## Example 1:

Rat throws a baseball with a horizontal speed of $15 \mathrm{~m} / \mathrm{s}$ from a building that is 35 m tall.
a.) What is the initial velocity in the $x$-direction?
b.) What is the initial velocity in the $y$-direction?
c.) How much time does it take the ball to hit the ground?
d.) How far from the base of the building does the ball strike the ground?
e.) Find the magnitude and direction of the velocity of the ball just before it hits the ground.

Example 1:

$$
y_{i}=35 \mathrm{~m}, \theta_{i}=0, \text { and } v_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

d.) How far from the base of the building does the ball strike the ground?

$$
y=0 \text { so } \Delta y=y-y_{i}=0-35 \mathrm{~m}=-35 \mathrm{~m}, t=2.67 \mathrm{~s}, \Delta x=?
$$

$$
\Delta x=v_{x} t=\left(15 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(2.67 \mathrm{~s})=40 \mathrm{~m}
$$

e.) Find the magnitude and direction of the velocity of the ball just before
it hits the ground $\dot{y}_{i}=0-35 \mathrm{~m}=-35 \mathrm{~m}, t=2.67 \mathrm{~s}, v=?, \theta=$ ?
$v_{x}=15 \frac{\mathrm{~m}}{\mathrm{~s}}$ and $v_{y}=-g t+v_{y_{i}}=-\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(2.67 \mathrm{~s})+0=-26.2 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{x}=15 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\bar{\theta} \quad v=\sqrt{v_{x}{ }^{2}+v_{y}{ }^{2}}=\sqrt{\left(15 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(-26.2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=30 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{y}=-26.2 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\quad \theta=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)=\tan ^{-1}\left(\frac{-26.2 \frac{\mathrm{~m}}{\mathrm{~s}}}{15 \frac{\mathrm{~m}}{\mathrm{~s}}}\right)=-60^{\circ}$

Example 1:

$$
y_{i}=35 \mathrm{~m}, \theta_{i}=0, \text { and } v_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

a.) What is the initial velocity in the $x$-direction?

$$
v_{x}=v_{x_{i}}=v_{i} \cos \theta_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}} \cos 0=15 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

b.) What is the initial velocity in the $y$-direction?

$$
v_{y_{i}}=v_{i} \sin \theta_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}} \sin 0=0
$$

c.) How much time does it take the ball to reach the ground?

$$
\begin{gathered}
y=0 \text { so } \Delta y=y-y_{i}=0-35 \mathrm{~m}=-35 \mathrm{~m}, t=? \\
\Delta y=-\frac{1}{2} g t^{2}+v / y_{i} t=-\frac{1}{2} g t^{2} \\
t=\sqrt{\frac{-2 \Delta y}{g}}=\sqrt{\frac{-2(-35 \mathrm{~m})}{9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}}=2.67 \mathrm{~s}
\end{gathered}
$$

Example 1: $\quad y_{i}=35 \mathrm{~m}, \theta_{i}=0$, and $v_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}}$
e.) Find the magnitude and direction of the velocity of the ball just before
it hits the ground.
alternatively $v_{y}^{2}=v_{y_{i}}^{2}-2 g \Delta y$

$$
\begin{gathered}
\text { so } v_{y}= \pm \sqrt{v_{y_{i}}^{2}-2 g \Delta y} \\
v_{y}=-\sqrt{0-2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(-35 \mathrm{~m})} \\
v_{y}=-26.2 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

Example 2: $\quad y_{i}=60 \mathrm{~m}, \theta_{i}=0$, and $\Delta x=80 \mathrm{~m}$ when $y=0$
a.) What is the initial velocity in the $x$-direction?

