1.) $m = 1.81 \text{ x } 10^{-3}$ kg, $q = 1.22 \text{ x } 10^{-8}$ C, $\vec{v} = \left(3.00 \text{ x } 10^{5} \text{ m/s}\right)\hat{j}$, and $\vec{B} = -\left(0.815 \text{ T}\right)\hat{i}$

The magnetic force is: $\vec{F} = q\vec{v} \times \vec{B} = \left(1.22 \text{ x } 10^{-8}\text{C}\right)\left(3.00 \text{ x } 10^{5} \text{ m/s}\right)\hat{j} \times -\left(0.815 \text{ T}\right)\hat{i}$

$$\vec{F} = -\left(2.98 \text{ x } 10^{-3}\text{N}\right)\left(\hat{j} \times \hat{i}\right) = -\left(2.98 \text{ x } 10^{-3}\text{N}\right)\left(-\hat{k}\right) = \left(2.98 \text{ x } 10^{-3}\text{N}\right)\hat{k}$$

$$\vec{F} = m\vec{a} \qquad \text{so} \qquad \vec{a} = \frac{\vec{F}}{m} = \frac{\left(2.98 \text{ x } 10^{-3}\text{N}\right)\hat{k}}{1.81 \text{ x } 10^{-3}\text{kg}} = \boxed{\left(1.65 \ \frac{\text{m}}{\text{s}^2}\right)\hat{k}}$$

2.) $q = -2.48 \text{ x } 10^{-8}$ C and $\vec{v} = \left(-3.85 \text{ x } 10^{4} \text{ m/s}\right)\hat{i} + \left(4.19 \text{ x } 10^{4} \text{ m/s}\right)\hat{j}$

a.) $\vec{B} = \left(1.40 \text{ T}\right)\hat{i}$

$$\vec{F} = q\vec{v} \times \vec{B} = \left(-2.48 \text{ x } 10^{-8}\text{C}\right)\left(\left(-3.85 \text{ x } 10^{4} \text{ m/s}\right)\hat{i} + \left(4.19 \text{ x } 10^{4} \text{ m/s}\right)\hat{j}\right) \times \left(1.40 \text{ T}\right)\hat{i}$$

$$\vec{F} = q\vec{v} \times \vec{B} = \left(1.34 \text{ x } 10^{-3}\text{N}\right)\left(\hat{i} \times \hat{i}\right) + \left(1.45 \text{ x } 10^{-3}\text{N}\right)\left(-\hat{j} \times \hat{i}\right) = \boxed{\left(1.45 \text{ x } 10^{-3}\text{N}\right)\hat{k}}$$

b.) $\vec{B} = \left(1.40 \text{ T}\right)\hat{k}$

$$\vec{F} = q\vec{v} \times \vec{B} = \left(-2.48 \text{ x } 10^{-8}\text{C}\right)\left(\left(-3.85 \text{ x } 10^{4} \text{ m/s}\right)\hat{i} + \left(4.19 \text{ x } 10^{4} \text{ m/s}\right)\hat{j}\right) \times \left(1.40 \text{ T}\right)\hat{k}$$

$$\vec{F} = \left(1.34 \text{ x } 10^{-3}\text{N}\right)\left(\hat{i} \times \hat{k}\right) + \left(1.45 \text{ x } 10^{-3}\text{N}\right)\left(-\hat{j} \times \hat{k}\right) = -\left(1.34 \text{ x } 10^{-3}\text{N}\right)\hat{j} - \left(1.45 \text{ x } 10^{-3}\text{N}\right)\hat{i}$$

$$\vec{F} = \boxed{-\left(1.45 \text{ x } 10^{-3}\text{N}\right)\hat{i} - \left(1.34 \text{ x } 10^{-3}\text{N}\right)\hat{j}}$$

3.) $F = 4.60 \text{ x } 10^{-15}$ N, $B = 3.50 \text{ x } 10^{-3}$ T, $\theta = 40°$, and $q = -1.60 \text{ x } 10^{-19}$ C

$$\vec{F} = q\vec{v} \times \vec{B} = qvB\sin\theta \qquad \text{so} \qquad v = \frac{F}{qB\sin\theta} = \frac{4.6 \text{ x } 10^{-15}\text{N}}{\left(1.6 \text{ x } 10^{-19}\text{C}\right)\left(3.5 \text{ x } 10^{-3}\text{T}\right)\sin\left(40°\right)} = \boxed{1.28 \text{ x } 10^{7} \ \frac{\text{m}}{\text{s}}}$$

4.) circular area $R = 0.374$ m in the x-y plane

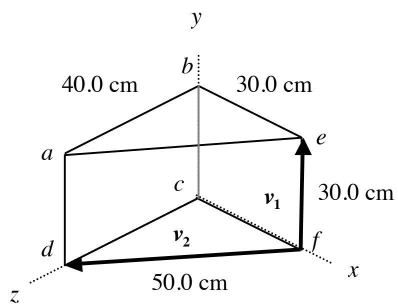
a.) $\vec{B} = \left(1.16 \text{ T}\right)\hat{k}$

$$\phi_m = \vec{B} \cdot \vec{A} = \left(1.16 \text{ T}\right)\hat{k} \cdot \left(\pi\left(0.374 \text{ m}\right)^2\right)\hat{k} = \boxed{0.510 \text{ Wb}}$$

b.) $\vec{B} = \left(1.16 \text{ T}\right)\hat{j}$

$$\phi_m = \vec{B} \cdot \vec{A} = \left(1.16 \text{ T}\right)\hat{j} \cdot \left(\pi\left(0.374 \text{ m}\right)^2\right)\hat{k} = \boxed{0}$$

5.)



$$\vec{B} = (0.385 \text{ T})\hat{i}$$

a.) surface *abcd* $\quad \vec{A}_{abcd} = (0.40 \text{ m})(0.30 \text{ m})(-\hat{i}) = -(0.12 \text{ m}^2)\hat{i}$

$$\phi_m = \vec{B}\cdot\vec{A} = (0.385 \text{ T})\hat{i}\cdot -(0.12 \text{ m}^2)\hat{i} = \ -0.0462 \text{ Wb}$$

b.) surface *befc* $\quad \vec{A}_{befc} = (0.30 \text{ m})(0.30 \text{ m})(-\hat{k}) = -(0.09 \text{ m}^2)\hat{k}$

$$\phi_m = \vec{B}\cdot\vec{A} = (0.385 \text{ T})\hat{i}\cdot -(0.09 \text{ m}^2)\hat{k} = \ 0$$

c.) surface *aefd* $\quad \vec{A}_{aefd} = (0.30 \text{ m})(0.50 \text{ m})\hat{a} = (0.15 \text{ m}^2)\hat{a}$

The direction is the same as the cross-product of two vectors that lie in the plane of the surface.

$$\vec{v}_1 = (0.30 \text{ m})\hat{j} \quad \text{and} \quad \vec{v}_2 = -(0.30 \text{ m})\hat{i} + (0.40 \text{ m})\hat{k}$$

$$\hat{a} = \frac{(\vec{v}_1\times\vec{v}_2)}{|\vec{v}_1\times\vec{v}_2|} \quad \text{and} \quad \vec{v}_1\times\vec{v}_2 = (0.30 \text{ m})\hat{j}\times -(0.30 \text{ m})\hat{i} + (0.40 \text{ m})\hat{k} = -(0.09 \text{ m}^2)(\hat{j}\times\hat{i}) + (0.12 \text{ m}^2)(\hat{j}\times\hat{k})$$

$$\vec{v}_1\times\vec{v}_2 = -(0.09 \text{ m}^2)(-\hat{k}) + (0.12 \text{ m}^2)(\hat{i}) = (0.12 \text{ m}^2)\hat{i} + (0.09 \text{ m}^2)\hat{k}$$

$$|\vec{v}_1\times\vec{v}_2| = \sqrt{(0.12 \text{ m}^2)^2 + (0.09 \text{ m}^2)^2} = 0.15 \text{ m}^2$$

$$\hat{a} = \frac{(\vec{v}_1\times\vec{v}_2)}{|\vec{v}_1\times\vec{v}_2|} = \frac{(0.12 \text{ m}^2)\hat{i} + (0.09 \text{ m}^2)\hat{k}}{0.15 \text{ m}^2} = 0.8\hat{i} + 0.6\hat{k}$$

$$\phi_m = \vec{B}\cdot\vec{A} = (0.385 \text{ T})\hat{i}\cdot(0.15 \text{ m}^2)\hat{a} = (0.385 \text{ T})\hat{i}\cdot(0.15 \text{ m}^2)(0.8\hat{i} + 0.6\hat{k}) = \ 0.0462 \text{ Wb}$$

d.) No flux passes through surfaces *abe* and *dcf* so the net flux is:

$$\phi_{net} = \phi_{abcd} + \phi_{befc} + \phi_{aefd} + \phi_{abe} + \phi_{dcf}$$

$$\phi_{net} = -0.0462 \text{ Wb} + 0 + 0.0462 \text{ Wb} + 0 + 0 = \ 0$$

This agrees with Gauss's Law for Magnetism which states that: $\oint \vec{B}\cdot d\vec{A} = 0$

