1.) $A = 0.0900\ \text{m}^2$, $\theta = 0°$, $B(0) = 3.80\ \text{T}$, $\dfrac{dB}{dt} = -0.190\ \dfrac{\text{T}}{\text{s}}$, $R = 0.300\ \Omega$, $N = 1$

$$\Phi_m = \vec{B} \cdot \vec{A} = BA\cos\theta = BA$$

$$\frac{d\Phi_m}{dt} = \frac{dBA}{dt} = A\frac{dB}{dt}$$

Faraday's Law $\quad \mathcal{E} = -N\dfrac{d\Phi_m}{dt} = -NA\dfrac{dB}{dt} = -(1)\left(0.0900\ \text{m}^2\right)\left(-0.190\ \dfrac{\text{T}}{\text{s}}\right) = 0.0171\ \text{V}$

Ohm's Law $\quad V = IR \quad$ so $\quad I = \dfrac{V}{R} = \dfrac{\mathcal{E}}{R} = \dfrac{(0.0171\ \text{V})}{(0.3\ \Omega)} = \boxed{0.0570\ \text{A}}$

2.) Rectangular coil 12.0 cm x 25.0 cm, $N = 50$ turns, $\Delta t = 0.0800\ \text{s}$, $B = 0.975\ \text{T}$

at $t = 0$ the plane of the coil has an angle of 45° with the magnetic field and

$$\Phi_{m1} = \vec{B} \cdot \vec{A} = BA\cos\theta = (0.975\ \text{T})(0.12\ \text{m})(0.25\ \text{m})\cos 45° = 0.02068\ \text{T} \cdot \text{m}^2$$

at $t = 0.08\ \text{s}$ the plane of the coil has an angle of 0° with the magnetic field and

$$\Phi_{m2} = \vec{B} \cdot \vec{A} = BA\cos\theta = (0.975\ \text{T})(0.12\ \text{m})(0.25\ \text{m})\cos 0° = 0.02925\ \text{T} \cdot \text{m}^2$$

Induced *emf* is $\quad \mathcal{E} = -N\dfrac{d\Phi_m}{dt} = -N\dfrac{\Delta\Phi_m}{\Delta t} = -(50)\dfrac{\left(0.02925\ \text{T}\cdot\text{m}^2 - 0.02068\ \text{T}\cdot\text{m}^2\right)}{(0.08\ \text{s} - 0)} = \boxed{-5.36\ \text{V}}$

3.) $r_{coil} = 2.00\ \text{cm}$, $N = 400$ turns, $B = \left(0.0100\ \dfrac{\text{T}}{\text{s}}\right)t + \left(2.00 \times 10^{-4}\ \dfrac{\text{T}}{\text{s}^3}\right)t^3$, $\theta = 0°$ and connected to $R = 500\ \Omega$,

a.)

$$\mathcal{E} = -N\frac{d\Phi_m}{dt} = -N\frac{dBA\cos\theta}{dt} = -NA\cos\theta\frac{dB}{dt} = -NA\cos\theta\frac{d\left(\left(0.0100\ \frac{\text{T}}{\text{s}}\right)t + \left(2.00 \times 10^{-4}\ \frac{\text{T}}{\text{s}^3}\right)t^3\right)}{dt}$$

$$\mathcal{E} = -(400)\left(\pi(0.02\ \text{m})^2\right)\cos(0)\left(\left(0.0100\ \frac{\text{T}}{\text{s}}\right) + 3\left(2.00 \times 10^{-4}\ \frac{\text{T}}{\text{s}^3}\right)t^2\right)$$

$$\boxed{\mathcal{E} = -\left(5.04 \times 10^{-3}\ \text{V} + \left(3.02 \times 10^{-4}\ \frac{\text{V}}{\text{s}^2}\right)t^2\right)}$$

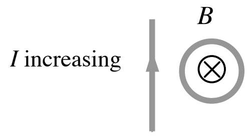
b.) at $t = 10.0\ \text{s}$

$$\mathcal{E} = -\left(5.04 \times 10^{-3}\ \text{V} + \left(3.02 \times 10^{-4}\ \frac{\text{V}}{\text{s}^2}\right)(10\ \text{s})^2\right) = -0.0352\ \text{V}$$

$$I = \frac{V}{R} = \frac{\mathcal{E}}{R} = \frac{(0.0352\ \text{V})}{(500\ \Omega)} = \boxed{7.04 \times 10^{-5}\ \text{A}}$$

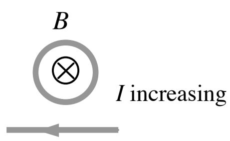
4.)

a.)