$$
\begin{gathered}
v_{x_{i}}=v_{x}=? \\
\Delta x=v_{x} t \text { so } v_{x}=\frac{\Delta x}{t} \text { where } t \text { is the time in the air } \\
\Delta y=y-y_{i}=0-60 \mathrm{~m}=-60 \mathrm{~m}, t=? \\
\Delta y=-\frac{1}{2} g t^{2}+v_{y_{i}} t=-\frac{1}{2} g t^{2} \\
v_{y_{i}}=0 \\
t=\sqrt{\frac{-2 \Delta y}{g}}=\sqrt{\frac{-2(-60 \mathrm{~m})}{9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}}=3.50 \mathrm{~s} \\
v_{x}=\frac{\Delta x}{t}=\frac{80 \mathrm{~m}}{3.50 \mathrm{~s}}=22.9 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

Example 2: $\quad y_{i}=60 \mathrm{~m}, \theta_{i}=0$, and $\Delta x=80 \mathrm{~m}$ when $y=0$
b.) What is the initial velocity in the $y$-direction?

$$
v_{y_{i}}=? \quad \text { horizontal projectile } v_{y_{i}}=0 \quad\left(\theta_{i}=0 \text { so } v_{i} \sin \theta_{i}=0\right)
$$

c.) Find the magnitude and direction of the velocity of the ball just before it hits the ground.
$\Delta y=y-y_{i}=0-60 \mathrm{~m}=-60 \mathrm{~m}, t=3.50 \mathrm{~s}, v=?, \theta=?$
$v_{x}=22.9 \frac{\mathrm{~m}}{\mathrm{~s}}$ and $v_{y}=-g t+v_{y_{i}}=-\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(3.50 \mathrm{~s})+0=-34.3 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{x}=22.9 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\bar{\theta}{ }^{-} v_{y}=-34.3 \frac{\mathrm{~m}}{\mathrm{~s}} \quad v=\sqrt{v_{x}{ }^{2}+v_{y}{ }^{2}}=\sqrt{\left(22.9 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(-34.3 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=41.2 \frac{\mathrm{~m}}{\mathrm{~s}}$

$$
\theta=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)=\tan ^{-1}\left(\frac{-34.3 \frac{\mathrm{~m}}{\mathrm{~s}}}{22.9 \frac{\mathrm{~m}}{\mathrm{~s}}}\right)=-56.3^{\circ}
$$

## Example 3:

Rat is flying a plane and is traveling horizontally at 40 $\mathrm{m} / \mathrm{s}$ at a height of 100 m above the ground when she drops a package of emergency supplies to a group on the ground.
a.) Where does the package strike the ground relative to the point at which it was released?
b.) Find the magnitude and direction of the velocity of the package just before it hits the ground.

Example 3:

$$
\theta_{i}=0, v_{i}=40 \frac{\mathrm{~m}}{\mathrm{~s}}, y_{i}=100 \mathrm{~m}, \text { and } y=0
$$

a.) Where does the package strike the ground relative to the point at which it was released?

$$
\begin{gathered}
\Delta x=? \\
\Delta x=v_{x} t
\end{gathered}
$$

horizontal projectile $\left(\theta_{i}=0\right)$ so $v_{x}=v_{i}=40 \frac{\mathrm{~m}}{\mathrm{~s}}$ and $v_{y_{i}}=0$

$$
\begin{aligned}
& \Delta y=y-y_{i}=0-100 \mathrm{~m}=-100 \mathrm{~m}, t=? \\
& \Delta y=-\frac{1}{2} g t^{2}+\underset{v_{y_{i}}=0}{v / t}=-\frac{1}{2} g t^{2} \\
& t=\sqrt{\frac{-2 \Delta y}{g}}=\sqrt{\frac{-2(-100 \mathrm{~m})}{9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}}=4.52 \mathrm{~s}
\end{aligned}
$$

Example 3: $\quad \theta_{i}=0, v_{i}=40 \frac{\mathrm{~m}}{\mathrm{~s}}, y_{i}=100 \mathrm{~m}$, and $y=0$
b.) Find the magnitude and direction of the velocity of the package just before it hits the ground.

