57\text{ A}} = \boxed{4.20\ \Omega}$$

(Solution using Mesh Currents)

b.) Closing the switch the circuit becomes:





The circuit is like the *bridge* circuit on HO 36 with nothing in parallel or series. Using Loop Method:

Applying Kirchhoff's Loop rule on Loop 1 (the loop containing I_1):

$$36\text{ V} = I_1(9\ \Omega) - I_2(6\ \Omega) - I_3(3\ \Omega) \quad (1)$$

Applying Kirchhoff's Loop rule on Loop 2 (the loop containing I_2):

$$0 = -I_1(6\ \Omega) + I_2(12\ \Omega) - I_3(3\ \Omega) \quad (2)$$

Applying Kirchhoff's Loop rule on Loop 3 (the loop containing I_3):

$$0 = -I_1(3\ \Omega) - I_2(3\ \Omega) + I_3(12\ \Omega) \quad (3)$$

We have three equations with three unknowns. Setting up a matrix gives:

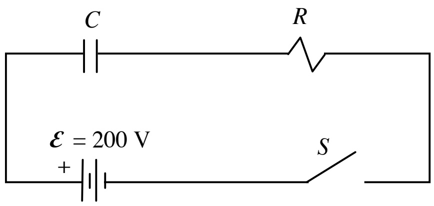
$$\begin{bmatrix} 9\ \Omega & -6\ \Omega & -3\ \Omega \\ -6\ \Omega & 12\ \Omega & -3\ \Omega \\ -3\ \Omega & -3\ \Omega & 12\ \Omega \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 36\text{ V} \\ 0 \\ 0 \end{bmatrix}$$

using matrix operations on a calculator: $I_1 = 8.57$ A, $I_2 = 5.14$ A, and $I_3 = 3.43$ A

The current through the switch is : $I_{switch} = I_2 - I_3 = 5.14\text{ A} - 3.43\text{ A} = \boxed{1.71\text{ A}}$

7.) $RC\ [=]\ \Omega \cdot \text{F}\ [=]\ \dfrac{\text{V}}{\text{A}}\dfrac{\text{C}}{\text{V}}\ [=]\ \dfrac{\text{C}}{\left(\frac{\text{C}}{\text{s}}\right)}\ [=]\ \text{s}$

8.)



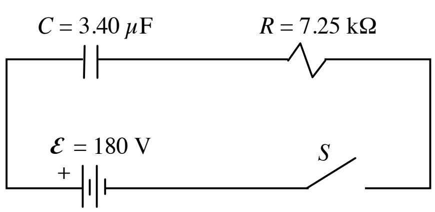
At time $t = 0$, $I_o = 8.6 \times 10^{-4}$ A, $Q_o = 0$, and $RC = 5.7\text{ s} = \tau$.

When switch is closed uncharged capacitor has no resistance and

$\mathcal{E} = I_o R$ so $R = \dfrac{\mathcal{E}}{I_o} = \dfrac{200\text{ V}}{8.6 \times 10^{-4}\text{ A}} = \boxed{2.33 \times 10^{5}\,\Omega}$

$RC = \tau$ so $C = \dfrac{\tau}{R} = \dfrac{5.7\text{ s}}{2.33 \times 10^{5}\,\Omega} = \boxed{2.45 \times 10^{-5}\text{ F}}$

9.)



$Q_o = 0$

Find the time for the current to decay to $i = 0.0185$ A.

For charging: $i(t) = I_o e^{-\frac{t}{RC}} = \dfrac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$

The time that the current is 0.0185 A is:

$$t = -RC \ln\left(\frac{Ri(t)}{\mathcal{E}}\right) = -(7.25 \times 10^{3}\,\Omega)(3.4 \times 10^{-6}\text{ F}) \ln\left(\frac{(7.25 \times 10^{3}\,\Omega)(0.0185\text{ A})}{180\text{ V}}\right) = 7.25 \times 10^{-3}\text{ s}$$

The charge on the capacitor is:

$$q(t) = Q_f\left(1 - e^{-\frac{t}{RC}}\right) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) = (3.4 \times 10^{-6}\text{ F})(180\text{ V})\left(1 - e^{-\frac{7.25 \times 10^{-3}\text{s}}{(7.25 \times 10^{3}\Omega)(3.4 \times 10^{-6}\text{F})}}\right) = \boxed{1.56 \times 10^{-4}\text{ C}}$$

9.)