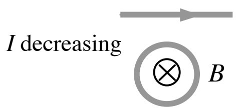
B is increasing into the page (RHR) so by Lenz's Law the induced current will produce a magnetic field out of the page. Therefore, the induced current is **counterclockwise**. (Right Hand Rule: thumb points in direction of magnetic field and fingers wrap around in the direction of the current.)

b.)



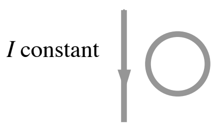
B is increasing into the page (RHR) so by Lenz's Law the induced current will produce a magnetic field out of the page. Therefore, the induced current is **counterclockwise**. (Right Hand Rule: thumb points in direction of magnetic field and fingers wrap around in the direction of the current.)

c.)



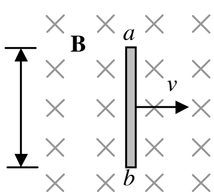
B is decreasing into the page (RHR) so by Lenz's Law the induced current will produce a magnetic field into the page. Therefore, the induced current is **clockwise**. (Right Hand Rule: thumb points in direction of magnetic field and fingers wrap around in the direction of the current.)

d.)



B is constant so there will be no change in magnetic flux in the loop and **no induced current** in the loop.

5.)



$L = 0.190$ m, $v = 6.00$ m/s, $\mathcal{E} = 2.00$ V

a.) The electric field in the rod is $E = \dfrac{\mathcal{E}}{L} = \dfrac{(2.0\ \text{V})}{(0.19\ \text{m})} = 10.53\dfrac{\text{V}}{\text{m}}$

The force on positive charges in the rod is $\vec{F} = q\vec{v} \times \vec{B}$. The velocity is in the x-direction $(+\hat{i})$ and the magnetic field is in the negative z-direction $(-\hat{k})$. Therefore the force on (+) charges in the rod is in the y-direction $\hat{i} \times (-\hat{k}) = \hat{j}$.

The force on the charges in the rod is due to the induced electric field in the rod.

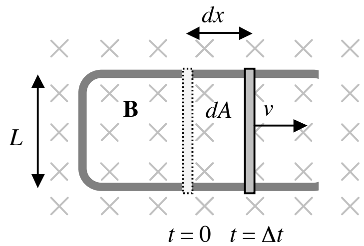
$$\vec{F} = q\vec{E} = q\vec{v} \times \vec{B} \qquad \text{so} \qquad B = \frac{E}{v} = \frac{\left(10.53\ \frac{\text{V}}{\text{m}}\right)}{\left(6.00\ \frac{\text{m}}{\text{s}}\right)} = \boxed{1.75\ \text{T}}$$

b.) the electrons in the rod are forced downward (opposite of a positive charge) so the top of the of the rod is positive with respect to the bottom of the rod and $\boxed{V_a > V_b.}$

6.) $L = 0.055$ m, $v = 6.00$ m/s, $B = 0.500$ T

$$F = qE = q\frac{\mathcal{E}}{L} = qvB \qquad \text{so} \qquad \mathcal{E} = vBL = \left(6.0\ \frac{\text{m}}{\text{s}}\right)(0.50\ \text{T})(0.055\ \text{m}) = \boxed{0.165\ \text{V}}$$

7.)



$L = 0.650$ m, $B = 1.20$ T, and $\mathcal{E} = 2.00$ V

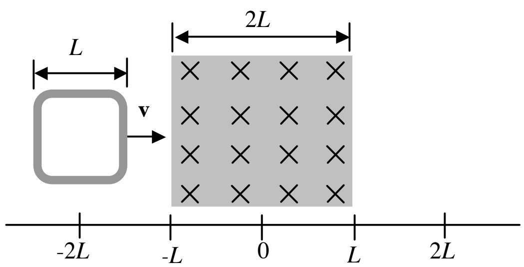
a.) $dA = Ldx$ and $\mathcal{E} = -\dfrac{d\Phi_m}{dt} = \dfrac{dBA}{dt} = B\dfrac{dA}{dt} = B\dfrac{Ldx}{dt} = BL\dfrac{dx}{dt} = BLv$

$$v = \frac{\mathcal{E}}{BL} = \frac{(0.320\ \text{V})}{(1.20\ \text{T})(0.0650\ \text{m})} = \boxed{4.10\ \frac{\text{m}}{\text{s}}}$$

b.) $I = \dfrac{V}{R} = \dfrac{\mathcal{E}}{R} = \dfrac{(0.320\ \text{V})}{(0.8\ \Omega)} = \boxed{0.4\ \text{A}}$

c.) $\vec{F} = I\vec{\ell} \times \vec{B} = (0.4\ \text{A})(0.065\ \text{m})\hat{j} \times (1.2\ \text{T})(-\hat{k}) = \boxed{(-0.0312\ \text{N})\hat{i}}$

8.)