6.) $q = 4.80 \text{ x } 10^{-19}$ C travels in circular orbit $R = 0.468$ m due to force from $B = 1.65$ T perpendicular to its orbit

a.)

$$\vec{F} = q\vec{v}\times\vec{B} = m\vec{a} = m\frac{v^2}{R}\hat{a} \quad \text{since } v \text{ and } B \text{ are perpendicular} \quad qvB = m\frac{v^2}{R} \quad \text{and} \quad qBR = mv = p$$

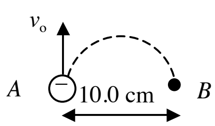
$$p = qBR = \left(4.8 \text{ x } 10^{-19}\text{C}\right)\left(1.65 \text{ T}\right)\left(0.468 \text{ m}\right) = \boxed{3.71 \text{ x } 10^{-19}\text{kg}\cdot\frac{\text{m}}{\text{s}}}$$

check units: $$p = qBR\,[=]\,(\text{C})(\text{T})(\text{m})\,[=]\,(\text{C})\left(\frac{\text{N}}{\text{A}\cdot\text{m}}\right)(\text{m})\,[=]\,(\text{C})\left(\frac{\text{kg}\frac{\text{m}}{\text{s}^2}}{\frac{\text{C}}{\text{s}}}\right)[=]\,\text{kg}\cdot\frac{\text{m}}{\text{s}}$$

b.)

$$\vec{L} = \vec{r}\times\vec{p} \quad \text{so} \quad L = R\,p = \left(0.468 \text{ m}\right)\left(3.71 \text{ x } 10^{-19}\text{kg}\cdot\frac{\text{m}}{\text{s}}\right) = \boxed{1.73 \text{ x } 10^{-19}\text{kg}\cdot\frac{\text{m}^2}{\text{s}}}$$

7.)



$$\vec{v}_o = \left(2.94 \text{ x } 10^6\,\frac{\text{m}}{\text{s}}\right)\hat{j} \text{ and } R = 0.05 \text{ m}$$

for an electron $q = -1.60 \text{ x } 10^{-19}$ C and $m_e = 9.11 \text{ x } 10^{-31}$ kg

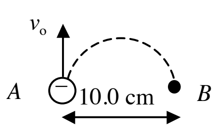
$$\vec{F} = q\vec{v}\times\vec{B} = m\vec{a} = m\frac{v^2}{R}\hat{a} \quad \text{and} \quad qvB = m\frac{v^2}{R}$$

The magnitude of B is: $$B = \frac{mv}{qR} = \frac{\left(9.11 \text{ x } 10^{-31}\text{kg}\right)\left(2.94 \text{ x } 10^6\,\frac{\text{m}}{\text{s}}\right)}{\left(1.60 \text{ x } 10^{-19}\text{C}\right)\left(0.05 \text{ m}\right)} = 3.35 \text{ x } 10^{-4}\,\text{T}$$

The direction of B is determined using Right-Hand-Rule. At point A the force is to the right and the velocity is upward. So the magnetic field must point outward for positive charges and inward for negative charges.

Therefore: $$\vec{B} = \boxed{-\left(3.35 \text{ x } 10^{-4}\,\text{T}\right)\hat{k}}$$

8.)



$$\vec{v}_o = \left(2.94 \text{ x } 10^6\,\frac{\text{m}}{\text{s}}\right)\hat{j} \text{ and } R = 0.05 \text{ m}$$

for a proton $q = 1.60 \text{ x } 10^{-19}$ C and $m_p = 1.67 \text{ x } 10^{-27}$ kg

$$\vec{F} = q\vec{v}\times\vec{B} = m\vec{a} = m\frac{v^2}{R}\hat{a} \quad \text{and} \quad qvB = m\frac{v^2}{R}$$

The magnitude of B is: $$B = \frac{mv}{qR} = \frac{\left(1.67 \text{ x } 10^{-27}\text{kg}\right)\left(2.94 \text{ x } 10^6\,\frac{\text{m}}{\text{s}}\right)}{\left(1.60 \text{ x } 10^{-19}\text{C}\right)\left(0.05 \text{ m}\right)} = 0.614 \text{ T}$$

The direction of B is determined using Right-Hand-Rule. At point A the force is to the right and the velocity is upward. So the magnetic field must point outward for positive charges and inward for negative charges.

Therefore: $$\vec{B} = \boxed{\left(0.614 \text{ T}\right)\hat{k}}$$

9.) $m = 1.16 \text{ x } 10^{-26}$ kg, $q = 1.60 \text{ x } 10^{-19}$ C, $\Delta V = 450$ V, $B = 0.723$ T (perpendicular to v)

Use energy conservation to get the speed of the particle when it enters the magnetic field.

$$\Delta K = \Delta U = q\Delta V \quad \text{so} \quad \frac{1}{2}mv^2 = q\Delta V \quad \text{and} \quad v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6 \text{ x } 10^{-19}\text{C})(450 \text{ V})}{(1.16 \text{ x } 10^{-26}\text{kg})}} = 1.11 \text{ x } 10^5 \frac{\text{m}}{\text{s}}$$

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a} = m\frac{v^2}{R}\hat{a} \quad \text{so} \quad qvB = m\frac{v^2}{R} \quad \text{and} \quad R = \frac{mv}{qB}$$

$$R = \frac{mv}{qB} = \frac{(1.16 \text{ x } 10^{-26}\text{kg})\left(1.11 \text{ x } 10^5 \frac{\text{m}}{\text{s}}\right)}{(1.6 \text{ x } 10^{-19}\text{C})(0.723 \text{ T})} = \boxed{0.011 \text{ m}}$$

$$\text{check units:} \quad R = \frac{mv}{qB} [=] \frac{(\text{kg})\left(\frac{\text{m}}{\text{s}}\right)}{(\text{C})(\text{T})} [=] \frac{(\text{kg})\left(\frac{\text{m}}{\text{s}}\right)}{(\text{C})\left(\frac{\text{N}}{\text{A} \cdot \text{m}}\right)} [=] \frac{(\text{kg})\left(\frac{\text{m}}{\text{s}}\right)}{(\text{C})\left(\frac{\text{kg}\frac{\text{m}}{\text{s}^2}}{\frac{\text{C}}{\text{s}} \cdot \text{m}}\right)} [=] \text{ m}$$

10.) $m = 9.11 \text{ x } 10^{-31}$ kg, $q = -1.60 \text{ x } 10^{-19}$ C, $\Delta V = 20{,}000$ V

Use energy conservation to get the speed of the electron when it enters the magnetic field.

$$\Delta K = \Delta U = q\Delta V \quad \text{so} \quad \frac{1}{2}mv^2 = q\Delta V \quad \text{and} \quad v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6 \text{ x } 10^{-19}\text{C})(20{,}000 \text{ V})}{(9.11 \text{ x } 10^{-31}\text{kg})}} = 8.38 \text{ x } 10^7 \frac{\text{m}}{\text{s}}$$

Electron enters magnetic field in a circular arc $R = 0.13$ m.

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a} = m\frac{v^2}{R}\hat{a} \quad \text{so} \quad qvB = m\frac{v^2}{R} \quad \text{and} \quad B = \frac{mv}{qR}$$

$$B = \frac{mv}{qR} = \frac{(9.11 \text{ x } 10^{-31}\text{kg})\left(8.38 \text{ x } 10^7 \frac{\text{m}}{\text{s}}\right)}{(1.6 \text{ x } 10^{-19}\text{C})(0.13 \text{ m})} = \boxed{3.67 \text{ x } 10^{-3}\text{T}}$$

11.) $m = 9.11 \text{ x } 10^{-31}$ kg, $q = -1.60 \text{ x } 10^{-19}$ C, $\vec{v} = \left(40 \ \frac{\text{km}}{\text{s}}\right)\hat{i} + \left(35 \ \frac{\text{km}}{\text{s}}\right)\hat{j}$, $\vec{F} = -\left(4.2 \text{ fN}\right)\hat{i} + \left(4.8 \text{ fN}\right)\hat{j}$

$B_x = 0$

$$\vec{F} = q\vec{v} \times \vec{B} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} \quad \text{so} \quad \vec{F} = -\left(4.2 \text{ fN}\right)\hat{i} + \left(4.8 \text{ fN}\right)\hat{j} = \left(-1.6 \text{ x } 10^{-19}\text{C}\right)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 40\frac{\text{km}}{\text{s}} & 35\frac{\text{km}}{\text{s}} & 0 \\ 0 & B_y & B_z \end{vmatrix}$$