Method 2:

Realizing that the sum of the voltage on the capacitor and resistor must always be 180 V and at the time when $i = 0.0185$ A:

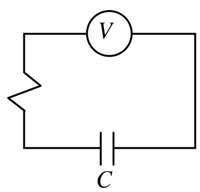
$$v_R = iR = (0.0185\text{ A})(7.25 \text{ x } 10^3\Omega) = 134.1\text{ V}$$

$$\mathcal{E} = v_R + v_C \quad \text{so} \quad v_C = \mathcal{E} - v_R v_C = 180\text{ V} - 134.1\text{ V} = 45.9\text{ V}$$

The charge is $q_C = Cv_C = (3.4 \text{ x } 10^{-6}\text{F})(45.9\text{ V}) = 1.56 \text{ x } 10^{-4}\text{C}$

10.)

$R_s = 2.25$ MΩ



On capacitor $V_o = 15$ V and after 5.00 s voltmeter reads 5.0 V.

The voltmeter reads the voltage on the capacitor so at $t = 5$ s, $v_C = 5.0$ V.

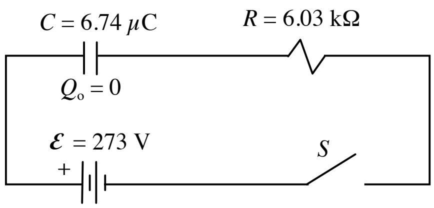
While discharging the voltage on the capacitor is: $v(t) = V_o e^{-\frac{t}{RC}}$

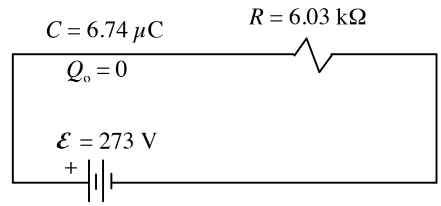
Solving for C:

$$\ln\left(\frac{v(t)}{V_o}\right) = -\frac{t}{RC}$$

so $$C = \frac{-t}{R\ln\left(\frac{v(t)}{V_o}\right)} = \frac{-5\text{ s}}{(2.25 \text{ x } 10^6\Omega)\ln\left(\frac{5\text{ V}}{15\text{ V}}\right)} = \boxed{2.02 \text{ x } 10^{-6}\text{F}}$$

1.)





Right after the switch is closed.

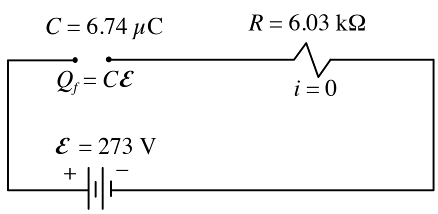
a.) The capacitor is uncharged and $v_C = \dfrac{Q}{C} = \boxed{0}$

b.) The voltage across the resistor $v_R = \mathcal{E} = \boxed{273\text{ V}}$

c.) The charge on the capacitor $q = Q_o = \boxed{0}$

d.) The current through the resistor $i_R = \dfrac{v_R}{R} = \dfrac{\mathcal{E}}{R} = \dfrac{273\text{ V}}{6.03 \text{ x } 10^3\,\Omega} = \boxed{0.0453\text{ A}}$

e.)



A long time after the switch is closed.

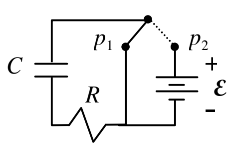
a.) The voltage across the capacitor $v_C = \mathcal{E} = \boxed{273\text{ V}}$

b.) The voltage across the resistor $v_R = iR = \boxed{0}$

c.) The charge on the capacitor $q = Q_f = C\mathcal{E} = \left(6.74 \text{ x } 10^{-6}\text{F}\right)\left(273\text{ V}\right) = \boxed{1.84 \text{ x } 10^{-3}\text{C}}$

d.) The current through the resistor $i_R = \boxed{0}$

2.)



$C = 7.50\,\mu\text{F}, \mathcal{E} = 36.0\text{ V}$

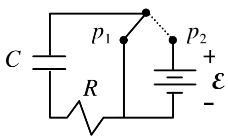
a.) $Q_f = C\mathcal{E} = \left(7.5 \text{ x } 10^{-6}\text{F}\right)\left(36\text{ V}\right) = \boxed{2.7 \text{ x } 10^{-4}\text{C}}$

b.) $q(t) = Q_f\left(1 - e^{\frac{-t}{RC}}\right)$ so $\ln\left(1 - \dfrac{q(t)}{Q_f}\right) = \dfrac{-t}{RC}$

and $R = \dfrac{-t}{C\ln\left(1 - \dfrac{q(t)}{Q_f}\right)} = \dfrac{-\left(3 \text{ x } 10^{-3}\text{s}\right)}{\left(7.5 \text{ x } 10^{-6}\text{F}\right)\ln\left(1 - \dfrac{\left(225 \text{ x } 10^{-6}\text{C}\right)}{\left(270 \text{ x } 10^{-6}\text{C}\right)}\right)} = \boxed{223\,\Omega}$

c.) $q(t) = Q_f\left(1 - e^{\frac{-t}{RC}}\right)$ so $\ln\left(1 - \dfrac{q(t)}{Q_f}\right) = \dfrac{-t}{RC}$

and $t = -RC\ln\left(1 - \dfrac{q(t)}{Q_f}\right) = -\left(223\,\Omega\right)\left(7.5 \text{ x } 10^{-6}\text{F}\right)\ln\left(1 - \dfrac{0.99Q_f}{Q_f}\right) = \boxed{7.70 \text{ x } 10^{-3}\text{s}}$

3.) 