There is no induced current (and no force needed) until the right edge of the loop reaches the region of magnetic field.

When the right edge reaches the region containing the magnetic field, the magnetic flux within the loop begins to increase resulting in an induced *emf* and an induced current.

The induced *emf* is $\mathcal{E} = \dfrac{d\Phi_m}{dt} = \dfrac{dBA}{dt} = B\dfrac{dA}{dt} B\dfrac{Ldx}{dt} = BL\dfrac{dx}{dt} = BLv$

This creates an induced current $I = \dfrac{V}{R} = \dfrac{\mathcal{E}}{R} = \dfrac{BLv}{R}$ (counterclockwise)

This current is created by an increasing magnetic field going into the plane of the page so by Lenz's Law the current be counterclockwise direction creating a magnetic field going out of the plane of the page (RHR).

Since this current is moving through a magnetic field, there will be forces $\left(\vec{F} = I\vec{\ell} \times \vec{B}\right)$ on the parts of the loop that have reached the magnetic field. This includes the entire right side of the loop and a section of the top and bottom.

The force on the right edge is $F = ILB = \dfrac{BLv}{R}LB = \dfrac{B^2L^2v}{R}$. Since the current is counterclockwise it is moving up and the magnetic field creating the force is into the page. Using the Right-hand Rule the direction of this force is to the left.

The force on the top of the loop is down and is cancelled by the force on the bottom of the loop which is up.

So the net force on the loop due to the magnetic field is directed to the left. In order for the loop to continuing moving at a constant velocity an opposing force of the same magnitude must be applied to the right (+).

This magnetic force will be present until the entire loop has entered the field. When the entire loop has entered the region of magnetic field, the magnetic flux within the loop is no longer changing and the induced *emf* is zero. As a result, there is no induced current and the force from the magnetic field is zero. Therefore, no external force is needed to keep the loop moving at a constant velocity.

When the right edge of the loop reaches the end of the magnetic field the flux begins to decrease (into the page) as the loop exits. This creates an induced current in the loop in the clockwise in order to generate a magnetic field which is into the page (Lenz's Law). This current results in forces on parts of the loop that are still in the magnetic field.

The force on the top and bottom sections again cancel each other out while that on the left side of the loop is directed to the left, once again opposing the motion of the loop. Therefore, an applied force is needed to the left in order for the loop to continue to move at a constant speed.

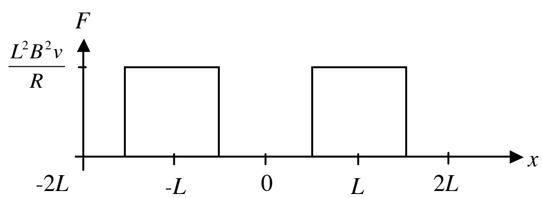
Once again, $\mathcal{E} = \frac{d\Phi_m}{dt} = \frac{dBA}{dt} = B\frac{dA}{dt}B\frac{Ldx}{dt} = BL\frac{dx}{dt} = BLv$

$$I = \frac{V}{R} = \frac{\mathcal{E}}{R} = \frac{BLv}{R} \quad \text{(clockwise)}$$

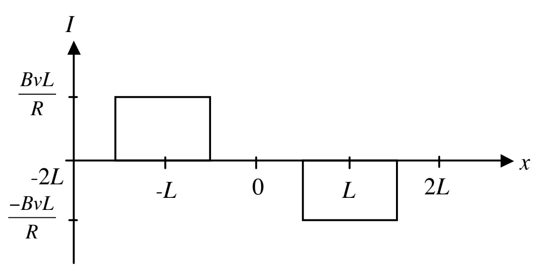
and $$F = ILB = \frac{BLv}{R}LB = \frac{B^2L^2v}{R}$$

Using the center of the loop as a reference point ($x = 0$)

a.)



b.)