Expanding determinant:

$$\vec{F} = -\left(4.2 \text{ fN}\right)\hat{i} + \left(4.8 \text{ fN}\right)\hat{j} = \left(-1.6 \text{ x } 10^{-19}\text{C}\right)\left(\hat{i}\begin{vmatrix} 35\frac{\text{km}}{\text{s}} & 0 \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} 40\frac{\text{km}}{\text{s}} & 0 \\ 0 & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} 40\frac{\text{km}}{\text{s}} & 35\frac{\text{km}}{\text{s}} \\ 0 & B_y \end{vmatrix}\right)$$

$$-\left(4.2 \text{ fN}\right)\hat{i} = \left(-1.6 \text{ x } 10^{-19}\text{C}\right)\left(\hat{i}\begin{vmatrix} 35\frac{\text{km}}{\text{s}} & 0 \\ B_y & B_z \end{vmatrix}\right) = \left(-1.6 \text{ x } 10^{-19}\text{C}\right)\left(35\frac{\text{km}}{\text{s}}B_z - 0\right)\hat{i}$$

$$B_z = \frac{-\left(4.2 \text{ x } 10^{-15}\text{N}\right)}{\left(-1.6 \text{ x } 10^{-19}\text{C}\right)\left(35 \text{ x } 10^3 \ \frac{\text{m}}{\text{s}}\right)} = 0.75 \text{ T}$$

$$\left(4.8 \text{ fN}\right)\hat{j} = \left(-1.6 \text{ x } 10^{-19}\text{C}\right)\left(-\hat{j}\begin{vmatrix} 35\frac{\text{km}}{\text{s}} & 0 \\ B_y & B_z \end{vmatrix}\right) = \left(1.6 \text{ x } 10^{-19}\text{C}\right)\left(35\frac{\text{km}}{\text{s}}B_z - 0\right)\hat{j}$$

$$B_z = \frac{\left(4.8 \text{ x } 10^{-15}\text{N}\right)}{\left(1.6 \text{ x } 10^{-19}\text{C}\right)\left(40 \text{ x } 10^3 \ \frac{\text{m}}{\text{s}}\right)} = 0.75 \text{ T}$$

$$0 = \left(-1.6 \text{ x } 10^{-19}\text{C}\right)\left(\hat{k}\begin{vmatrix} 40\frac{\text{km}}{\text{s}} & 35\frac{\text{km}}{\text{s}} \\ 0 & B_y \end{vmatrix}\right) = \left(-1.6 \text{ x } 10^{-19}\text{C}\right)\left(40\frac{\text{km}}{\text{s}}B_y - 0\right) \quad \text{so} \quad B_y = 0$$

Therefore: $\boxed{\vec{B} = \left(0.75 \text{ T}\right)\hat{k}}$

1.) $\ell = 0.200\text{ m}, B_\perp = 0.087\text{ T}, F = 0.22\text{ N}$

$$\vec{F} = I\vec{\ell}\times\vec{B} = I\ell B_\perp \quad \text{so} \quad I = \frac{F}{\ell B_\perp} = \frac{0.22\text{ N}}{(0.2\text{ m})(0.087\text{ T})} = \boxed{13\text{ A}}$$

2.) $I = 7.00\text{ A}, \ \vec{\ell} = (0.01\text{ m})\hat{i}$

a.) $\vec{B} = -(0.65\text{ T})\hat{j}$

$$\vec{F} = I\vec{\ell}\times\vec{B} = (7\text{ A})\left((0.01\text{ m})\hat{i}\times -(0.65\text{ T})\hat{j}\right) = -(0.0455\text{ N})\left(\hat{i}\times\hat{j}\right) = \boxed{-(0.0455\text{ N})\hat{k}}$$

b.) $\vec{B} = (0.56\text{ T})\hat{k}$

$$\vec{F} = I\vec{\ell}\times\vec{B} = (7\text{ A})\left((0.01\text{ m})\hat{i}\times(0.56\text{ T})\hat{k}\right) = (0.0392\text{ N})\left(\hat{i}\times\hat{k}\right) = \boxed{-(0.0392\text{ N})\hat{j}}$$

c.) $\vec{B} = -(0.31\text{ T})\hat{i}$

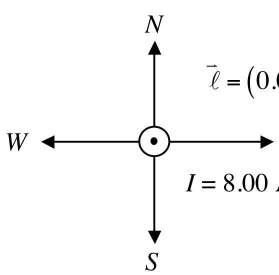
$$\vec{F} = I\vec{\ell}\times\vec{B} = (7\text{ A})\left((0.01\text{ m})\hat{i}\times -(0.31\text{ T})\hat{i}\right) = -(0.0217\text{ N})\left(\hat{i}\times\hat{i}\right) = \boxed{0}$$

d.) $\vec{B} = (0.74\text{ T})\hat{j} - (0.36\text{ T})\hat{k}$

$$\vec{F} = I\vec{\ell}\times\vec{B} = (7\text{ A})\left((0.01\text{ m})\hat{i}\times\left((0.74\text{ T})\hat{j} - (0.36\text{ T})\hat{k}\right)\right) = (0.0518\text{ N})\left(\hat{i}\times\hat{j}\right) - (0.0252\text{ N})\left(\hat{i}\times\hat{k}\right)$$

$$\vec{F} = (0.0518\text{ N})\hat{k} + (0.0252\text{ N})\hat{j} = \boxed{(0.0252\text{ N})\hat{j} + (0.0518\text{ N})\hat{k}}$$

3.)



$\vec{\ell} = (0.01\text{ m})\hat{k}$

$I = 8.00$ A (out of the page)

a.) $B = 6.72\text{ T, east} \qquad \vec{B} = (6.72\text{ T})\hat{i}$

$$\vec{F} = I\vec{\ell}\times\vec{B} = (8\text{ A})\left((0.01\text{ m})\hat{k}\times(6.72\text{ T})\hat{i}\right)$$

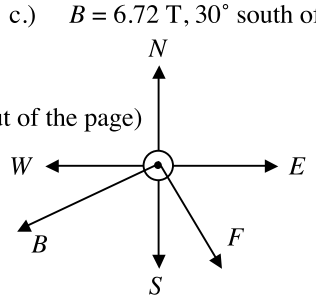
$$\vec{F} = (0.538\text{ N})\left(\hat{k}\times\hat{i}\right) = \boxed{(0.538\text{ N})\hat{j}\text{ (north)}}$$

b.) $B = 6.72\text{ T, south} \qquad \vec{B} = -(6.72\text{ T})\hat{j}$

$$\vec{F} = I\vec{\ell}\times\vec{B} = (8\text{ A})\left((0.01\text{ m})\hat{k}\times -(6.72\text{ T})\hat{j}\right) = -(0.538\text{ N})\left(\hat{k}\times\hat{j}\right) = \boxed{(0.538\text{ N})\hat{i}\text{ (east)}}$$

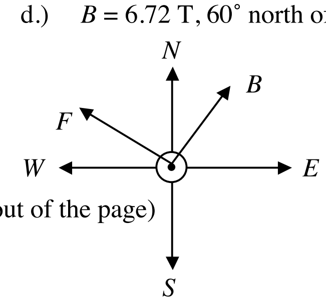
c.) $B = 6.72\text{ T}, 30^\circ$ south of west

$I = 8.00$ A (out of the page)



$F = \boxed{0.538\text{ N }(60^\circ\text{ south of east})}$

d.) $B = 6.72\text{ T}, 60^\circ$ north of east



$I = 8.00$ A (out of the page)

$F = \boxed{0.538\text{ N }(60^\circ\text{ west of north})}$

4.) $D = 6.5$ cm, $N = 12, I = 2.7$ A, $B = 0.56$ T

a.) Maximum torque occurs when magnetic field is perpendicular to the axis of the coils.

$$\tau = NIBA\sin\phi = 12(2.7\text{ A})(0.56\text{ T})\left(\pi\left(3.25 \text{ x } 10^{-2}\text{ m}\right)^2\right)\sin(90°) = \boxed{0.0602\text{ N}\cdot\text{m}}$$

b.) Occurs when $\sin\phi = 0.5$ or when $\phi = \boxed{30°}$

5.) 5 cm x 12 cm rectangular coil, $N = 600, I = 0.0613$ A, $B = 0.267$ T

$$\tau = NIBA\sin\phi = 600(0.0613\text{ A})(0.267\text{ T})\left((0.05\text{ m})(0.12\text{ m})\right)\sin(90°) = \boxed{0.0589\text{ N}\cdot\text{m}}$$