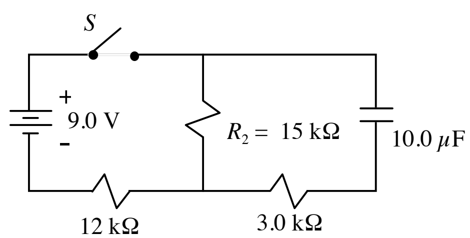
$C = 40.0\ \mu\text{F}$, $\mathcal{E} = 24.0\ \text{V}$, and $R = 950\ \Omega$

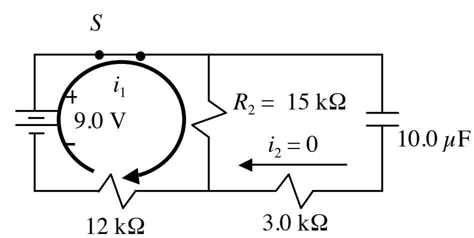
a.) $$q(t) = Q_f\left(1 - e^{\frac{-t}{RC}}\right) = C\mathcal{E}\left(1 - e^{\frac{-t}{RC}}\right) = \left(40 \times 10^{-6}\,\text{F}\right)\left(24\ \text{V}\right)\left(1 - e^{\frac{-(0.05\ \text{s})}{(950\ \Omega)\left(40 \times 10^{-6}\,\text{F}\right)}}\right) = \boxed{7.02 \times 10^{-4}\,\text{C}}$$

b.) $$v_C = \frac{q}{C} = \frac{\left(7.02 \times 10^{-4}\,\text{C}\right)}{\left(40 \times 10^{-6}\,\text{F}\right)} = \boxed{17.6\ \text{V}} \quad \text{and} \quad \mathcal{E} = v_R + v_C \quad \text{or} \quad v_R = \mathcal{E} - v_C = 24\ \text{V} - 17.6\ \text{V} = \boxed{6.4\ \text{V}}$$

c.) $$v_C = \frac{q}{C} = \frac{\left(7.02 \times 10^{-4}\,\text{C}\right)}{\left(40 \times 10^{-6}\,\text{F}\right)} = \boxed{17.6\ \text{V}} \quad \text{and} \quad 0 = v_R + v_C \quad \text{or} \quad v_R = -v_C = \boxed{-17.6\ \text{V}}$$

d.) $$q(t) = Q_o e^{\frac{-t}{RC}} = \left(7.02 \times 10^{-4}\,\text{C}\right)e^{\frac{-(0.05\ \text{s})}{(950\ \Omega)\left(40 \times 10^{-6}\,\text{F}\right)}} = \boxed{1.88 \times 10^{-4}\,\text{C}}$$

4.) 



At steady state.

a.) At steady state the capacitor is fully charged and current has decayed to zero.

The current through the 3.0 kΩ resistor $\boxed{i_2 = 0}$

The current through the 15 kΩ and the 12 kΩ resistors is the same since they are in series.

$$i_1 = \frac{V_{9V}}{R_{eq}} = \frac{9\ \text{V}}{\left(15{,}000\ \Omega + 12{,}000\ \Omega\right)} = \boxed{3.33 \times 10^{-4}\,\text{A}}$$

b.) The voltage on the capacitor is the voltage on 15 kΩ resistor. (No voltage drop on 3 kΩ resistor.)

$$v_C = v_{R2} = i_1 R_2 = \left(3.33 \times 10^{-4}\,\text{A}\right)\left(15{,}000\ \Omega\right) = 5\ \text{V}$$

The charge is: $$q_C = C v_C = \left(10 \times 10^{-6}\,\text{F}\right)\left(5\ \text{V}\right) = \boxed{5.0 \times 10^{-5}\,\text{C}}$$

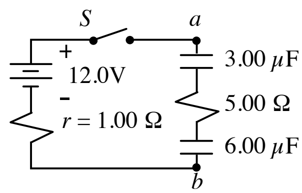
c.) When the switch is open the capacitor discharges into the series combination of the 3 kΩ and 15 kΩ resistor.