$$
\Delta y=-100 \mathrm{~m}, t=4,52 \mathrm{~s}, v=?, \theta=?
$$

$$
v_{x}=40 \frac{\mathrm{~m}}{\mathrm{~s}} \text { and } v_{y}=-g t+v_{y_{i}}=-\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(4.52 \mathrm{~s})+0=-44.3 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

$$
\begin{aligned}
& v_{x}=40 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \bar{\theta} \theta_{y}=-44.3 \frac{\mathrm{~m}}{\mathrm{~s}} \quad v=\sqrt{v_{x}{ }^{2}+v_{y}{ }^{2}}=\sqrt{\left(40 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(-44.3 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=60 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \theta=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)=\tan ^{-1}\left(\frac{-44.3 \frac{\mathrm{~m}}{\mathrm{~s}}}{40 \frac{\mathrm{~m}}{\mathrm{~s}}}\right)=-48^{\circ}
\end{aligned}
$$

Example 4: $\quad y_{i}=0, \theta_{i}=53.13^{\circ}$, and $v_{i}=20 \frac{\mathrm{~m}}{\mathrm{~s}}$
a.) What is the initial velocity in the $x$-direction?

$$
v_{x}=v_{x_{i}}=v_{i} \cos \theta_{i}=20 \frac{\mathrm{~m}}{\mathrm{~s}} \cos \left(53.13^{\circ}\right)=12 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

b.) What is the initial velocity in the $y$-direction?

$$
v_{y_{i}}=v_{i} \sin \theta_{i}=20 \frac{\mathrm{~m}}{\mathrm{~s}} \sin \left(53.13^{\circ}\right)=16 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

c.) What is the velocity of the ball just before it hits the ground?

$$
y=0 \text { so } \Delta y=y-y_{i}=0-0=0, v=?, \theta=?
$$

$$
v_{x}=12 \frac{\mathrm{~m}}{\mathrm{~s}} \quad v_{y}^{2}=v_{y_{i}}^{2}-2 g \Delta y \text { so } v_{y}= \pm \sqrt{v_{y_{i}}^{2}-2 g \Delta y}
$$

$$
v_{y}=-\sqrt{\left(16 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(0)}=-16 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

$$
\begin{aligned}
v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{\left(12 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(-16 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=20 \frac{\mathrm{~m}}{\mathrm{~s}} \quad & \theta=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)=\tan ^{-1}\left(\frac{-16 \frac{\mathrm{~m}}{\mathrm{~s}}}{12 \frac{\mathrm{~m}}{\mathrm{~s}}}\right) \\
\theta & \theta-53.13^{\circ}
\end{aligned}
$$

Example 4:

$$
y_{i}=0, \theta_{i}=53.13^{\circ}, \text { and } v_{i}=20 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

d.) How much time does it take the ball to reach the ground?

$$
\begin{aligned}
& y=0 \text { so } \Delta y=y-y_{i}=0-0=0, v_{y_{i}}=16 \frac{\mathrm{~m}}{\mathrm{~s}}, v_{y}=-16 \frac{\mathrm{~m}}{\mathrm{~s}}, t=? \\
& v_{y}=-g t+v_{y_{i}} \text { so } t=\frac{v_{y}-v_{y_{i}}}{-g}=\frac{-16 \frac{\mathrm{~m}}{\mathrm{~s}}-16 \frac{\mathrm{~m}}{\mathrm{~s}}}{-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=3.27 \mathrm{~s} \\
& \text { alternatively } \Delta y=-\frac{1}{2} g t^{2}+v_{y_{i}} t=0 \text { so } 0=t\left(-\frac{1}{2} g t+v_{y_{i}}\right) \\
& \text { solutions are when } t=0 \text { and }-\frac{1}{2} g t+v_{y_{i}}=0 \\
& \text { so } t=\frac{2 v_{y_{i}}}{g}=\frac{2\left(16 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=3.27 \mathrm{~s}
\end{aligned}
$$

## Example 5:

Rat kicks a soccer ball from ground level at an angle of $36.87^{\circ}$ with respect to the horizontal. The initial speed of the ball is $16 \mathrm{~m} / \mathrm{s}$.
a.) How much time does it take the ball to reach its maximum height?
b.) What is the velocity of the ball at its maximum height?
c.) What is the velocity of the ball when its strikes the ground?
d.) How far does the ball travel in the horizontal direction?