6.) $q = 4.97$ nC, when $\vec{v}_1 = 3.57 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\angle 45°$ force F_1 is in the $-\hat{k}$ direction

when $\vec{v}_2 = \left(1.62 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\right)\hat{k}$ force is $F_2 = 4.00 \text{ x } 10^{-5}$ N along the x-axis

$$\vec{v}_1 = 3.57 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\angle 45° = \left(3.57 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\right)\cos(45°)\hat{i} + \left(3.57 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\right)\sin(45°)\hat{j} = \left(2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\right)\hat{i} + \left(2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\right)\hat{j}$$

$$\vec{F} = q\vec{v}\times\vec{B} = q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ v_x & v_y & v_z\\ B_x & B_y & B_z\end{vmatrix} \quad \text{so} \quad \vec{F}_1 = -F_z\hat{k} = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}} & 2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}} & 0\\ B_x & B_y & B_z\end{vmatrix}$$

$$-F_z\hat{k} = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\left(\begin{vmatrix}2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}} & 0\\ B_y & B_z\end{vmatrix}\hat{i} - \begin{vmatrix}2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}} & 0\\ B_x & B_z\end{vmatrix}\hat{j} + \begin{vmatrix}2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}} & 2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\\ B_x & B_y\end{vmatrix}\hat{k}\right)$$

F_1 has no x or y-component which means first two terms are zero and $B_z = 0$

$$-F_z = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\begin{vmatrix}2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}} & 2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\\ B_x & B_z\end{vmatrix} = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\left(2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}B_y - 2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}B_x\right)$$

$$-F_z = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\begin{vmatrix}2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}} & 2.52 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\\ B_x & B_z\end{vmatrix} = \left(1.25 \text{ x } 10^{-4}\ \frac{\text{C}\cdot\text{m}}{\text{s}}\right)\left(B_y - B_x\right)$$

for this component to be negative $\left(B_y - B_x\right) < 0$

$$\vec{F}_2 = \pm F_x\hat{i} = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1.62 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\\ B_x & B_y & B_z\end{vmatrix}$$

$$\pm\left(4.00 \text{ x } 10^{-5}\text{N}\right)\hat{i} = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\left(\begin{vmatrix}0 & 1.62 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\\ B_y & B_z\end{vmatrix}\hat{i} - \begin{vmatrix}0 & 1.62 \text{ x } 10^4\ \frac{\text{m}}{\text{s}}\\ B_x & B_z\end{vmatrix}\hat{j} + \begin{vmatrix}0 & 0\\ B_x & B_y\end{vmatrix}\hat{k}\right)$$

F_2 has no y-component so middle term is zero and $B_x = 0$

For the x-component: $\pm\left(4.00 \text{ x } 10^{-5}\text{N}\right) = \left(4.97 \text{ x } 10^{-9}\text{C}\right)\left(0 - \left(1.62 \text{ x } 10^{4}\ \frac{\text{m}}{\text{s}}\right)B_y\right)$

and $$B_y = \pm\frac{\left(4.00 \text{ x } 10^{-5}\text{N}\right)}{\left(4.97 \text{ x } 10^{-9}\text{C}\right)\left(1.62 \text{ x } 10^{4}\ \frac{\text{m}}{\text{s}}\right)} = \pm 0.498 \text{ T}$$

since $\left(B_y - B_x\right) < 0$ and $B_x = 0$ it follows that $B_y = -0.498$ T

Therefore: $\vec{B} = \boxed{-\left(0.498 \text{ N}\right)\hat{j}}$

7.) $q = 35$ nC, $\vec{v} = -\left(5.89 \text{ x } 10^{5}\ \frac{\text{m}}{\text{s}}\right)\hat{i}$ in a uniform magnetic field with $B_x = 0.202$ T, $B_y = -0.522$ T, and $B_z = 0.322$ T

$$\vec{F} = q\vec{v}\times\vec{B} = q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ v_x & v_y & v_z\\ B_x & B_y & B_z\end{vmatrix} = \left(35 \text{ x } 10^{-9}\text{C}\right)\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -5.89 \text{ x } 10^{5}\ \frac{\text{m}}{\text{s}} & 0 & 0\\ 0.202 \text{ T} & -0.522 \text{ T} & 0.322 \text{ T}\end{vmatrix}$$

$$\vec{F} = \left(35 \text{ x } 10^{-9}\text{C}\right)\left(\begin{vmatrix}0 & 0\\ 0.202 \text{ T} & 0.322 \text{ T}\end{vmatrix}\hat{i} - \begin{vmatrix}-5.89 \text{ x } 10^{5}\ \frac{\text{m}}{\text{s}} & 0\\ 0.202 \text{ T} & 0.322 \text{ T}\end{vmatrix}\hat{j} + \begin{vmatrix}-5.89 \text{ x } 10^{5}\ \frac{\text{m}}{\text{s}} & 0\\ 0.202 \text{ T} & -0.522 \text{ T}\end{vmatrix}\hat{k}\right)$$

$$\vec{F} = \left(35 \text{ x } 10^{-9}\text{C}\right)\left(0\hat{i} - \left(-5.89 \text{ x } 10^{5}\ \frac{\text{m}}{\text{s}}\left(0.322 \text{ T}\right) - 0\right)\hat{j} + \left(-5.89 \text{ x } 10^{5}\ \frac{\text{m}}{\text{s}}\left(-0.522 \text{ T}\right) - 0\right)\hat{k}\right)$$

$\vec{F} = \left(0.00644 \text{ N}\right)\hat{j} + \left(0.0108 \text{ N}\right)\hat{k}$ and $\boxed{F_x = 0,\ F_y = 0.00664 \text{ N, and } F_z = 0.0108 \text{ N}}$

8.) $\ell = 0.200$ m, $I = 8.00$ A in the $+y$-direction in a uniform magnetic field with $B_x = 0.107$ T, $B_y = -1.13$ T, and $B_z = 0.538$ T

a.)

$$\vec{F} = I\vec{\ell}\times\vec{B} = I\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \ell_x & \ell_y & \ell_z\\ B_x & B_y & B_z\end{vmatrix} = \left(8 \text{ A}\right)\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0.2 \text{ m} & 0\\ 0.107 \text{ T} & -1.13 \text{ T} & 0.538 \text{ T}\end{vmatrix}$$

$$\vec{F} = \left(8 \text{ A}\right)\left(\begin{vmatrix}0.2 \text{ m} & 0\\ -1.13 \text{ T} & 0.538 \text{ T}\end{vmatrix}\hat{i} - \begin{vmatrix}0 & 0\\ 0.107 \text{ T} & 0.538 \text{ T}\end{vmatrix}\hat{j} + \begin{vmatrix}0 & 0.2 \text{ m}\\ 0.107 \text{ T} & -1.13 \text{ T}\end{vmatrix}\hat{k}\right)$$

$$\vec{F} = \left(8 \text{ A}\right)\left(\left(0.2 \text{ m}\left(0.538 \text{ T}\right) - 0\right)\hat{i} - 0\hat{j} + \left(0 - 0.2 \text{ m}\left(0.107 \text{ T}\right)\right)\hat{k}\right)$$

$\vec{F} = \left(0.6456 \text{ N}\right)\hat{i} - \left(0.1284 \text{ N}\right)\hat{k}$ and $\boxed{F_x = 0.6456 \text{ N},\ F_y = 0, \text{ and } F_z = -0.1284 \text{ N}}$

b.)

$$F = \sqrt{F_x^{\,2} + F_z^{\,2}} = \sqrt{\left(0.6456 \text{ N}\right)^2 + \left(-0.1284 \text{ N}\right)^2} = \boxed{0.658 \text{ N}}$$

$$\theta = \tan^{-1}\frac{F_z}{F_x} = \tan^{-1}\frac{-0.1284 \text{ N}}{0.6456 \text{ N}} = \boxed{-11.25°}$$

9.) $q = \text{-}1.6 \text{ x } 10^{-19}\text{ C}$, $v_1 = 0$, $\Delta V = 350\text{ V}$, $B_\perp = 200\text{ mT}$

a.)

Conservation of Energy

$$K_1 + U_1 = K_2 + U_2 \quad \text{so} \quad K_2 = -\Delta U = -q\Delta V \quad \text{or} \quad \frac{1}{2}m{v_2}^2 = q\Delta V$$

$$\text{Therefore:} \quad v_2 = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(\text{-}1.6 \text{ x } 10^{-19}\text{C})(\text{-}350\text{ V})}{(9.11 \text{ x } 10^{-31}\text{kg})}} = \boxed{1.11 \text{ x } 10^{7}\ \frac{\text{m}}{\text{s}}}$$

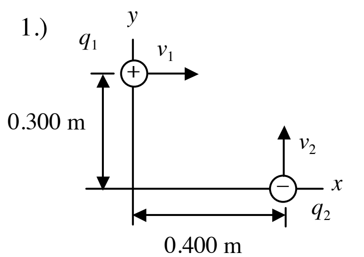
b.)

$$F = qvB_\perp = ma = m\frac{v^2}{R} \quad \text{so} \quad R = \frac{mv}{qB_\perp} = \frac{(9.11 \text{ x } 10^{-31}\text{kg})\left(1.11 \text{ x } 10^{7}\ \frac{\text{m}}{\text{s}}\right)}{(1.6 \text{ x } 10^{-19}\text{C})(200 \text{ x } 10^{-3}\text{T})} = \boxed{3.16 \text{ x } 10^{-4}\text{ m}}$$

10.) $\ell = 1.80\text{ m}$, $I = 13.0\text{ A}$, $\theta = 35^\circ$, $B = 1.50\text{ T}$

$$\vec{F} = I\vec{\ell} \times \vec{B} \quad \text{so} \quad F = I\ell B\sin\theta = (13\text{ A})(1.8\text{ m})(1.5\text{ T})\sin(35^\circ) = \boxed{20.1\text{ N}}$$

1.)