HO 45 Solutions

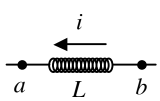
1.) $\dfrac{di}{dt} = 0.0600\ \dfrac{\text{A}}{\text{s}}$ and $\mathcal{E} = 0.0180\ \text{V}$

a.) for inductors $\mathcal{E} = -L\dfrac{di}{dt}$ so $L = \dfrac{\mathcal{E}}{\left(\dfrac{di}{dt}\right)} = \dfrac{(0.0180\ \text{V})}{\left(0.0600\ \dfrac{\text{A}}{\text{s}}\right)} = \boxed{0.30\ \text{H}}$

b.) $i = 0.80\ \text{A}$ $L = \dfrac{N\Phi_B}{i}$ so $\Phi_B = \dfrac{Li}{N} = \dfrac{(0.30\ \text{H})(0.8\ \text{A})}{(300)} = \boxed{8.0 \text{ x } 10^{-4}\ \text{Wb}}$

2.) $\dfrac{di}{dt} = -0.0350\ \dfrac{\text{A}}{\text{s}}$, $\mathcal{E} = 8.40\ \text{mV}$, $i = 1.25\ \text{A}$, and $\Phi_B = 0.00375\ \text{Wb}$

$\mathcal{E} = -L\dfrac{di}{dt}$ and $L = \dfrac{N\Phi_B}{i}$ therefore $N = \dfrac{iL}{\Phi_B} = \dfrac{i\mathcal{E}}{\Phi_B\left(\dfrac{di}{dt}\right)} = \dfrac{(1.25\ \text{A})(8.4 \text{ x } 10^{-3}\ \text{V})}{(0.00375\ \text{Wb})\left(0.0350\ \dfrac{\text{T}}{\text{s}}\right)} = \boxed{80\ \text{turns}}$

3.) 

$L = 0.540\ \text{H}$, $\dfrac{di}{dt} = -0.0300\ \dfrac{\text{A}}{\text{s}}$

a.) $\mathcal{E} = -L\dfrac{di}{dt} = (0.540\ \text{H})\left(0.030\ \dfrac{\text{T}}{\text{s}}\right) = \boxed{0.0162\ \text{V}}$

b.) The current is decreasing with time to the left so the self-induced *emf* is positive from *a* to *b* and as a result $\boxed{V_a > V_b.}$

4.) $L = 16.0\ \text{H}$, $R = 200\ \Omega$, and $i = 0.350\ \text{A}$

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(16\ \text{H})(0.35\ \text{A})^2 = \boxed{0.98\ \text{J}}$$

5.) $\dfrac{L}{R}\ [=]\ \dfrac{\text{H}}{\Omega}\ [=]\ \dfrac{\frac{\text{Wb}}{\text{A}}}{\Omega}\ [=]\ \dfrac{\frac{\text{T}\cdot\text{m}^2}{\text{A}}}{\Omega}\ [=]\ \dfrac{\frac{\text{N}\cdot\text{m}^2}{\text{A}^2\cdot\text{m}}}{\Omega}\ [=]\ \dfrac{\frac{\text{N}\cdot\text{m}}{\text{A}^2}}{\Omega}\ [=]\ \dfrac{\frac{\text{J}}{\text{A}^2}}{\Omega}\ [=]\ \dfrac{\frac{\text{J}\cdot\text{s}}{\text{C}\cdot\text{A}}}{\Omega}\ [=]\ \dfrac{\frac{\text{V}\cdot\text{s}}{\text{A}}}{\Omega}\ [=]\ \dfrac{\Omega\cdot\text{s}}{\Omega}\ [=]\ \text{s}$

6.) $L = 3.00\ \text{H}$, $R = 7.00\ \Omega$, and $\mathcal{E} = 12.0\ \text{V}$

a.) Initially all the battery *emf* is across the inductor and $\mathcal{E}_L = -L\dfrac{di}{dt}$ so $\dfrac{di}{dt} = \dfrac{\mathcal{E}_L}{L} = \dfrac{(12\ \text{V})}{(3\ \text{H})} = \boxed{4\ \dfrac{\text{A}}{\text{s}}}$

b.) When the current is 1.00 A the voltage across the resistor is $v_R = iR = (1.0\ \text{A})(7\ \Omega) = 7\ \text{V}$.
Therefore, the voltage across the inductor is $\mathcal{E}_L = \mathcal{E} - v_R = 12\ \text{V} - 7\ \text{V} = 5\ \text{V}$

For an inductor $\mathcal{E}_L = -L\dfrac{di}{dt}$ so $\dfrac{di}{dt} = \dfrac{\mathcal{E}_L}{L} = \dfrac{(5\ \text{V})}{(3\ \text{H})} = \boxed{1.67\ \dfrac{\text{A}}{\text{s}}}$

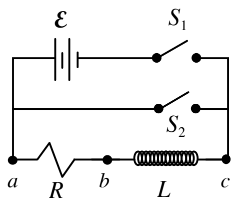
6.) (cont'd)

c.) for RL circuits when magnetizing the inductor $i(t)=\frac{\mathcal{E}}{R}\left(1-e^{-\frac{R}{L}t}\right)=\left(\frac{12\text{ V}}{7\ \Omega}\right)\left(1-e^{-\frac{7\ \Omega}{3\text{ H}}t}\right)=1.714\text{ A}\left(1-e^{-\left(2.33\text{ s}^{-1}\right)t}\right)$