Example 4: $\quad y_{i}=0, \theta_{i}=53.13^{\circ}$, and $v_{i}=20 \frac{\mathrm{~m}}{\mathrm{~s}}$
e.) How far does the ball travel in the horizontal direction?

$$
\begin{gathered}
v_{x}=12 \frac{\mathrm{~m}}{\mathrm{~s}}, t=3.27 \mathrm{~s}, \Delta x=? \\
\Delta x=v_{x} t=\left(12 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(3.27 \mathrm{~s})=39.2 \mathrm{~m}
\end{gathered}
$$

f.) What is the maximum height of the ball?

$$
\begin{aligned}
& v_{y_{i}}=16 \frac{\mathrm{~m}}{\mathrm{~s}}, v_{y}=0 \text { (max height), } y_{i}=0, y=? \\
& v_{y}^{2}=v_{y_{i}}^{2}-2 g \Delta y \text { so } \Delta y=y-y_{i}=\frac{v_{y}^{2}-v_{y_{i}}^{2}}{-2 g} \\
& y=\frac{v_{y}^{2}-v_{y_{i}}^{2}}{-2 g}+y_{i}=\frac{0-\left(16 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{-2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}+0=13.1 \mathrm{~m}
\end{aligned}
$$

Example 5:

$$
\begin{gathered}
y_{i}=0, \theta_{i}=36.87^{\circ}, \text { and } v_{i}=16 \frac{\mathrm{~m}}{\mathrm{~s}} \\
v_{x}=v_{x_{i}}=v_{i} \cos \theta_{i}=16 \frac{\mathrm{~m}}{\mathrm{~s}} \cos \left(36.87^{\circ}\right)=12.8 \frac{\mathrm{~m}}{\mathrm{~s}} \\
v_{y_{i}}=v_{i} \sin \theta_{i}=16 \frac{\mathrm{~m}}{\mathrm{~s}} \sin \left(36.87^{\circ}\right)=9.6 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

a.) How much time does it take the ball to reach its maximum height?

$$
v_{y}=0(\max \text { height }), t=?
$$

$$
v_{y}=-g t+v_{y_{i}} \text { so } t=\frac{v_{y}-v_{y_{i}}}{-g}=\frac{0-9.6 \frac{\mathrm{~m}}{\mathrm{~s}}}{-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=0.98 \mathrm{~s}
$$

b.) What is the velocity of the ball at its maximum height?

$$
v_{x}=12.8 \frac{\mathrm{~m}}{\mathrm{~s}} \text { and } v_{y}=0, v=?, \theta=?
$$

$$
\begin{aligned}
& v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{\left(12.8 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+0}=12.8 \frac{\mathrm{~m}}{\mathrm{~s}} \theta \\
&=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)=\tan ^{-1}\left(\frac{0}{12.8 \frac{\mathrm{~m}}{\mathrm{~s}}}\right) \\
& \theta=0
\end{aligned}
$$

Example 5: $\quad y_{i}=0, \theta_{i}=36.87^{\circ}$, and $v_{i}=16 \frac{\mathrm{~m}}{\mathrm{~s}}$
c.) What is the velocity of the ball when it strikes the ground?

$$
\Delta y=0, v_{x}=12.8 \frac{\mathrm{~m}}{\mathrm{~s}} \text { and } v_{y_{i}}=9.6 \frac{\mathrm{~m}}{\mathrm{~s}}, v=?, \theta=?
$$

ground - to - ground is symmetric so $v=16 \frac{\mathrm{~m}}{\mathrm{~s}}$ and $\theta=-\theta_{i}=-36.87^{\circ}$
d.) How far does the ball travel in the horizontal direction?