$$i_2 = I_o e^{\frac{-t}{R_{eq}C}} = \frac{v_C(0)}{R_{eq}} e^{\frac{-t}{R_{eq}C}} = \frac{5\ \text{V}}{\left(15{,}000\ \Omega + 3000\ \Omega\right)} e^{\frac{-t}{(15{,}000\ \Omega + 3000\ \Omega)\left(10 \times 10^{-6}\,\text{F}\right)}} = \boxed{\left(2.78 \times 10^{-4}\,\text{A}\right)e^{\frac{-t}{(0.18\ \text{s})}}}$$

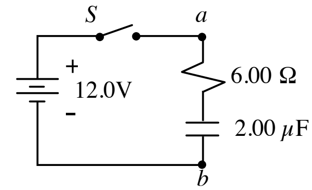
d.) $$q(t) = Q_o e^{\frac{-t}{R_{eq}C}} \quad \text{so} \quad \ln\left(\frac{q(t)}{Q_o}\right) = \frac{-t}{R_{eq}C}$$

and $$t = -R_{eq}C \ln\left(\frac{q(t)}{Q_o}\right) = -\left(18{,}000\ \Omega\right)\left(10 \times 10^{-6}\,\text{F}\right)\ln\left(\frac{0.2Q_o}{Q_o}\right) = \boxed{0.29\ \text{s}}$$

5.)



The circuit is equivalent to:
(resistors in series capacitors in series.)



a.) time constant is $\tau = R_{eq}C_{eq} = (6\ \Omega)(2 \text{ x } 10^{-6}\text{F}) = \boxed{1.2 \text{ x } 10^{-5}\text{s}}$

b.) The final charge on the 3 μC capacitor is the same as the final charge on its equivalent.

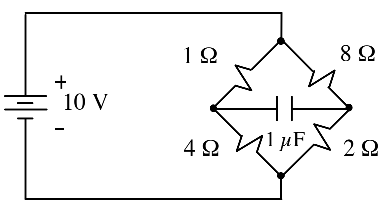
$$Q_f = C_{eq}V = (2 \text{ x } 10^{-6}\text{F})(12 \text{ V}) = 2.4 \text{ x } 10^{-5}\text{C}$$

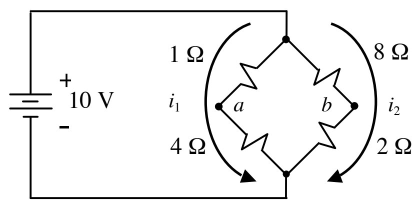
The charge at any time is: $q(t) = Q_f\left(1 - e^{\frac{-t}{R_{eq}C_{eq}}}\right)$ (both capacitors have same charge at all times)

So the voltage on the 3 μC capacitor at any time is: $v(t) = \dfrac{Q_f}{C}\left(1 - e^{\frac{-t}{R_{eq}C_{eq}}}\right)$

When $t = \tau$ the voltage on the 3 μC capacitor is: $v(\tau) = \dfrac{(2.4 \text{ x } 10^{-5}\text{C})}{(3 \text{ x } 10^{-6}\text{F})}(1 - e^{-1}) = \boxed{5.06 \text{ V}}$

6.)





After a long time, capacitor is fully charged.

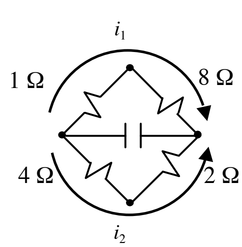
a.) When the capacitor is fully charged, no current passes through it.

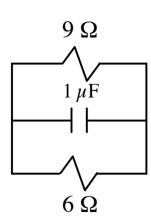
$$i_1 = \frac{V}{R} = \frac{10 \text{ V}}{(1\ \Omega + 4\ \Omega)} = 2 \text{ A} \quad \text{and} \quad i_2 = \frac{V}{R} = \frac{10 \text{ V}}{(8\ \Omega + 2\ \Omega)} = 1 \text{ A}$$

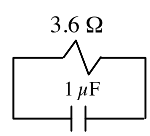
$$V_a = V_{10V} - V_{1\Omega} = 10 \text{ V} - (2 \text{ A})(1\ \Omega) = 8 \text{ V} \quad \text{and} \quad V_b = V_{10V} - V_{8\Omega} = 10 \text{ V} - (1 \text{ A})(8\ \Omega) = 2 \text{ V}$$

The voltage on the capacitor is: $V_a - V_b = 8 \text{ V} - 2 \text{ V} = \boxed{6 \text{ V}}$

b.)







The equivalent circuit while discharging.

$$v(t) = V_o e^{\frac{-t}{R_{eq}C}} \quad \text{or} \quad t = -R_{eq}C \ln\left(\frac{v(t)}{V_o}\right) = -(3.6\ \Omega)(1 \text{ x } 10^{-6}\text{F})\ln\left(\frac{0.1V_o}{V_o}\right) = \boxed{8.29 \text{ x } 10^{-6}\text{s}}$$