$i(0.2\text{ s})=1.714\text{ A}\left(1-e^{-\left(2.33\text{ s}^{-1}\right)(0.2\text{ s})}\right)=\boxed{0.639\text{ A}}$

d.) $i(\infty)=1.714\text{ A}\left(1-e^{-\infty}\right)=\ 1.714\text{ A}(1-0)=\boxed{1.714\text{ A}}$

7.)



$L = 0.200$ H, $R = 400\ \Omega$, and $\mathcal{E} = 120$ V

a.) After switch S_1 has been closed for a long time $v_R = 120$ V and $v_L = 0$. The current is $i=\frac{\mathcal{E}}{R}=\frac{(120\text{ V})}{(400\ \Omega)}=0.30\text{ A}$. When switch S_1 is opened and switch S_2 is closed the current remains at 0.30 A and begins to decay. So $i(0)=I_o=\boxed{0.30\text{ A}}$

b.) During current decay $i(t)=I_o e^{-\frac{R}{L}t}=(0.30\text{ A})e^{-\frac{400\ \Omega}{0.200\text{ H}}t}=(0.30\text{ A})e^{-\left(2000\text{ s}^{-1}\right)t}$ and $i(0.0002)=(0.30\text{ A})e^{-\left(2000\text{ s}^{-1}\right)(0.0002\text{ s})}=\boxed{0.201\text{ A}}$

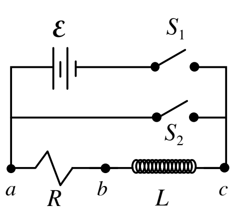
c.) $v_{bc}=v_L=v_R=iR=(0.201\text{ A})(400\ \Omega)=\boxed{80.4\text{ V}}$

d.) $i(t)=(0.30\text{ A})e^{-\left(2000\text{ s}^{-1}\right)t}=\ 0.15\text{ A}$ so $e^{-\left(2000\text{ s}^{-1}\right)t}=\ 0.5$

$$-\left(2000\text{ s}^{-1}\right)t=\ \ln(0.5)$$

and $t=\frac{-\ln(0.5)}{\left(2000\text{ s}^{-1}\right)}=\boxed{3.47\text{ x }10^{-4}\text{ s}}$

8.)



$L = 0.200$ H, $R = 400\ \Omega$, $\mathcal{E} = 80$ V, $i(0) = 0$

a.) Switch S_2 is opened and switch S_1 is closed. Right after the switch is closed $i = 0$ and $v_{ab}=v_R=iR=\boxed{0}$ and $v_{bc}=v_L=\mathcal{E}=\boxed{80\text{ V}}$.

b.) After a long time the current in the circuit is $i(\infty)=\frac{\mathcal{E}}{R}=\frac{(80\text{ V})}{(400\ \Omega)}=0.20\text{ A}$

Therefore, $v_{ab}=v_R=iR=(0.20\text{ A})(400\ \Omega)=\boxed{80\text{ V}}$ and $v_{bc}=v_L=\mathcal{E}-v_R=\ 80\text{ V}-80\text{ V}=\boxed{0}$

c.) $v_{ab}=v_R=iR=(0.50\text{ A})(400\ \Omega)=\boxed{20\text{ V}}$ and $v_{bc}=v_L=\mathcal{E}-v_R=\ 80\text{ V}-20\text{ V}=\boxed{60\text{ V}}$

9.) $\sqrt{LC}\ [=]\ \sqrt{\text{H}\cdot\text{F}}\ [=]\ \sqrt{\left(\frac{\text{Wb}}{\text{A}}\right)\left(\frac{\text{C}}{\text{V}}\right)}\ [=]\ \sqrt{\left(\frac{\text{T}\cdot\text{m}^2}{\text{A}}\right)\left(\frac{\text{C}^2}{\text{J}}\right)}\ [=]\ \sqrt{\left(\frac{\text{N}\cdot\text{m}^2}{\text{A}^2\cdot\text{m}}\right)\left(\frac{\text{C}^2}{\text{J}}\right)}\ [=]\ \sqrt{\left(\frac{\text{J}\cdot\text{s}^2}{\text{C}^2}\right)\left(\frac{\text{C}^2}{\text{J}}\right)}\ [=]\ \sqrt{\text{s}^2}\ [=]\ \text{s}$