$$
\begin{gathered}
\Delta y=0, v_{x}=12.8 \frac{\mathrm{~m}}{\mathrm{~s}} \text { and } v_{y_{i}}=9.6 \frac{\mathrm{~m}}{\mathrm{~s}}, \Delta x=? \\
v_{y}^{2}=v_{y_{i}}^{2}-2 g \Delta y \text { so } v_{y}= \pm \sqrt{v_{y_{i}}^{2}-2 g \Delta y}=-\sqrt{\left(9.6 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(0)}=-9.6 \frac{\mathrm{~m}}{\mathrm{~s}} \\
v_{y}=-g t+v_{y_{i}} \text { so } t=\frac{v_{y}-v_{y_{i}}}{-g}=\frac{-9.6 \frac{\mathrm{~m}}{\mathrm{~s}}-9.6 \frac{\mathrm{~m}}{\mathrm{~s}}}{-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=1.96 \mathrm{~s} \quad \Delta x=v_{x} t=\left(12.8 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(1.96 \mathrm{~s}) \\
\Delta x=25.1 \mathrm{~m}
\end{gathered}
$$

## Example 6:

Larry kicks a soccer ball with a speed of $15 \mathrm{~m} / \mathrm{s}$ and angle of $36.87^{\circ}$ above the horizontal from a building that is 35 m tall.
a.) What is the magnitude and direction of the velocity of the ball 0.5 s after it is kicked?
b.) What is the maximum height of the ball?
c.) What is the magnitude and direction of the velocity of the ball when it hits the ground?
d.) How much time does it take the ball to hit the ground?
e.) How far from the base of the building does the ball strike the ground?

Example 6:

$$
y_{i}=35 \mathrm{~m}, \theta_{i}=36.87^{\circ}, \text { and } v_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

$$
\begin{aligned}
& v_{x}=v_{x_{i}}=v_{i} \cos \theta_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}} \cos \left(36.87^{\circ}\right)=12 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{y_{i}}=v_{i} \sin \theta_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}} \sin \left(36.87^{\circ}\right)=9 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

a.) What is the magnitude and direction of the velocity of the ball 0.5 s after the ball is kicked? $\quad t=0.5 \mathrm{~s}, v=?, \theta=$ ?

$$
v_{x}=12 \frac{\mathrm{~m}}{\mathrm{~s}} \text { and } v_{y}=-g t+v_{y_{i}}=-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}(0.5 \mathrm{~s})+9 \frac{\mathrm{~m}}{\mathrm{~s}}=4.1 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

$\vec{v} \quad v_{y}=4.1 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{x}=12 \frac{\mathrm{~m}}{\mathrm{~s}} \quad \sqrt{v_{x}{ }^{2}+v_{y}^{2}}=\sqrt{\left(12 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(4.1 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=12.7 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\theta$
Example 6: $\quad y_{i}=35 \mathrm{~m}, \theta_{i}=36.87^{\circ}$, and $v_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}}$
b.) What is the maximum height of the ball?

$$
\begin{gathered}
v_{y_{i}}=9 \frac{\mathrm{~m}}{\mathrm{~s}}, v_{y}=0(\max \text { height }), y=? \\
v_{y}^{2}=v_{y_{i}}^{2}-2 g \Delta y \text { so } \Delta y=y-y_{i}=\frac{v_{y}^{2}-v_{y_{i}}^{2}}{-2 g} \\
y=\frac{v_{y}^{2}-v_{y_{i}}^{2}}{-2 g}+y_{i}=\frac{0-\left(9 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{-2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}+35 \mathrm{~m}=39.1 \mathrm{~m}
\end{gathered}
$$

c.) What is the magnitude and direction of the velocity of the ball when it strikes the ground?

$$
\begin{gathered}
v_{x}=12 \frac{\mathrm{~m}}{\mathrm{~s}}, v_{y_{i}}=9 \frac{\mathrm{~m}}{\mathrm{~s}}, y_{i}=35 \mathrm{~m}, y=0, \Delta y=y-y_{i}=-35 \mathrm{~m}, v=?, \theta=? \\
v_{y}^{2}=v_{y_{i}}^{2}-2 g \Delta y \text { so } v_{y}= \pm \sqrt{v_{y_{i}}^{2}-2 g \Delta y}=-\sqrt{\left(9 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-2\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(-35 \mathrm{~m})}=-27.7 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}
$$