$q_1 = 5.00\ \mu\text{C}$ $\quad$ $q_2 = -3.00\ \mu\text{C}$

$r_1 = 0.300\text{ m}$ $\quad$ $r_2 = 0.400\text{ m}$

$\hat{r}_1 = -\hat{j}$ $\quad$ $\hat{r}_2 = -\hat{i}$

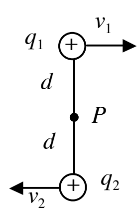
$\vec{v}_1 = \left(6.00 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\hat{i}$ $\quad$ $\vec{v}_2 = \left(8.00 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\hat{j}$

$$\vec{B}_1 = \frac{\mu_o q_1 \vec{v}_1 \times \hat{r}_1}{4\pi r_1^2} = \left(10^{-7}\ \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)\frac{\left(5 \text{ x } 10^{-6}\text{C}\right)\left(6 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\hat{i} \times \left(-\hat{j}\right)}{\left(0.3\text{ m}\right)^2} = -\left(3.33 \text{ x } 10^{-6}\text{T}\right)\hat{k}$$

$$\vec{B}_2 = \frac{\mu_o q_1 \vec{v}_2 \times \hat{r}_2}{4\pi r_2^2} = \left(10^{-7}\ \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)\frac{\left(-3 \text{ x } 10^{-6}\text{C}\right)\left(8 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\hat{j} \times \left(-\hat{i}\right)}{\left(0.4\text{ m}\right)^2} = -\left(1.5 \text{ x } 10^{-6}\text{T}\right)\hat{k}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \boxed{-\left(4.83 \text{ x } 10^{-6}\text{T}\right)\hat{k}}$$

2.)



$q_1 = 4.00\ \mu\text{C}$ $\quad$ $q_2 = 6.00\ \mu\text{C}$

$r_1 = d = 0.150\text{ m}$ $\quad$ $r_2 = d = 0.150\text{ m}$

$\hat{r}_1 = -\hat{j}$ $\quad$ $\hat{r}_2 = \hat{j}$

$\vec{v}_1 = \left(7.50 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\hat{i}$ $\quad$ $\vec{v}_2 = -\left(2.50 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\hat{i}$

$$\vec{B}_1 = \frac{\mu_o q_1 \vec{v}_1 \times \hat{r}_1}{4\pi r_1^2} = \left(10^{-7}\ \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)\frac{\left(4 \text{ x } 10^{-6}\text{C}\right)\left(7.5 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\hat{i} \times \left(-\hat{j}\right)}{\left(0.15\text{ m}\right)^2} = -\left(1.33 \text{ x } 10^{-5}\text{T}\right)\hat{k}$$

$$\vec{B}_2 = \frac{\mu_o q_1 \vec{v}_2 \times \hat{r}_2}{4\pi r_2^2} = \left(10^{-7}\ \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)\frac{\left(6 \text{ x } 10^{-6}\text{C}\right)\left(2.5 \text{ x } 10^5\ \frac{\text{m}}{\text{s}}\right)\left(-\hat{i}\right) \times \left(\hat{j}\right)}{\left(0.15\text{ m}\right)^2} = -\left(6.66 \text{ x } 10^{-6}\text{T}\right)\hat{k}$$

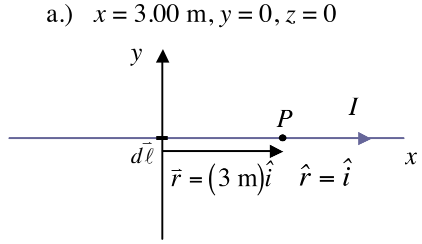
$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \boxed{-\left(2.00 \text{ x } 10^{-5}\text{T}\right)\hat{k}}$$

3.) long wire along x-axis with current $I = 8.00$ A in the x-direction

For small segments of wire the equation for an infinitesimal current element can be used with $d\vec{\ell} = \left(2.00\text{ mm}\right)\hat{i}$.

The magnetic field of a current element is: $\quad d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{\ell} \times \hat{r}}{r^2}$ $\quad \hat{r} = \hat{i}$

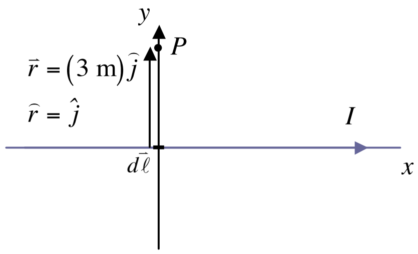
a.) $x = 3.00\text{ m}, y = 0, z = 0$



$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{\ell} \times \hat{r}}{r^2} = \left(10^{-7}\ \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)\frac{\left(8\text{ A}\right)\left(2 \text{ x } 10^{-3}\text{m}\right)\hat{i} \times \hat{i}}{\left(3\text{ m}\right)^2} = \boxed{0}$$

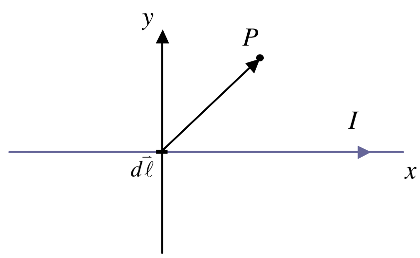
3.) (cont'd)

b.) $x = 0, y = 3.00, z = 0$



$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{4\pi}\frac{(8\text{ A})(2\text{ x }10^{-3}\text{m})\hat{i}\times\hat{j}}{(3\text{ m})^2} = \boxed{(1.78\text{ x }10^{-10}\text{T})\hat{k}}$$

c.) $x = 3.00\text{ m}, y = 3.00, z = 0$



$$\vec{r} = (3\text{ m})\hat{i} + (3\text{ m})\hat{j}$$
$$r = \sqrt{(3\text{ m})^2 + (3\text{ m})^2} = \sqrt{18}\text{ m}$$
$$\hat{r} = \frac{\vec{r}}{r} = \frac{(3\text{ m})\hat{i} + (3\text{ m})\hat{j}}{\sqrt{18}\text{ m}} = (0.7071)\hat{i} + (0.7071)\hat{j}$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{4\pi}\frac{(8\text{ A})(2\text{ x }10^{-3}\text{m})\hat{i}\times\left((0.7071)\hat{i} + (0.7071)\hat{j}\right)}{(\sqrt{18}\text{ m})^2} = \boxed{(6.29\text{ x }10^{-11}\text{T})\hat{k}}$$

4.) $B = 7.50\text{ x }10^{-4}\text{ T}, r = 0.050\text{ m}$

a.)

for a long wire: $B = \frac{\mu_o}{2\pi}\frac{I}{r}$ so $I = \frac{2\pi rB}{\mu_o} = \frac{2\pi(0.05\text{ m})(7.5\text{ x }10^{-4}\text{T})}{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)} = \boxed{188\text{ A}}$

b.) $r = 0.100\text{ m}, I = 188\text{ A}$

$$B = \frac{\mu_o}{2\pi}\frac{I}{r} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(188\text{ A})}{(0.1\text{ m})} = \boxed{3.76\text{ x }10^{-4}\text{T}}$$

5.)

long wire $I = 900$ A from east to west $r = 5$ m above the ground

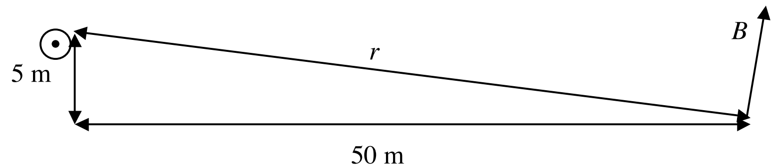
a.) directly under the wire $r = 5$ m

$$B = \frac{\mu_o}{2\pi}\frac{I}{r} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(900\text{ A})}{(5\text{ m})} = \boxed{3.6\text{ x }10^{-5}\text{T}}$$

Using Right-hand Rule the direction is south.