Example 6:

$$
y_{i}=35 \mathrm{~m}, \theta_{i}=36.87^{\circ}, \text { and } v_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

c.) What is the magnitude and direction of the velocity of the ball when it strikes the ground?

$$
\begin{gathered}
v_{x}=12 \frac{\mathrm{~m}}{\mathrm{~s}}, v_{y_{i}}=9 \frac{\mathrm{~m}}{\mathrm{~s}}, y_{i}=35 \mathrm{~m}, y=0, \Delta y=y-y_{i}=0-35 \mathrm{~m}=-35 \mathrm{~m}, v=?, \theta=? \\
v_{x}=12 \frac{\mathrm{~m}}{\mathrm{~s}} \\
\bar{\theta} \\
\quad v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{\left(12 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(-27.7 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=30.2 \frac{\mathrm{~m}}{\mathrm{~s}} \\
\theta=-27.7 \frac{\mathrm{~m}}{\mathrm{~s}} \\
\operatorname{van}^{-1}\left(\frac{v_{y}}{v_{x}}\right)=\tan ^{-1}\left(\frac{-27.7 \frac{\mathrm{~m}}{\mathrm{~s}}}{12 \frac{\mathrm{~m}}{\mathrm{~s}}}\right)=-66.6^{\circ}
\end{gathered}
$$

Example 6:

$$
y_{i}=35 \mathrm{~m}, \theta_{i}=36.87^{\circ}, \text { and } v_{i}=15 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

d.) How much time does it take the ball to hit the ground?

$$
\begin{gathered}
v_{y_{i}}=9 \frac{\mathrm{~m}}{\mathrm{~s}}, v_{y}=-27.7 \frac{\mathrm{~m}}{\mathrm{~s}}, t=? \\
v_{y}=-g t+v_{y_{i}} \text { so } t=\frac{v_{y}-v_{y_{i}}}{-g}=\frac{-27.7 \frac{\mathrm{~m}}{\mathrm{~s}}-9 \frac{\mathrm{~m}}{\mathrm{~s}}}{-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=3.74 \mathrm{~s}
\end{gathered}
$$

e.) How far from the base of the building does the ball strike the ground?

$$
\begin{gathered}
\Delta x=? \\
\Delta x=v_{x} t=\left(12 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(3.74 \mathrm{~s}) \\
\Delta x=44.9 \mathrm{~m}
\end{gathered}
$$

Example 7: $\quad \Delta x=48 \mathrm{~m}, \theta_{i}=53.13^{\circ}$, and $t=4.0 \mathrm{~s}$
a.) What is the magnitude of the initial velocity of the ball?

$$
v_{i}=?
$$

$$
\Delta x=v_{x} t \text { so } v_{x}=\frac{\Delta x}{t}=v_{i} \cos \theta_{i}
$$

$$
v_{i}=\frac{\Delta x}{t \cos \theta_{i}}=\frac{48 \mathrm{~m}}{(4 \mathrm{~s}) \cos \left(53.13^{\circ}\right)}=20 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

b.) From what initial height was the ball kicked?

$$
v_{i}=20 \frac{\mathrm{~m}}{\mathrm{~s}}, \theta_{i}=53.13^{\circ}, t=4.0 \mathrm{~s}, \text { and } y=0, y_{i}=?
$$

$$
\Delta y=-\frac{1}{2} g t^{2}+v_{y_{i}} t=y-y_{i}
$$

$$
y_{i}=\frac{1}{2} g t^{2}-v_{y_{i}} t+y=\frac{1}{2} g t^{2}-v_{i} \sin \theta_{i} t+y
$$

$$
y_{i}=\frac{1}{2}\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(4 \mathrm{~s})^{2}-\left(20 \frac{\mathrm{~m}}{\mathrm{~s}}\right) \sin (4 \mathrm{~s})\left(53.13^{\circ}\right)+0=14.4 \mathrm{~m}
$$