5.) (cont'd)

b.) walking away from the wire

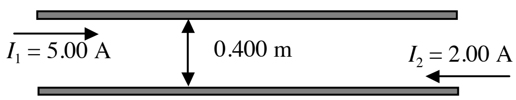


$$r = \sqrt{(50\text{ m})^2 + (5\text{ m})^2} = 50.25\text{ m}$$

$$B = \frac{\mu_o}{2\pi}\frac{I}{r} = \frac{\left(4\pi \text{ x } 10^{-7}\,\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(900\text{ A})}{(50.25\text{ m})} = \boxed{3.58 \text{ x } 10^{-6}\text{T}}$$

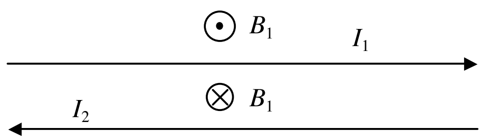
6.)

a.)



The magnetic field on wire 2 by wire 1 is: $B_1 = \frac{\mu_o}{2\pi}\frac{I_1}{r} = \frac{\left(4\pi \text{ x } 10^{-7}\,\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(5\text{ A})}{(0.4\text{ m})} = 2.5 \text{ x } 10^{-6}\text{T}$

Using Right-hand Rule the direction is into the page or $-\hat{k}$ below the wire.

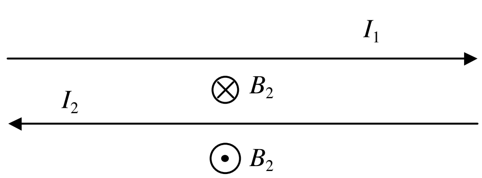


The magnetic force on wire 2 is: $\vec{F}_{21} = I_2\vec{\ell}_2 \times \vec{B}_1$ and the force on a 0.2 m length is:

$$\vec{F}_{21} = (2\text{ A})(0.2\text{ m})(-\hat{i}) \times (2.5 \text{ x } 10^{-6}\text{T})(-\hat{k}) = -(1.0 \text{ x } 10^{-6}\text{N})\hat{j}$$

The magnetic field on wire 1 by wire 2 is: $B_2 = \frac{\mu_o}{2\pi}\frac{I_2}{r} = \frac{\left(4\pi \text{ x } 10^{-7}\,\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(2\text{ A})}{(0.4\text{ m})} = 1.0 \text{ x } 10^{-6}\text{T}$

Using Right-hand Rule the direction is into the page or $-\hat{k}$ above the wire.



The magnetic force on wire 1 is: $\vec{F}_{12} = I_1\vec{\ell}_1 \times \vec{B}_2$ and the force on a 0.2 m length is:

$$\vec{F}_{12} = (5\text{ A})(0.2\text{ m})(\hat{i}) \times (1.0 \text{ x } 10^{-6}\text{T})(-\hat{k}) = (1.0 \text{ x } 10^{-6}\text{N})\hat{j}$$

$$\boxed{\text{So the wires are repelled by a force of } F = 1.0 \text{ x } 10^{-6}\text{N}}$$

6.) (cont'd)

b.) when I_1 = 15 A and I_2 = 6 A the force is still repulsive and on wire 2:

$$B_1 = \frac{\mu_o}{2\pi}\frac{I_1}{r} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(15\text{ A})}{(0.4\text{ m})} = 7.5 \text{ x } 10^{-6}\text{T}$$

$$\vec{F}_{21} = I_2\vec{\ell}_2 \times \vec{B}_1 = (6\text{ A})(0.2\text{ m})(-\hat{i}) \times (7.5 \text{ x } 10^{-6}\text{T})(-\hat{k}) = -(9.0 \text{ x } 10^{-6}\text{N})\hat{j}$$

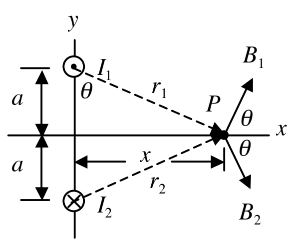
on wire 1:

$$B_2 = \frac{\mu_o}{2\pi}\frac{I_2}{r} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(6\text{ A})}{(0.4\text{ m})} = 3.0 \text{ x } 10^{-6}\text{T}$$

$$\vec{F}_{12} = I_1\vec{\ell}_1 \times \vec{B}_2 = (15\text{ A})(0.2\text{ m})(\hat{i}) \times (3.0 \text{ x } 10^{-6}\text{T})(-\hat{k}) = (9.0 \text{ x } 10^{-6}\text{N})\hat{j}$$

So the wires are repelled by a force of $F = 9.0 \text{ x } 10^{-6}\text{N}$

7.)



I = 9.00 A, a = 0.300 m, x = 0.400 m

at point P

$$B_2 = B_1 = \frac{\mu_o}{2\pi}\frac{I_1}{r_1} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(9\text{ A})}{(0.5\text{ m})} = 3.6 \text{ x } 10^{-6}\text{T}$$

y-components cancel and $B_{2_x} = B_{1_x} = B_1\cos\theta = (3.6 \text{ x } 10^{-6}\text{T})\frac{(0.3\text{ m})}{(0.5\text{ m})} = 2.16 \text{ x } 10^{-6}\text{T}$

$$B_P = B_{2_x} + B_{1_x} = 2(2.16 \text{ x } 10^{-6}\text{T}) = 4.32 \text{ x } 10^{-6}\text{T} \quad \text{and} \quad \vec{B}_P = (4.32 \text{ x } 10^{-6}\text{T})\hat{i}$$

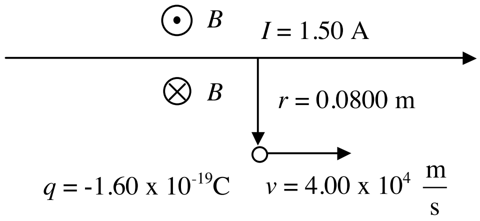
a.) I_P = 9.00 A into the page or $-\hat{k}$

$$\vec{F}_P = I_P\vec{\ell}_P \times \vec{B}_P \quad \text{and} \quad \frac{\vec{F}_P}{\ell_P} = I_P(-\hat{k}) \times \vec{B}_P = (9\text{ A})(-\hat{k}) \times (4.32 \text{ x } 10^{-6}\text{T})\hat{i} = \boxed{-\left(3.89 \text{ x } 10^{-5}\frac{\text{N}}{\text{m}}\right)\hat{j}}$$

b.) I_P = 9.00 A out of the page or $\hat{k}$

$$\vec{F}_P = I_P\vec{\ell}_P \times \vec{B}_P \quad \text{and} \quad \frac{\vec{F}_P}{\ell_P} = I_P(-\hat{k}) \times \vec{B}_P = (9\text{ A})\hat{k} \times (4.32 \text{ x } 10^{-6}\text{T})\hat{i} = \boxed{\left(3.89 \text{ x } 10^{-5}\frac{\text{N}}{\text{m}}\right)\hat{j}}$$

1.)



The magnetic field from the current in the wire is

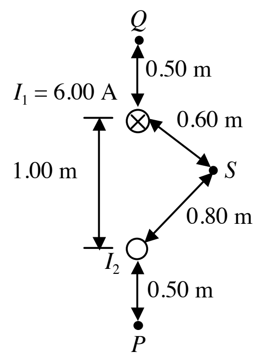
$$B = \frac{\mu_o}{2\pi}\frac{I}{r} = \frac{\left(4\pi \text{ x } 10^{-7} \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(1.5 \text{ A})}{(0.08 \text{ m})} = 3.75 \text{ x } 10^{-6}\text{T}$$

The magnetic field is into the page or in the $-\hat{k}$ direction.

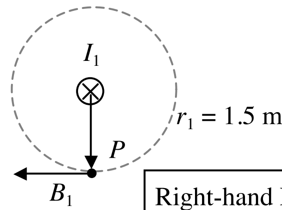
The force on the electron is $\vec{F} = q\vec{v}\times\vec{B} = \left(-1.60 \text{ x } 10^{-19}\text{C}\right)\left(4 \text{ x } 10^{4} \frac{\text{m}}{\text{s}}\right)\hat{i}\times\left(3.75 \text{ x } 10^{-6}\text{T}\right)\left(-\hat{k}\right) = \boxed{\left(-2.4 \text{ x } 10^{-20}\text{N}\right)\hat{j}}$

Note that this force is away from the wire and would be directed in the upward $\hat{j}$ direction if the electron were traveling above the wire.

2.)



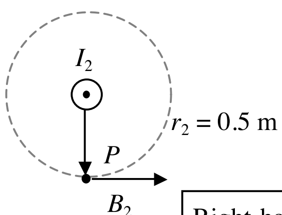
a.) At point P the magnetic field due to wire 1 is



$$B_1 = \frac{\mu_o}{2\pi}\frac{I_1}{r_1} = \frac{\left(4\pi \text{ x } 10^{-7} \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(6 \text{ A})}{(1.5 \text{ m})} = 8.0 \text{ x } 10^{-7}\text{T}$$

Right-hand Rule: Thumb *I* points into the page and fingers *B* wrap around clockwise.

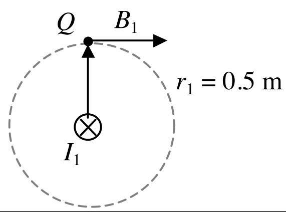
For the no net field at point P the current in wire 2 must be out of the page to create a magnetic field to the right and equal to B_1.



$$B_2 = \frac{\mu_o}{2\pi}\frac{I_2}{r_2} \quad \text{so} \quad I_2 = \frac{2\pi B_1 r_2}{\mu_o} = \frac{2\pi\left(8.0 \text{ x } 10^{-7}\text{T}\right)(0.5 \text{ m})}{\left(4\pi \text{ x } 10^{-7} \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)} = \boxed{2 \text{ A out of the page}}$$

Right-hand Rule: Thumb *I* points out of the page and fingers *B* wrap around counterclockwise.

b.)

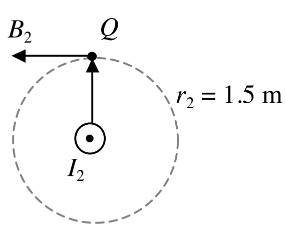


At point Q the magnetic field due to wire 1 is

$$B_1 = \frac{\mu_o}{2\pi}\frac{I_1}{r_1} = \frac{\left(4\pi \text{ x } 10^{-7} \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(6 \text{ A})}{(0.5 \text{ m})} = 2.4 \text{ x } 10^{-6}\text{T}$$

Right-hand Rule: Thumb *I* points into the page and fingers *B* wrap around clockwise.

At point Q the magnetic field due to wire 2 is



$$B_2 = \frac{\mu_o}{2\pi}\frac{I_2}{r_2} = \frac{\left(4\pi \text{ x } 10^{-7} \frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(2 \text{ A})}{(1.5 \text{ m})} = 2.67 \text{ x } 10^{-7}\text{T}$$

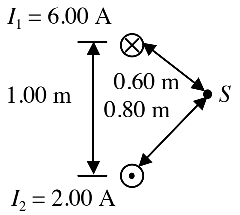
The total magnetic field at point Q is

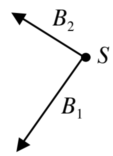
$$\vec{B}_Q = \vec{B}_1 + \vec{B}_2 = \left(2.4 \text{ x } 10^{-6}\text{T}\right)\hat{i} - \left(2.67 \text{ x } 10^{-7}\text{T}\right)\hat{i} = \boxed{\left(2.13 \text{ x } 10^{-6}\text{T}\right)\hat{i}}$$

Right-hand Rule: Thumb *I* points out of the page and fingers *B* wrap around counterclockwise.

2.) (cont'd)

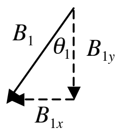
c.)

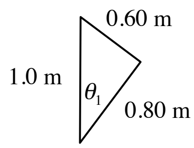




$$B_1 = \frac{\mu_o}{2\pi}\frac{I_1}{r_1} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(6\text{ A})}{(0.6\text{ m})} = 2.0 \text{ x } 10^{-6}\text{T}$$

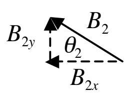
$$B_2 = \frac{\mu_o}{2\pi}\frac{I_2}{r_2} = \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(2\text{ A})}{(0.8\text{ m})} = 5.0 \text{ x } 10^{-7}\text{T}$$

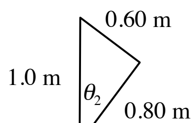




$$\cos\theta_1 = \frac{a}{h} = \frac{(0.8\text{ m})}{(1.0\text{ m})} = 0.8 \quad\text{and}\quad \sin\theta_1 = \frac{o}{h} = \frac{(0.6\text{ m})}{(1.0\text{ m})} = 0.6$$

$$B_{1x} = -B_1\sin\theta_1 = -\left(2.0 \text{ x } 10^{-6}\text{T}\right)(0.6) = -1.2 \text{ x } 10^{-6}\text{T} \quad\text{and}\quad B_{1y} = -B_1\cos\theta_1 = -\left(2.0 \text{ x } 10^{-6}\text{T}\right)(0.8) = -1.6 \text{ x } 10^{-6}\text{T}$$





$$\cos\theta_2 = \frac{a}{h} = \frac{(0.8\text{ m})}{(1.0\text{ m})} = 0.8 \quad\text{and}\quad \sin\theta_2 = \frac{o}{h} = \frac{(0.6\text{ m})}{(1.0\text{ m})} = 0.6$$

$$B_{2x} = -B_2\cos\theta_2 = -\left(5.0 \text{ x } 10^{-7}\text{T}\right)(0.8) = -4.0 \text{ x } 10^{-7}\text{T} \quad\text{and}\quad B_{2y} = B_2\sin\theta_2 = \left(5.0 \text{ x } 10^{-7}\text{T}\right)(0.6) = 3.0 \text{ x } 10^{-7}\text{T}$$

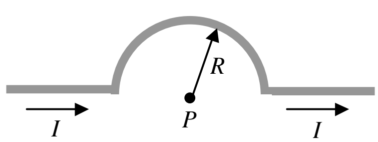
$$B_{Sx} = B_{1x} + B_{2x} = \left(-1.2 \text{ x } 10^{-6}\text{T}\right) + \left(-4.0 \text{ x } 10^{-7}\text{T}\right) = -1.6 \text{ x } 10^{-6}\text{T}$$
$$B_{Sy} = B_{1y} + B_{2y} = \left(-1.6 \text{ x } 10^{-6}\text{T}\right) + \left(3.0 \text{ x } 10^{-7}\text{T}\right) = -1.3 \text{ x } 10^{-6}\text{T}$$

$$B_S = \sqrt{B_{Sx}^2 + B_{Sy}^2} = \sqrt{\left(-1.6 \text{ x } 10^{-6}\text{T}\right)^2 + \left(-1.3 \text{ x } 10^{-6}\text{T}\right)^2} = 2.06 \text{ x } 10^{-6}\text{T}$$

$$\theta = \tan^{-1}\frac{B_{Sy}}{B_{Sx}} = \tan^{-1}\frac{\left(-1.3 \text{ x } 10^{-6}\text{T}\right)}{\left(-1.6 \text{ x } 10^{-6}\text{T}\right)} = 39.1° + 180° = 219.1°$$

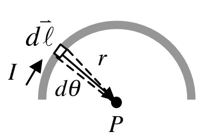
$$\boxed{\vec{B}_S = 2.06 \text{ x } 10^{-6}\text{T } \angle 219.1°}$$

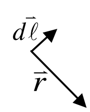
3.)



$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

For the straight sections of wire $d\vec{\ell} \times \hat{r} = 0$ and do not contribute to field at P.



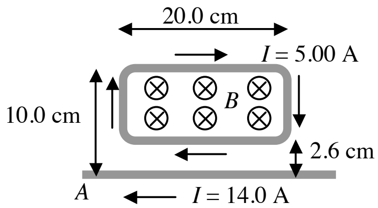


For circular section of the wire $d\vec{\ell}$ and $\vec{r}$ are always perpendicular and by the Right-hand Rule $d\vec{\ell} \times \hat{r}$ points into the plane of the page so $d\vec{\ell} \times \hat{r} = Rd\theta\left(-\hat{k}\right)$.

$$B_Z = \int_0^\pi \frac{\mu_o}{4\pi}\frac{IRd\theta}{R^2} = \frac{\mu_o I}{4\pi R}\theta\Big]_0^\pi = \frac{\mu_o I}{4\pi R}(\pi - 0) = \frac{\mu_o I}{4R}$$

so $\boxed{\vec{B} = \frac{\mu_o I}{4R}\left(-\hat{k}\right)}$

4.)



For left side of the loop is the current is going up $\hat{j}$ and the magnetic field points into the plane of the page $-\hat{k}$ so the force is to the left.

The direction of the force is determined by direction of $\vec{\ell} \times \vec{B} = \hat{j} \times \left(-\hat{k}\right) = -\hat{i}$.

For right side of the loop is the current is going down $-\hat{j}$ and the magnetic field points into the plane of the page $-\hat{k}$ so the force is to the right.

The direction of the force is determined by direction of $\vec{\ell} \times \vec{B} = \left(-\hat{j}\right) \times \left(-\hat{k}\right) = \hat{i}$.

These forces cancel each other out since they are the same distance from the wire.

The force on the top of the loop is

$$\vec{F} = I\vec{\ell} \times \vec{B} = I_{loop}\vec{\ell} \times \frac{\mu_o I_{wire}}{2\pi r}\hat{B} = (5\text{ A})(0.2\text{ m})\hat{i} \times \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(14\text{ A})}{(0.10\text{ m})}\left(-\hat{k}\right) = \left(2.8 \text{ x } 10^{-5}\text{N}\right)\hat{j}$$

The force on the bottom of the loop is

$$\vec{F} = I\vec{\ell} \times \vec{B} = I_{loop}\vec{\ell} \times \frac{\mu_o I_{wire}}{2\pi r}\hat{B} = (5\text{ A})(0.2\text{ m})\left(-\hat{i}\right) \times \frac{\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)}{2\pi}\frac{(14\text{ A})}{(0.026\text{ m})}\left(-\hat{k}\right) = -\left(1.08 \text{ x } 10^{-4}\text{N}\right)\hat{j}$$

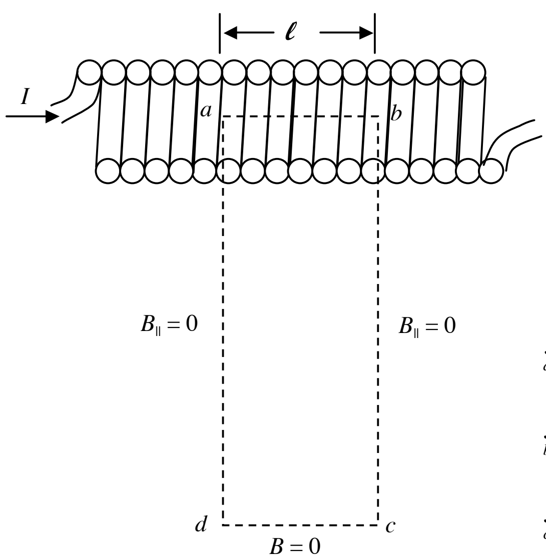
The total force on the loop is $\vec{F}_{top} + \vec{F}_{bottom} = \left(2.8 \text{ x } 10^{-5}\text{N}\right)\hat{j} - \left(1.08 \text{ x } 10^{-4}\text{N}\right)\hat{j} = \boxed{-\left(8.0 \text{ x } 10^{-5}\text{N}\right)\hat{j}}$

5.) Solenoid $N = 500$, $L = 20.0$ cm, $R = 3.00$ cm, and $I = 6.00$ A

Near the center of a solenoid $B_s = n\mu_o I$ where $n = \frac{\text{number of turns}}{\text{length}} = \frac{N}{L}$

Therefore $B_s = \frac{(500\text{ turns})}{(0.20\text{ m})}\left(4\pi \text{ x } 10^{-7}\frac{\text{N}\cdot\text{s}^2}{\text{C}^2}\right)\left(6\ \frac{\text{A}}{\text{turn}}\right) = 0.0189\text{ T}$

The formula for the magnetic field near the center of long solenoid can be found using Ampere's Law.



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

$$\int_{ab} \vec{B} \cdot d\vec{\ell} + \int_{bc} \vec{B} \cdot d\vec{\ell} + \int_{cd} \vec{B} \cdot d\vec{\ell} + \int_{da} \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

$$B\ell + 0 + 0 + 0 = \mu_o \ell n I$$

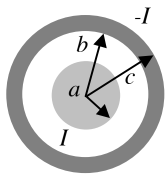
$$B = \mu_o n I$$

$\int_{ab} \vec{B} \cdot d\vec{\ell} = B\int d\ell = B\ell$ because B and $d\ell$ are parallel

$\int_{bc} \vec{B} \cdot d\vec{\ell} = 0$ and $\int_{da} \vec{B} \cdot d\vec{\ell} = 0$ because B and $d\ell$ are perpendicular

$\int_{cd} \vec{B} \cdot d\vec{\ell} = 0$ because $B = 0$ very far from the loop

6.)



a.) For inner conductor the current density is $J_{inner} = \frac{I}{A} = \frac{I}{\pi a^2}$

Using Ampere's Law and a circular path of radius $r < a$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

$$B 2\pi r = \mu_o J_{inner} A = \mu_o \frac{I}{\pi a^2} \pi r^2$$

$$\boxed{B = \frac{\mu_o I}{2\pi a^2} r \quad (r < a)}$$

b.) For $a < r < b$ the enclosed current is I. Using Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

$$B 2\pi r = \mu_o I$$

$$\boxed{B = \frac{\mu_o I}{2\pi r} \quad (a < r < b)}$$

c.) For the outer conductor the current density is $J = \frac{-I}{A} = \frac{-I}{\left(\pi c^2 - \pi b^2\right)}$

For $b < r < c$ the enclosed current includes that on the inner conductor and a portion of the current on the outer conductor. Using Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

$$B 2\pi r = \mu_o \left(I + J_{outer} A\right) = \mu_o \left(I + \frac{-I}{\left(\pi c^2 - \pi b^2\right)} \left(\pi r^2 - \pi b^2\right)\right) = \mu_o I \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$B = \frac{\mu_o I}{2\pi r} \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

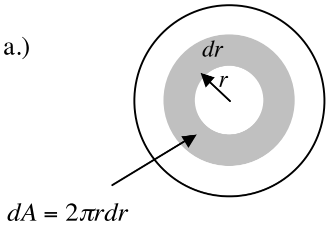
$$\boxed{B = \frac{\mu_o I}{2\pi r} \left(\frac{c^2 - r^2}{c^2 - b^2}\right) \quad (b < r < c)}$$

d.) When $r > c$ the enclosed current $I_{enclosed} = I - I = 0$ and using Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed} = 0 \quad \text{so} \quad \boxed{B = 0 \quad (c < r)}$$

7.) Long wire radius R and current I with $J = \alpha r$ where α is a constant.

a.)



$$I = \int J dA = \int_0^R \alpha r 2\pi r dr = 2\pi\alpha \int_0^R r^2 dr = 2\pi\alpha \frac{r^3}{3}\Big]_0^R = \frac{2\pi\alpha R^3}{3}$$

Therefore $\boxed{\alpha = \dfrac{3I}{2\pi R^3}}$

b.)

i.) for $r \le R$

$$I_{enclosed} = \int_0^r \vec{J} \cdot d\vec{A} = \int_0^r \alpha r 2\pi r dr = 2\pi\alpha \int_0^r r^2 dr = 2\pi\alpha \frac{r^3}{3}\Big]_0^r = \frac{2\pi\alpha}{3} r^3$$

using the result from (a.) $I_{enclosed} = \dfrac{I}{R^3} r^3$

Using Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

$$B 2\pi r = \mu_o \frac{I}{R^3} r^3$$

$$B = \frac{\mu_o}{2\pi r} \frac{I}{R^3} r^3 = \boxed{\frac{\mu_o}{2\pi} \frac{I}{R^3} r^2 \quad (r \le R)}$$

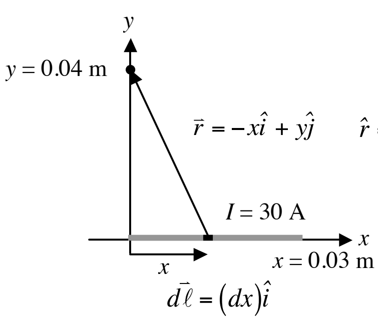
ii.) for $r > R$ $\quad I_{enclosed} = I$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enclosed}$$

$$B 2\pi r = \mu_o I$$

$$\boxed{B = \frac{\mu_o I}{2\pi r} \quad (r > R)}$$

8.)



$$\vec{r} = -x\hat{i} + y\hat{j} \qquad \hat{r} = \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$

$$d\vec{B} = \frac{\mu_o}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_o}{4\pi} \frac{I(dx)\hat{i}}{(x^2 + y^2)} \times \left(\frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}\right)$$

$$d\vec{B} = \frac{\mu_o}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_o}{4\pi} \frac{I\, ydx}{(x^2 + y^2)^{\frac{3}{2}}} \hat{k}$$

$$\vec{B} = \int \frac{\mu_o}{4\pi} \frac{I\, ydx}{(x^2 + y^2)^{\frac{3}{2}}} \hat{k}$$

from integral tables $\displaystyle\int \frac{dx}{(x^2 + y^2)^{\frac{3}{2}}} = \frac{x}{y^2\sqrt{x^2 + y^2}} + C$ so $\displaystyle\vec{B} = \frac{\mu_o}{4\pi} \frac{I\, yxdx}{y^2\sqrt{x^2 + y^2}} \hat{k} + C$

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{I\, x}{y\sqrt{x^2 + y^2}}\Bigg]_{x=0}^{x=0.03\text{ m}} \hat{k} = \left(10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}\right) \frac{(30\text{ A})\,(0.03\text{ m})}{(0.04\text{ m})\sqrt{(0.03\text{ m})^2 + (0.04\text{ m})^2}} \hat{k} = \boxed{\left(4.5 \text{ x } 10^{-5}\text{T}\right)\hat{k}}$$